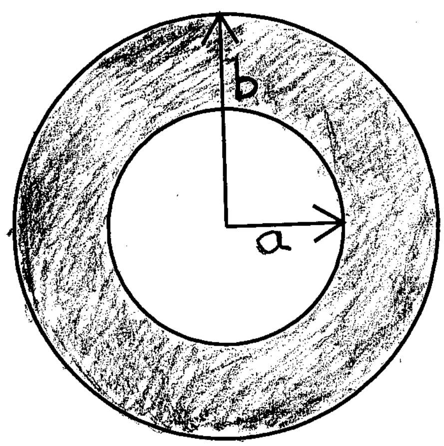
Physics 102– Pledged Problem 3

Time allowed: **2 hours at a single sitting**
Due 5PM Monday, February 6, 2006, in the boxes marked Phys 101-102 in the physics lounge. You may use your own textbook, your notes, and a non-programmed calculator. You may also consult the on-line solutions to the corresponding suggested problems. You should consult no other help. Show how you arrived at your answer; the correct answer by itself may not be sufficient.
Further instructions:
(a) Write legibly on **one** side of 8.5" x 11" white or lightly tinted paper.
(b) Staple all sheets together, including this one, in the upper left corner and make one vertical fold.
(c) On the outside, staple side up, print your name in capital letters, your LAST NAME first followed by your FIRST NAME.
(d) Below your name, print the phrase "Pledged Problem 3", followed by the due date.
(e) Also indicate **start time** and **end time**.
(f) Write and sign the pledge, with the understanding that you may consult the materials noted above.

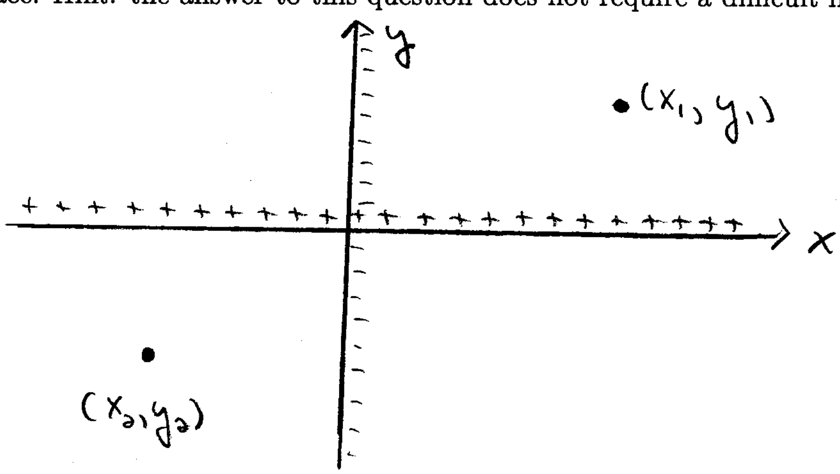
I. A hollow spherical shell has an inner radius a and an outer radius b. The shell is made of an insulating material that has a total charge $+Q$ uniformly distributed throughout its volume.

(a) Determine the volume charge density ρ for the shell.
(b) Determine the electric field $\vec{E}(r)$ in the region $r < a$.
(c) Determine the electric field $\vec{E}(r)$ in the region $b > r > a$.
(d) Determine the electric field $\vec{E}(r)$ in the region $r > b$.



II. A very long, thin insulating rod carries a uniform linear charge density $+\lambda$ along its entire length. The rod is situated along the x-axis as shown below. An second rod carries a uniform, negative linear charge density $-\lambda$, and is situated along the y-axis.

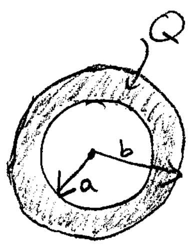
(a) Determine the electric field $\vec{E}(x, y)$ for an arbitrary point (x_1, y_1) located in the first quadrant of this coordinate system $(x > 0, y > 0)$.
(b) Determine the electric field $\vec{E}(x, y)$ for an arbitrary point (x_2, y_2) located in the third quadrant $(x < 0, y < 0)$.
(c) Consider a Gaussian surface consisting of a sphere of radius R centered at the origin. Determine the electric flux Φ_E through this surface. Hint: the answer to this question does not require a difficult integral!



Physics 102

Pledged Problem 3.

I.



(a) $\rho = \frac{Q}{\text{Volume}}$ Volume $= \frac{4}{3}\pi(b^3 - a^3)$

$$\boxed{\rho = \frac{3Q}{4\pi(b^3 - a^3)}}$$

(b) $\vec{E}$ for $r < a$

Consider a Gaussian surface concentric with the shell and with $r < a$. By symmetry, $\vec{E}$ must be constant on the Gaussian surface & normal to the surface. By Gauss' law we have.

$$\int \vec{E} \cdot d\vec{A} = \frac{Q_{encl}}{\varepsilon_0}$$

$$\int E\,dA = 4\pi r^2 E = \frac{Q_{encl}}{\varepsilon_0} = 0$$

$\boxed{\vec{E} = 0 \text{ for } r < a}$ $\vec{E} = 0$ everywhere inside the sphere!

(c) $\vec{E}$ for $a < r < b$.

Now consider a Gaussian sphere with $a < r < b$. Symmetry still requires $\vec{E}$ to be constant on the surface & radially outward.

$$\int E \cdot d\vec{A} = \frac{Q_{encl}}{\varepsilon_0} = \frac{\rho}{\varepsilon_0}(\text{volume})$$

$$4\pi r^2 E = \frac{\rho}{\varepsilon_0}\left[\underbrace{\tfrac{4}{3}\pi\,(r^3 - a^3)}_{\text{volume containing charge}}\right]$$

$$\boxed{\vec{E} = \frac{\rho(r^3 - a^3)}{3\varepsilon_0 r^2}\,\hat{r}}$$

Direction is radially outward.

Or we can substitute back in for ρ:

$$\vec{E} = \frac{\cancel{3}Q\,(r^3 - a^3)}{\cancel{3}(4\pi)\varepsilon_0 r^2 (b^3 - a^3)}$$

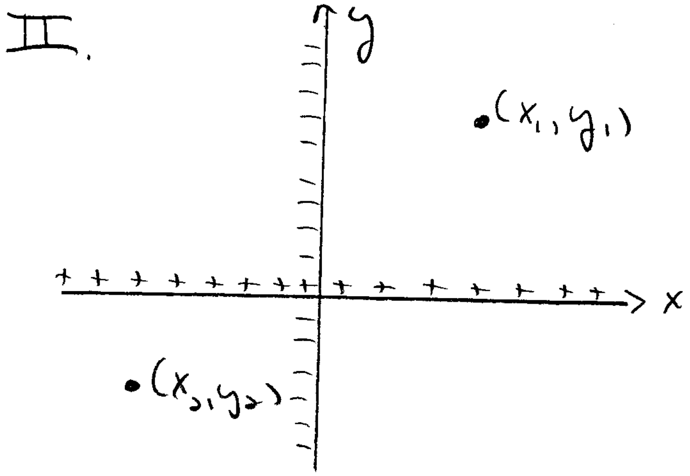
$$\boxed{\vec{E} = \frac{Q\,(r^3 - a^3)}{4\pi\varepsilon_0 r^2 (b^3 - a^3)}\,\hat{r}}$$

(d) $\vec{E}$ for $r > b$

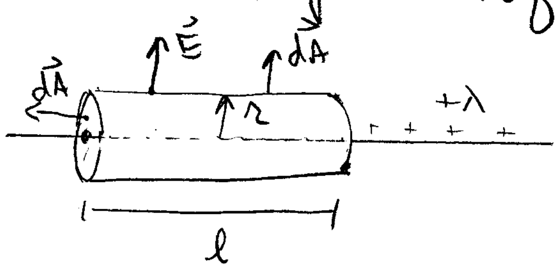
Now <u>all</u> the charge Q is enclosed, and the electric field looks the same as a point charge at the origin.

$$\boxed{\vec{E} = \frac{Q}{4\pi\varepsilon_0 r^2}\,\hat{r}}$$

II.



(a) First determine the field due to the positive line of charge on the x-axis. Use Gauss' Law with cylindrical symmetry. The Gaussian surface is a concentric cylinder of radius r & length l



By symmetry, E is radially outward from the line of charge.

$$\int \vec{E}\cdot d\vec{A} = \underbrace{\int \vec{E}\cdot d\vec{A}}_{\text{ends of cylinder}} + \underbrace{\int \vec{E}\cdot d\vec{A}}_{\text{curved side}} = \frac{Q_{encl}}{\varepsilon_0} = \frac{\lambda l}{\varepsilon_0}$$

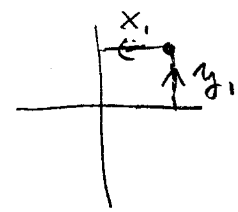
The integral over the ends $= 0$ since $d\vec{A} \perp \vec{E}$.

Over the curved sides, $\vec{E}$ and $d\vec{A}$ are $\parallel$ and $|\vec{E}|$ is constant.

$$\int \vec{E}\cdot d\vec{A} = 2\pi r l E = \frac{\lambda l}{\varepsilon_0}$$ where λl = charge enclosed.

$$\vec{E} = \frac{\lambda}{2\pi\varepsilon_0 r}\,\hat{r}$$ Direction is radially outward from line of charge.

For the negative line of charge, the Gaussian cylinder sits on the y-axis, and $\lambda \to -\lambda$ so the electric field is radially inward. For the point (x_1, y_1) in quadrant I,



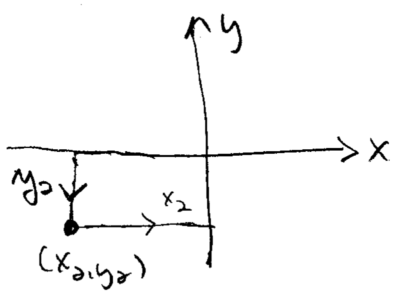
$$\vec{E}_1 = \frac{\lambda}{2\pi\varepsilon_0 y_1}\hat{j} \qquad \vec{E}_2 = \frac{-\lambda}{2\pi\varepsilon_0 x_1}\hat{\imath}$$

(due to $+\lambda$) (due to $-\lambda$)

$$\boxed{\vec{E}_{TOT} = \frac{\lambda}{2\pi\varepsilon_0}\left(\frac{-\hat{\imath}}{x_1} + \frac{\hat{j}}{y_1}\right)}$$

Total field is the sum of the two contributions

(b) For a point in quadrant III, the direction of both components changes



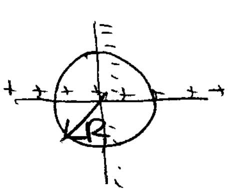
$$\vec{E}_1 = \frac{-\lambda}{2\pi\varepsilon_0 |y_2|}\hat{j} \qquad \vec{E}_2 = \frac{\lambda}{2\pi\varepsilon_0 |x_2|}\hat{\imath}$$

$$\boxed{\vec{E}_{TOT} = \frac{\lambda}{2\pi\varepsilon_0}\left(\frac{\hat{\imath}}{|x_2|} - \frac{\hat{j}}{|y_2|}\right)}$$

Or, since x_2 & y_2 are both negative, we can write

$$\boxed{\vec{E}_{TOT} = \frac{\lambda}{2\pi\varepsilon_0}\left(\frac{-\hat{\imath}}{x_2} + \frac{\hat{j}}{y_2}\right).}$$

(c) A Gaussian surface at the origin of radius R encloses $2R(\lambda) + 2R(-\lambda)$ charge $= 0$! By Gauss law



$$\Phi_E = \int \vec{E} \cdot d\vec{A} = \frac{Q_{encl}}{\varepsilon_0} = 0$$

$$\boxed{\Phi_E = 0}$$

This does not mean that $\vec{E} = 0$ on the surface!