Physics 102 – Pledged Problem 5

Time allowed: **2 hours at a single sitting**

DUE 4PM MONDAY, February 25, 2008, in the boxes marked Phys 101-102 in the physics lounge. You may use your own textbook, your notes, and a non-programmed calculator. You may also consult the on-line solutions to the corresponding suggested problems. You should consult no other help. Show how you arrived at your answer; the correct answer by itself may not be sufficient.

Further instructions:

(a) Write legibly on **one** side of 8.5" x 11" white or lightly tinted paper.
(b) Staple all sheets together, including this one, in the upper left corner and make one vertical fold.
(c) On the outside, staple side up. Print your name in capital letters, your LAST NAME first followed by your FIRST NAME.
(d) Below your name, print the phrase "Pledged Problem 5", followed by the due date.
(e) Also indicate **start time** and **end time.**
(f) Write and sign the pledge, with the understanding you may consult the materials noted above.

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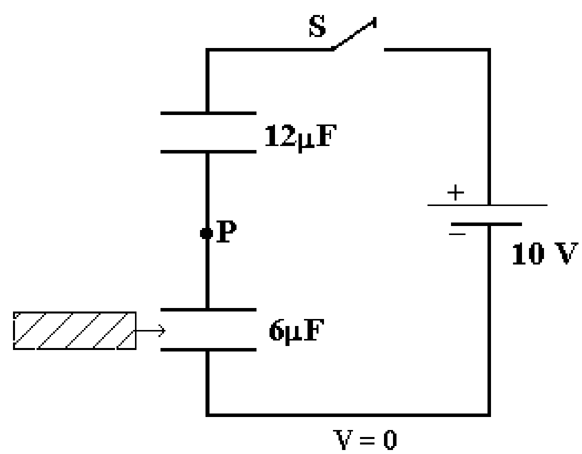
1. (5 pts) Two capacitors (12μF and 6μF), initially uncharged, an open switch, and a battery (10 V) are connected as shown. The potential at the negative side of the battery is chosen to be zero.

   (A) The switch is closed, and the capacitors become charged. Determine the potential at P.

   (B) With the switch still closed, a slab of dielectric constant 2.0 is inserted between the plates of the capacitor, filling the space between the plates. Determine the potential at P.

   (C) With the slab still in place, the switch is opened. The slab is then removed. Determine the potential at P.



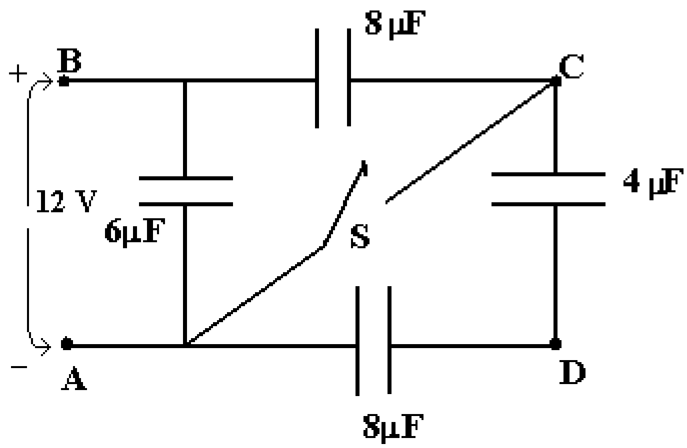
2. A potential difference of 12 V is initially applied across A-B ($V_B - V_A = 12$ V) and the source of this potential difference is then removed, leaving the system of capacitors charged but isolated, as shown below.

(A) Calculate the charge on each capacitor.

<u>The switch is now closed.</u>

(B) Calculate the potential difference between $V_B - V_A$ and the potential difference $V_D - V_A$.

(C) Calculate the amount of charge which flowed through the switch S after it was closed.



Solution – Pledged Problems #5

1. (A) Note – there are a number of ways to solve this problem. The following is one way to approach and solve it. After the switch is closed, an amount of charge ±Q will accumulate on each of the two capacitors (+ on the upper plate, - on the lower plate). Q can be determined by calculating the value for C_{eq}, the single capacitor equivalent to the two original capacitors in series, and then applying the equation $Q = C_{eq}V_{tot}$ (where V_{tot} = 10V). The voltage across each capacitor can then be determined by applying the equation $Q = C_iV_i$ (i = 1, 2). So,

$$\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} = \frac{1}{12 \text{ x } 10^{-6}} + \frac{1}{6 \text{ x } 10^{-6}} = \frac{3}{12 \text{ x } 10^{-6}} = \frac{1}{4 \text{ x } 10^{-6}}$$

or $C_{eq} = 4\mu F$. Therefore, $Q = C_{eq}V_{tot}$ = (4 x 10^{-6} C/V)(10V) = 40μC, and

$V_2 = Q/C_2$ = 40μC/6μF = **6.67V**.

(B) The same approach is used to determine the potential at P after the dielectric has been inserted, recognizing that C_2 will increase proportionally with the dielectric constant κ of the inserted material, or $C_2' = \kappa C_2 = 2 \cdot 6\mu F = 12\mu F$. Therefore,

$$\frac{1}{C'_{eq}} = \frac{1}{C_1} + \frac{1}{C'_2} = \frac{1}{12 \text{ x } 10^{-6}} + \frac{1}{12 \text{ x } 10^{-6}} = \frac{2}{12 \text{ x } 10^{-6}} = \frac{1}{6 \text{ x } 10^{-6}}$$

or $C'_{eq} = 6\mu F$. Therefore, $Q' = C'_{eq}V_{tot}$ = (6 x 10^{-6} C/V)(10V) = 60μC, and

$V'_2 = Q'/C'_2$ = 40μC/6μF = **5.0V**. One could have also noted that because the 2 capacitors now have the same 12μF capacitance, the voltage drop across each will be the same, or ½ of the total 10V.

(C) Opening the switch has the effect of eliminating the flow of charge, so that Q on the lower capacitor before and after removal of the dielectric remains the same. In part (B), Q was determined to be 60μC. The voltage across the capacitor after the dielectric is removed will be related to the charge Q and original capacitance by the equation

$V'_2 = Q'/C_2$ = 60μC/6μF = **10V.**

2. (A) The charge on the 6μF capacitor (which we will call Q_1) is related to the 12 V potential drop across it by the equation Q = CV, so that Q_1 = 6μF x 12V = **72μC.** Because the remaining capacitors are in series, the charge on each of them (which we will call Q_2) will be the same, and can be determined by calculating the equivalent capacitance of a single capacitor -

$$\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} = \frac{1}{8 \text{ x } 10^{-6}} + \frac{1}{4 \text{ x } 10^{-6}} + \frac{1}{8 \text{ x } 10^{-6}} = \frac{4}{8 \text{ x } 10^{-6}} = \frac{1}{2 \text{ x } 10^{-6}}$$

or C_{eq} = 2μF. Therefore, $Q_2 = C_{eq}V$ = (2 x 10^{-6} C/V)(12V) = **24μC**

(B) After the switch is closed, Points A and C will be at the same potential. Therefore, there will be no potential drop across the 2 capacitors on the right side of the system (the 4μF capacitor between C and D, and the 8μF capacitor between D and A); Point D will be at the same potential as Point A, and therefore, the potential difference **$V_D - V_A$ = 0**.

The potential difference $V_B - V_A$ can be determined by recognizing that, because the system is isolated, the total charge on the tops and bottoms of the two remaining capacitors ($Q_1 + Q_2$ = 72μC + 24μC = 96μC) will remain constant (although it will redistribute itself between the two capacitors because of the closing of the switch). To determine the new potential difference, one first calculates the equivalent capacitance for the two capacitors, and then uses that, together with the known, total charge of 96μC, to derive the new voltage difference across the two capacitors. Because the capacitors are parallel, C_{eq} = 6μF + 8μF = 14μF, and **$V_{new} = Q/C_{eq}$ = 96μC/14μC = 6.9V**.

(C) The amount of charge which flowed through the switch S after it was closed will have 2 components; the charge that flowed as a result of discharging the two capacitors on the right side of the circuit and the charge that flowed as part of the redistribution of the 96μC on the two capacitors on the left side of the circuit.

As determined in part (A), the amount of charge on the two capacitors on the right equaled +/- 24μC. That charge would have flowed from C to A, to discharge those two capacitors.

The new charge distribution for the capacitors on the left can be determined by using the voltage V_{BA} found in part (B) and the equation Q = CV. Thus, the charge on the 8μF capacitor will be **$C_{8\mu F}$ = 8μF x 6.9V = 55μC** and the charge on the 6μF capacitor will be **$C_{6\mu F}$ = 6μF x 6.9V = 41μC**. (A quick check confirms that the total charge remains 96μC (= 55μC + 41μC).) The initial charge on the 8μF capacitor was 24μC (as determined in part (A)), so the amount of charge that flowed through the switch associated with the left two capacitors will be 55μC - 24μC = 31μC, flowing from A to C.

The net charge flowing from A to C will, therefore, be **31μC - 24μC = 7μC**.