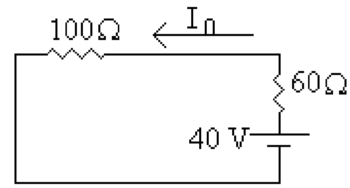
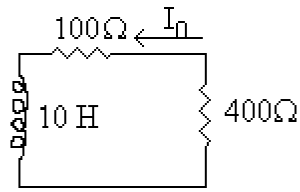
Solution – Pledged Problems #9



1. (a) If the switch has been closed for a long time, then the current has settled down to a constant value and the inductor plays no role in the circuit. The relevant circuit is shown in the figure to the right, and the current through the inductor will be

$$I_0 = \frac{V}{R_{eq}} = \frac{40V}{100\Omega + 60\Omega} = 0.25A$$



(b) When the switch is thrown to the position B, then the circuit is shown in the figure to the right, and the initial current through the inductor will be the I_0 calculated in part 1(a). Therefore, the potential drop across the 100Ω resistor will be

$$V_1 = (100\Omega)(0.25A) = 25V.$$

And since current can't accumulate, the same I_0 will initially flow across the 400Ω resistor, so the potential drop across it will be

$$V_2 = (400\Omega)(0.25A) = 100V.$$

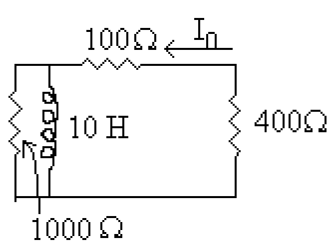
Finally, Kirchhoff's Loop Rule dictates that the potentials will add to zero when following a loop from a point back to that point, so the potential drop across the 10H inductor must be $V_1 + V_2 = 125V$

(c)The energy in the inductor at any given time is $U(t) = ½ L(I(t))^2$, where $I(t) = I_0 e^{-Rt/L}$. Therefore, $U(t) = ½ LI_0^2 e^{-2Rt/L}$. The initial rate of energy-loss is the time derivative of U(t) evaluated at t = 0.

$$\frac{dU(t)}{dt} = ½ LI_0^2 \left(\frac{-2R}{L}\right) e^{-2R(t=0)/L} = ½ (10H)(0.25A)^2 \left(\frac{-2(500\Omega)}{10H}\right) = \mathbf{-31.25W}$$

Alternatively, the power dissipation associated with the inductor will equal the power dissipated through the resistors = $I^2R = (0.25A)^2 500\Omega = 31.25W$.

Modifying the circuit by adding a resistor in parallel with the inductor will not affect the answer to (a), since the inductor essentially acts as a short across the 1000Ω resistor after the switch S has been in position A for a long time. After the switch is thrown to position B, the circuit will be as shown to the right. The initial current through the 100Ω and 500Ω resistors will remain 0.25A, for the reasons given in the earlier discussion. However, the power being dissipated includes not only the -31.25W associated with the



0.25A flowing through the original resistors, but also the power being dissipated through the 1000Ω resistor, also initially held at 125V, which equals $\frac{V^2}{R} = \frac{125^2}{1000} = 15.625W$. Therefore, the total initial rate of energy loss from the inductor will be

$$31.25W + 15.625W = \mathbf{46.875W}.$$

2. (a) The magnitude of the magnetic induction $\vec{B}$ at the center of the coil can be derived using the Biot-Savart Law, with each element of the coil, *dl*, contributing $dB = \frac{\mu_0 N_1 I_1\, dl}{4\pi R_1^{\,2}}$ to the field. The total field then has magnitude –

$$B = \int dB = \int \frac{\mu_0 N_1 I_1}{4\pi R_1^{\,2}} dl = \frac{\mu_0 N_1 I_1}{4\pi R_1^{\,2}} \int dl = \frac{\mu_0 N_1 I_1}{4\pi R_1^{\,2}} 2\pi R_1 = \frac{\mu_0 N_1 I_1}{2R_1}$$

Given the direction of the current in the loop, the right hand rule indicates the direction of $\vec{B}$ is to the left.

(b) The torque experienced by the small coil can be approximately determined by assuming that $\vec{B}$ is constant in the vicinity of the small coil. The direction and magnitude of the torque on a current loop are given by $\vec{\tau} = \vec{\mu}_2 \times \vec{B}$, where $\vec{\mu}_2 = N_2 I_2 A_2 = N_2 I_2 \pi R_2^{\,2}$. Therefore, the torque experienced by the loop 2 is

$$\vec{\tau} = \vec{\mu}_2 \times \vec{B} = \left(N_2 I_2 \pi R_2^{\,2}\right) \frac{\mu_0 N_1 I_1}{2R_1} \sin\theta$$

(c) The mutual inductance between the two coils can be derived from the equation of the flux in the inner loop associated with the magnetic induction $\vec{B}$ produced by the outer loop.

$$\Phi_2 = \int \vec{B} \cdot d\vec{A}_2 = BA_2 \cos\theta = \frac{\mu_0 N_1 I_1}{2R_1} \pi N_2 R_2^{\,2} \cos\theta = \frac{\mu_0 \pi N_2 N_1 R_2^{\,2} \cos\theta}{2R_1} I_1$$

$$\rightarrow \qquad \boldsymbol{M = \frac{\mu_0 \pi N_2 N_1 R_2^{\,2} \cos\theta}{2R_1}}$$

since, by definition, the mutual inductance is the constant of proportionality between the current I_1 and flux Φ_2.