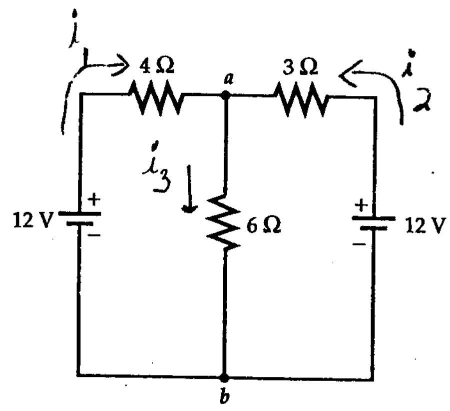
Physics 102– Pledged Problem 6 Solution

In the circuit shown below, determine the following:

(a) The current through each resistor.
(b) The potential difference between points a and b.
(c) The power supplied by each battery.



(a) This circuit can be solved using Kirchhoff's rules. We can define i_1 as the current through the 4Ω resistor, with the positive direction as clockwise. Define i_2 as the current through the 3Ω resistor, with the positive direction being counterclockwise. The third current i_3 is through the 6Ω resistor, with the positive direction being down in the drawing. If the current is in fact flowing in the opposite direction from these choices, the number we get for the current will be negative.

We can now write down the relationships between these currents from Kirchhoff's rules. We have three unknown currents, so we need three equations. One relationship comes from the junction at point a, which is $i_1 + i_2 = i_3$. A second relation comes from the loop that passes (clockwise) through the battery on the left, the 4Ω resistor, and the 6Ω resistor. For this loop Kirchoff's rule gives $12 - 4i_1 - 6i_3 = 0$. For the third relationship we can use the loop that passes counterclockwise through the battery on the right, the 3Ω resistor, and the 6Ω resistor. For this loop the equation is $12 - 3i_2 - 6i_3 = 0$.

To summarize, the three equations we have to solve are:

$i_1 + i_2 = i_3$
$12 = 4i_1 + 6i_3$ which reduces to $6 = 2i_1 + 3i_3$
$12 = 3i_2 + 6i_3$ which reduces to $4 = i_2 + 2i_3$.

There are many ways to solve this set of equations. One easy way is to solve the first one for i_2 in terms of i_1 and i_3, and substitute this result into the third equation. Then the second and third equations can be solved for i_1 and i_3.

$i_2 = i_3 - i_1$
$6 = 2i_1 + 3i_3$
$4 = -i_1 + 3i_3$

Multiply the third equation by 2 and add to the first to get i_3. Substitute back into the third and first equations to get i_1 and i_2. The resulting currents are

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$$i_1 = 2/3 \text{ A}$$
$$i_2 = 8/9 = 0.89 \text{ A}$$
$$i_3 = 14/9 = 1.56 \text{ A}$$

2 pts each

(b) The potential difference between points a and b is the same as the voltage drop across the 6Ω resistor. From (a) we know i_3, so

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$$V_{ab} = V_a - V_b = (6)(14/9) = 28/3 = 9.3V$$

Note that V_{ab} is positive, indicating that the potential is higher (more positive) at point a than point b.

(c) The power supplied by a battery is given by IV, the product of the current and the voltage. The units are watts, or Joules/sec.

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$$P_{left} = i_1 V = (12)(2/3) = 8 \text{ watts}$$
$$P_{right} = i_2 V = (12)(8/9) = 32/3 = 10.7 \text{ watts}$$

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II. In the circuit shown below, the switch is initially opened.

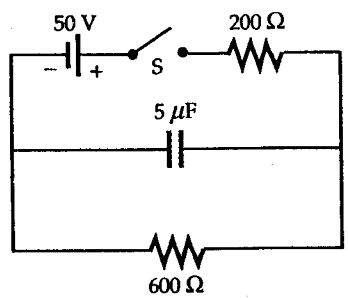
3 (a) Determine the current through the battery immediately after the switch is closed.

3 (b) Determine the current through the battery a long time after the switch is closed.

4 (c) Determine the voltage across the capacitor a long time after the switch is closed.

After the switch has been closed for a long time, it is opened again.

5 (d) Determine the current through the 600Ω resistor as a function of time $I(t)$ after the switch is opened. Sketch $I(t)$ vs. t.



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(a) Immediately after the switch is closed, the capacitor acts like a short circuit, that is, a wire with zero resistance. Then the 600Ω resistor is effectively out of the circuit, and the current is determined only by

$$I = 50V/200\Omega = 0.25A$$

(b) A long time after the switch is closed, the capacitor is fully charged and acts like an open circuit, that is, no current flows through it. Then the two resistors are effectively in series, and the current is determined by the series combination of the resistors:

$$I = 50V/(200\Omega + 600\Omega) = 1/16 = 0.0625\text{ A.}$$

capacitor discharges through the 600Ω resistor. The initial voltage across the capacitor is 37.5V, giving an initial current of $I_o = 0.0625$ A. The current decays with the characteristic RC time constant

$$I(t) = I_o e^{-t/RC}$$

with $RC = (600\Omega)(5 \times 10^{-6}F) = 3 \times 10^{-3}\text{sec} = 3\text{msec}$ and $I_o = 0.0625A$.

See sketch below.