

# Physics 102– Pledged Problem 1

Time allowed: 2 hours at a single sitting

Due 5PM Monday, January 22, 2007, in the boxes marked Phys 101-102 in the physics lounge. You may use your own textbook, your notes, and a non-programmed calculator. You may also consult the on-line solutions to the corresponding suggested problems. You should consult no other help. Show how you arrived at your answer; the correct answer by itself may not be sufficient.

Further instructions:

- Write legibly on **one** side of 8.5" x 11" white or lightly tinted paper.
- Staple all sheets together, including this one, in the upper left corner.
- Make one vertical fold.
- On the outside, staple side up, print your name in capital letter, your LAST NAME first followed by your FIRST NAME.
- Below your name, print the phrase "Pledged Problem 1", followed by the due date.
- Also indicate **start time** and **end time**.
- Write and sign the pledge, with the understanding that you may consult the materials noted above.

I. In class we discussed the fact that the electron and proton appear to have exactly the same charge, so that matter is electrically neutral. Suppose that is not the case, but rather that the proton's charge is slightly larger in magnitude. For purposes of this problem, assume the following:

The charge of the electron is  $-1.6022 \times 10^{-19} \text{C}$  and the charge of the proton is  $+1.6023 \times 10^{-19} \text{C}$ .

The mass of the earth is  $5.98 \times 10^{24} \text{ kg}$  and the mass of the sun is  $1.99 \times 10^{30} \text{ kg}$ .

The mass of the proton is  $1.67 \times 10^{-27} \text{ kg}$

The masses of the sun and earth are due entirely to the mass of the protons that make them up (that is, we will neglect the mass of the electrons).

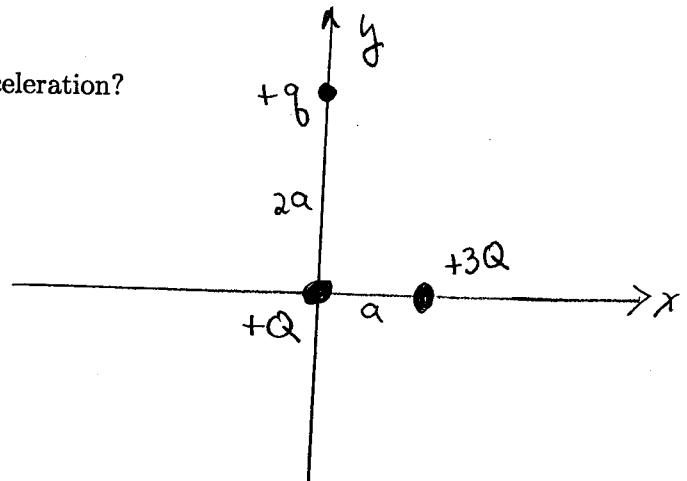
The number of protons is equal to the number of electrons in both the earth and the sun.

Use the mean distance between the earth and the sun which is  $1.50 \times 10^{11} \text{ m}$ .

- How many protons and electrons are there in the sun?
- How many protons and electrons are there in the earth?
- Determine the net charge of the earth, under the assumptions above.
- Determine the net charge of the sun, under the assumptions above.
- Determine the electrical force between the earth and the sun. Be sure to indicate the direction as well as the magnitude.
- Determine the gravitational force between the earth and the sun.
- Could we have a stable solar system if the magnitudes of the proton and electron charge were even slightly different from each other?

II. A positive charge of  $+Q$  is located at the origin, and a second charge of  $+3Q$  is located on the  $x$ -axis at  $x = a$ . A third positive charge  $+q$  of mass  $m$  is located on the  $y$ -axis at  $y = 2a$ .

- Determine the net force on the positive charge  $+q$ .
- Determine the net force on the charge  $+Q$  at the origin.
- If the charge  $+q$  is released from rest, what is its initial acceleration?



# Phys 102

## Pledged Problem 1

I. Charge on electron =  $-1.6022 \times 10^{-19} \text{ C}$   
Charge on proton =  $+1.6023 \times 10^{-19} \text{ C}$

The slight charge difference will give a net charge to matter.

(a) Number of protons in the sun:

$$N_s = \frac{1.99 \times 10^{30} \text{ kg}}{1.67 \times 10^{-27} \text{ kg/proton}} = \boxed{1.19 \times 10^{57} \text{ protons}}$$

Number of electrons = number of protons.

(b) Number of protons in earth:

$$N_E = \frac{5.98 \times 10^{24} \text{ kg}}{1.67 \times 10^{-27} \text{ kg/proton}} = \boxed{3.58 \times 10^{51} \text{ protons in the earth}}$$

Number of electrons = number of protons

(c) Since, in our model, the proton's charge is slightly larger in magnitude, there will be a net charge given by

(2)

$$Q_E = (Q_{\text{proton}} - |Q_{\text{electron}}|) (N_E)$$

number of protons  
↓

$$= (1.6023 - 1.6022) \times 10^{-19} \text{ C} \quad (3.58 \times 10^{51})$$

$$Q_E = 10^{-23} \text{ C} (3.58 \times 10^{51}) = 3.58 \times 10^{28} \text{ C}$$

$$Q_E = 3.58 \times 10^{28} \text{ C}$$

(d) Same thing for the sun:

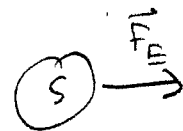
$$Q_S = (10^{-23} \text{ C}) (1.19 \times 10^{57})$$

$$Q_S = 1.19 \times 10^{34} \text{ C}$$

$$(e) \left| \frac{F}{E} \right| = \frac{k Q_E Q_S}{R^2} \quad \text{use } R = 1.50 \times 10^{11} \text{ m}$$

$$F_E = \frac{(9 \times 10^9 \text{ N m}^2/\text{C}^2) (3.58 \times 10^{28} \text{ C}) (1.19 \times 10^{34} \text{ C})}{(1.5 \times 10^{11} \text{ m})^2}$$

$$F_E = 17.0 \times 10^{49} \text{ N}$$



$$F_E = 1.7 \times 10^{50} \text{ N}$$

This force is repulsive since  $Q_E$  and  $Q_S$  are of the same sign. And it's huge!

(3)

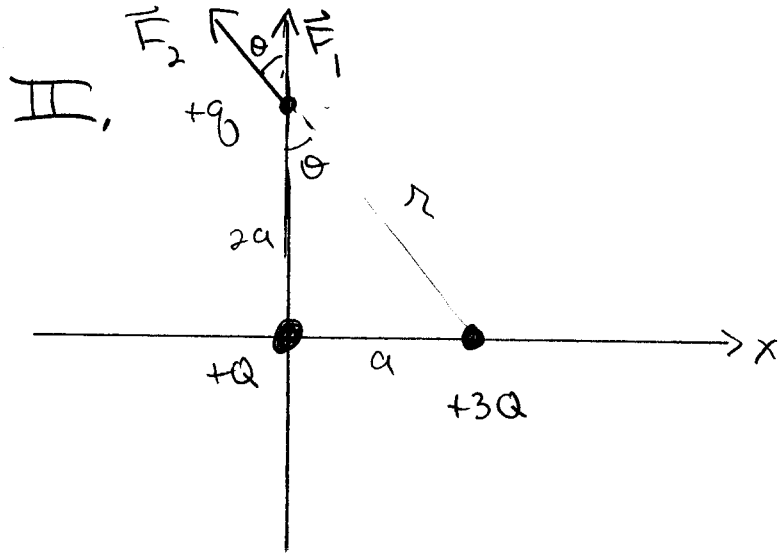
$$(f) F_G = \frac{G M_s M_E}{R^2} = \left( 6.67 \times 10^{-11} \frac{\text{Nm}^2}{\text{kg}^2} \right) \left( \frac{5.98 \times 10^{24} \text{ kg} (1.99 \times 10^{30} \text{ kg})}{(1.5 \times 10^{11} \text{ m})^2} \right)$$

$$F_G = 35.3 \times 10^{21} \text{ N}$$

$F_G = 3.53 \times 10^{22} \text{ N}$

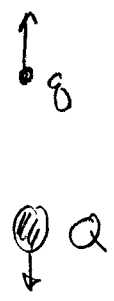
This force is attractive.

(g) With this small difference in charge, the electrical force totally overwhelms the gravitational force by 28 orders of magnitude! Since  $F_E$  is repulsive, even a small charge difference between protons & electrons would make a stable solar system impossible.



$$r = \sqrt{5} a$$

(a) The net force on  $q$  is the superposition of forces due to  $+Q$  and  $+3Q$



$\vec{F}_1 = \frac{k Q q}{4a^2} \hat{j}$  - force is in  $+y$  direction  
 (long due to  $+Q$ )

$|\vec{F}_2| = \frac{k 3Qq}{r^2} = \frac{3k Qq}{5a^2}$   
 (long due to  $+3Q$ )

The direction of  $\vec{F}_2$  is along the line joining  $3Q$  and  $q$ . We need to express  $\vec{F}_2$  in  $x$  &  $y$  components

$$F_{2x} = -F_2 \sin \theta \qquad F_{2y} = F_2 \cos \theta$$

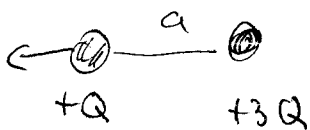
$$\sin \theta = \frac{a}{r} = \frac{1}{\sqrt{5}} \qquad \cos \theta = \frac{2a}{r} = \frac{2}{\sqrt{5}}$$

Now add  $\vec{F}_1$  &  $\vec{F}_2$  component-wise:

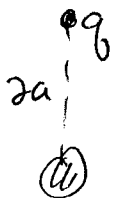
$$F_x = - \frac{3kqQ}{5a^2} \cdot \frac{1}{\sqrt{5}}$$

$$F_{m+q} = \frac{kQq}{a^2} \left[ - \frac{3}{5\sqrt{5}} \hat{i} + \left( \frac{6}{5\sqrt{5}} + \frac{1}{4} \right) \hat{j} \right]$$

(b) Force on Q:



$$F_1 = \frac{3kQ^2}{a^2} \hat{i}$$



$$F_2 = \frac{kqQ}{4a^2} \hat{j}$$

$$m\vec{a} = \frac{kQ}{a^2} \left[ -3Q \hat{i} - \frac{q}{4} \hat{j} \right]$$

(c) Release  $q$  from rest,  $\vec{a} = \vec{F}/m$   
Use the force from (a)

$$\vec{a} = \frac{kQq}{ma^2} \left[ -\frac{3}{5\sqrt{5}} \hat{i} + \left( \frac{6}{5\sqrt{5}} + \frac{1}{4} \right) \hat{j} \right]$$