Physics 102 – Pledged Problem 7

Time allowed: **2 hours at a single sitting**

**DUE 4PM MONDAY, March 24, 2008,** in the boxes marked Phys 101-102 in the physics lounge. You may use your own textbook, your notes, and a non-programmed calculator. You may also consult the on-line solutions to the corresponding suggested problems. You should consult no other help. Show how you arrived at your answer; the correct answer by itself may not be sufficient.

Further instructions:

(a) Write legibly on **one** side of 8.5” x 11” white or lightly tinted paper.
(b) Staple all sheets together, including this one, in the upper left corner and make one vertical fold.
(c) On the outside, staple side up. Print your name in capital letters, your LAST NAME first followed by your FIRST NAME.
(d) Below your name, print the phrase “Pledged Problem 7”, followed by the due date.
(e) Also indicate **start time** and **end time.**
(f) Write and sign the pledge, with the understanding you may consult the materials noted above.

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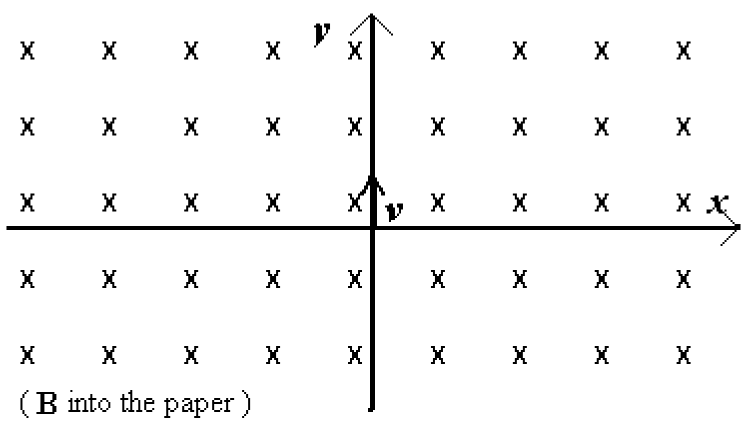
1. Two particles of the same charge, $q = 1.0 \times 10^{-10}$ C and different masses, $m_1 = 5 \times 10^{-15}$ kg and $m_2 = 6 \times 10^{-15}$ kg, are accelerated from rest through a potential difference $\Delta V = 1000$V. They are then injected simultaneously into a region of uniform magnetic field $B = 0.1$ T as shown:



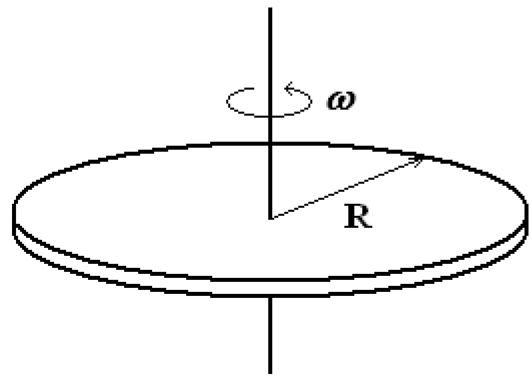

Assume that at t = 0, both particles are at the origin of the coordinate system, x = 0 and y = 0, and that the velocity of each is directed along the positive y-axis.

(a) Describe the subsequent orbits of the two particles, giving such quantities as radius, angular velocity, and period of revolution. [Draw a diagram, carefully labeled.]

(b) After what time interval will both particles first attain their maximum separation?

(c) After what time interval will both particles simultaneously return to the origin x = 0, y = 0 for the first time (t > 0)?

2. Consider a planar insulating disc of radius R with uniform surface charge density $\sigma$ ($C/m^2$) that is rotated about a perpendicular axis through its center with angular velocity $\omega$. (There is no charge on the lower face).




The aim of this problem is to calculate the total magnetic moment associated with the rotating disc.

(a) Derive an expression for the current dI associated with rotation of the charge in an elemental ring of radius r and thickness dr.

(b) What is the magnetic moment produced by this element?

(c) What is the total magnetic moment of the disc?

# Solution – Pledged Problems #7

1.(a) Because of the constant centripetal force provided by the charged particles' interaction with the uniform magnetic field, both particles will undergo uniform circular motion. The radius of motion is given by the equation $R = mv/qB$; the angular velocity by the equation $\omega = v/R$; and the period of revolution, $T = 2\pi m/qB = 2\pi/\omega$. The only variables missing are the particles' velocities, which can be determined by noting that conservation of energy dictates acceleration from rest through a potential difference of 1000V, will provide each particle with a kinetic energy $KE = \frac{1}{2}mv^2 = qV \rightarrow v = \sqrt{2(1000V)q/m}$.

So particle $m_1$, with a mass of $5 \times 10^{-15}kg$, will have

$v_1 = \sqrt{(2000)(1.0 \times 10^{-10})/5 \times 10^{-15}} = 6.3 \times 10^3 m/s,$

$R_1 = m_1 v_1/qB = (5 \times 10^{-15}kg)(6.3 \times 10^3 m/s)/(1.0 \times 10^{-10}C)(0.1T)$

$= \mathbf{3.15\ m}$

$\omega_1 = v_1/R_1 = 6.3 \times 10^3 ms^{-1}/3.15\ m$

$= \mathbf{2 \times 10^3\ rads^{-1}}$

$T_1 = 2\pi/\omega_1 = 2\pi/2 \times 10^3 = \mathbf{3.14 \times 10^{-3} s}$

Particle $m_2$, with a mass of $6 \times 10^{-15}kg$, will have

$v_2 = \sqrt{(2000)(1.0 \times 10^{-10})/6 \times 10^{-15}} = 5.8 \times 10^3 m/s,$

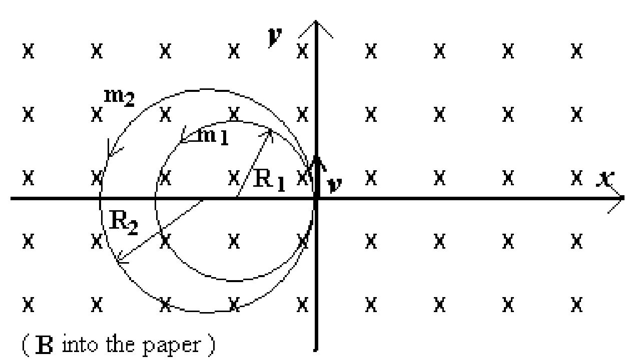
$R_2 = m_2 v_2/qB = (6 \times 10^{-15}kg)(5.8 \times 10^3 m/s)/(1.0 \times 10^{-10}C)(0.1T)$

$= \mathbf{3.48\ m}$

$\omega_2 = v_2/R_2 = 5.83 \times 10^3 ms^{-1}/3.48\ m$

$= \mathbf{1.67 \times 10^3\ rads^{-1}}$

$T_2 = 2\pi/\omega_2 = 2\pi/1.67 \times 10^3 = \mathbf{3.76 \times 10^{-3} s}$



(b) From looking at the particles' paths, it is clear the particles will attain their maximum separation when $m_1$ is at the origin and $m_2$ is halfway around its path, at the farthest distance from the origin. In determining the time interval when this will first occur, it should be noted that the times of revolution, $T_1$ and $T_2$, are proportional to the particle's masses ($T_1 = 5/6\ T_2$); they do not depend upon the particles' velocities. So the condition that must be satisfied is that $m_1$ must have traveled n times around its path while $m_2$ will have traveled only (n-1/2) times around its path.

$$nT_1 = \left(n - \frac{1}{2}\right)T_2 \rightarrow n\left(\frac{5}{6}T_2\right) = \left(n - \frac{1}{2}\right)T_2 \rightarrow \frac{5}{6}n = n - \frac{1}{2} \rightarrow n = 3$$

This will occur at time $nT_1 = (3)(3.14 \times 10^{-3}s\,) = \mathbf{9.42 \times 10^{-3}}\boldsymbol{s}$

(c) An approach similar to that described in (b) can be used to determine when the particles will simultaneously return to the origin for the first time. This will occur when the particle with the shorter time of revolution, $m_1$, travels one additional revolution, compared to the number of revolutions traveled by $m_2$.

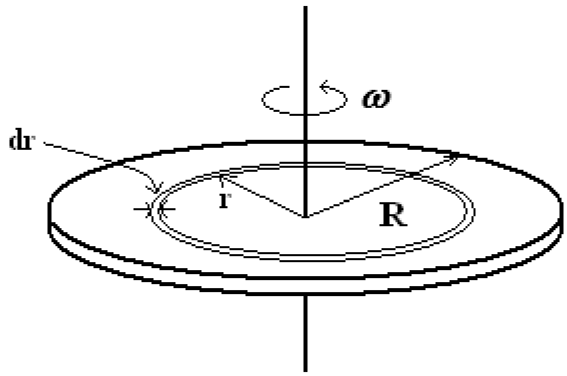
$$(m+1)T_1 = \mathrm{m}T_2 \rightarrow (m+1)\left(\frac{5}{6}T_2\right) = \mathrm{m}T_2 \rightarrow \frac{5}{6}(m+1) = m \rightarrow m = 5$$

This will occur at time $(m+1)T_1 = (6)(3.14 \times 10^{-3}s\,) = \mathbf{18.8 \times 10^{-3}}\boldsymbol{s}$

2. (a) Current is related to charge density by the equation I = σvA, where σ is the charge density, v is the velocity of the charged particles, and A is the cross-sectional area across which the charge flows. An elemental ring of radius r and width dr will therefore have a an incremental current of -

$$dI = \sigma v dA = \sigma(r\omega)(dr)$$

The velocity is related to the angular velocity by $v = r\omega$ and the cross-sectional area is the 1- dimensional face of width *dr* across which the charge flows.




(b) The magnetic moment associated with this ring is simply

$$d\mu = dIA = \sigma(r\omega)(dr)(\pi r^2) = \sigma\omega\pi r^3 dr$$

where A is the area enclosed by the elemental ring - $\pi r^2$.

(c) The total magnetic moment associated with the entire disk is obtained by adding the incremental magnetic moments associated with each elemental ring (which requires that we integrate over r from 0→R) –

$$\mu_{tot} = \int d\mu = \int_0^R \sigma\omega\pi r^3 dr = \frac{\sigma\omega\pi R^4}{4} = \boldsymbol{\frac{Q\omega R^2}{4}}$$

where $Q = \sigma\pi R^2$ is the total charge on the disc.