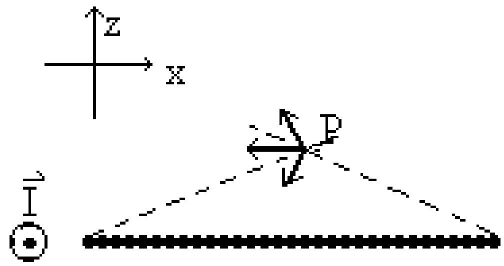
Solution – Pledged Problems #8

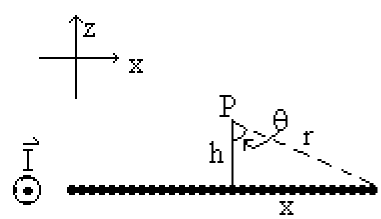
1. (a) Treat the infinitely wide stream of current as a combination of very thin wire-like ribbons of current, $dI = J_s dx$, all running in the negative y direction (out of the page). The total magnetic field at a given point simply will be the sum of the magnetic fields associated with each of the wire-like components of the current. In this section of the problem, we are just interested in a qualitative description of the fields. We know from Ampere's Law that at a given point the direction of the field for each component of current dI will be in a direction perpendicular to a line drawn from that point to the current component. Symmetry considerations indicate that, for any such point, the components of the field not parallel to the XY plane will cancel; there is a current component symmetrically positioned with respect to the perpendicular line from the sheet of current to the point in question that will produce a field of equal magnitude and pointing in a direction such that the components perpendicular to the surface will cancel. See the figure to the right. There will be no component of the fields parallel to the lines of current, again using Ampere's Law and the arguments of symmetry we used in discussing fields associated with current flowing through a wire.



Therefore, the fields associated with this sheet will all be parallel to the sheet of current and perpendicular to the flow of current. For the current direction shown, those magnetic field lines above the plane will point to the left; those on the bottom of the plane to the right.

(b) There are a couple of ways to solve for the field. We can extend the approach used in the first section by using a mathematical approach similar to that used earlier in determining the electric fields associated with linear charge distributions. The component of the magnetic field parallel to the XY plane associated with a given dI will be

$$dB = \frac{\mu_0 dI}{2\pi r}\cos\theta = \frac{\mu_0 \cos\theta}{2\pi r} J_s dx$$



where θ and r are as shown in the figure to the right and dx is the incremental step in the x direction associated with the current component dI. x is the horizontal distance along the current plane from the point P at which the field is being measured. We want to express all the related variables, r,dx, and θ in terms of a single variable.

$h/r = \cos\theta \rightarrow r = h/\cos\theta$; $x/h = \tan\theta \rightarrow dx = -h\sec^2\theta d\theta$. Substituting these into the equation for dB gives

$$dB = -\frac{\mu_0 \cos^2\theta}{2\pi h} J_s \, h \sec^2\theta \, d\theta = -\frac{\mu_0 J_s}{2\pi} d\theta$$

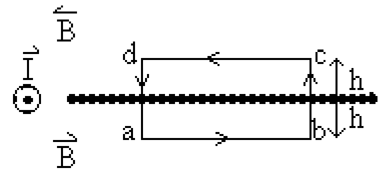
Therefore, the total magnetic field a distance *h* from the current sheet will be

$$B_{tot} = \int_{-\pi/2}^{\pi/2} dB = -\int_{-\pi/2}^{\pi/2} \frac{\mu_0 J_s}{2\pi} d\theta = \frac{\mu_0 J_s}{2\pi}(\pi/2 - (-\pi/2) = -\frac{\mu_0 J_s}{2}$$

pointing to the left above the current sheet and to the right below the current sheet. Note, similar to the electric fields associated with planes of charge particles, the magnetic fields associated with planes of currents are independent of distance from the current plane.

A simpler process is to use Ampere's Law, using a rectangle with the top and bottom at the same distance h from the current sheet.

$$\oint \vec{B} \cdot \vec{dl} = \int_a^b \vec{B} \cdot \vec{dl} + \int_b^c \vec{B} \cdot \vec{dl} + \int_c^d \vec{B} \cdot \vec{dl} + \int_d^a \vec{B} \cdot \vec{dl} = \mu_0 I$$



Because $\vec{B}$ is perpendicular to the sides, the second and fourth term equal zero; because $\vec{B}$ is parallel to the top and bottom $\vec{B} \cdot \vec{dl} = Bdl$-

$$\oint \vec{B} \cdot \vec{dl} = \int_a^b \vec{B} \cdot \vec{dl} + \int_c^d \vec{B} \cdot \vec{dl} = 2Bl = \mu_0 I = \mu_0 J_s l \rightarrow B = \frac{\mu_0 J_s}{2}$$

(c) For two infinite sheets positioned as shown in the problem, the fields add vectorally. Therefore, outside the sheets, the fields are of the same magnitude but point in opposite directions, producing a net field of zero; between the two sets of current sheets, the fields are of the same magnitude and point in the same direction, producing a net field pointing to the right of

$$B_{tot} = \frac{\mu_0 J_s}{2} \times 2 = \mu_0 J_s$$

2. (a) The net force on the sliding wire will equal the sum of the gravitational force and the magnetic force associated with the flow of current in the magnetic field. With the wire sliding downward, flux will be increasing and therefore Lenz' Law dictates the current will flow in the counterclockwise direction. This produces a force $\vec{F}_{mag} = I\vec{L} \times \vec{B}$ in the upward direction. The current I is generated by the emf ε produced according to Faraday's Law of induction, $\varepsilon = -\frac{d\Phi_B}{dt}$;

$$I = \frac{\varepsilon}{R} = -\frac{1}{R}\frac{d\Phi_B}{dt} = -\frac{B}{R}\frac{dA}{dt} = -\frac{BL}{R}\frac{dx}{dt} = -\frac{BLv}{R}$$

$$\vec{F}_{mag} = I\vec{L} \times \vec{B} = ILB = -\frac{B^2L^2v}{R}$$

Therefore, the net force $\vec{F}_{tot} = \vec{F}_{mag} + \vec{F}_{grav}$ will approach zero as v increases, and will reach a terminal velocity when the forces are of the same magnitude. When $\vec{F}_{tot} = 0$

$$\frac{B^2L^2v_{ter}}{R} = mg \rightarrow v_{ter} = \frac{mgR}{B^2L^2}$$

(b) The power dissipated is $P = I^2R = \left(\frac{BLv}{R}\right)^2 R = \frac{B^2L^2v^2}{R}$. At terminal velocity we can substitute $\frac{B^2L^2v_{ter}}{R} = mg$ and get $P = mgv_{ter} = mg\frac{dx}{dt} = \vec{F}_{grav}\frac{dx}{dt} = \frac{\vec{F}_{grav}d\vec{x}}{dt}$ or the rate at which gravity does work $W = \vec{F}_{grav} \cdot \vec{x}$.