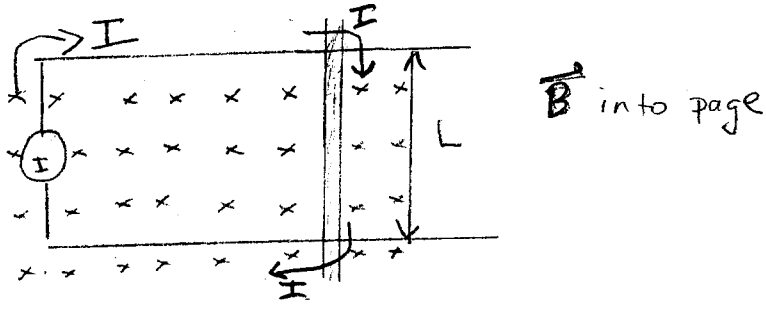
Physics 102– Pledged Problem 7 Solution

14 I. A metal crossbar of mass M slides without friction on a pair of long, horizontal conducting rails separated by a distance L. The rails are connected to a device that supplies a constant current I to the circuit. A uniform magnetic field B points into the page as shown in the figure below.

3 (a) In what direction will the bar move when the current source is turned on?

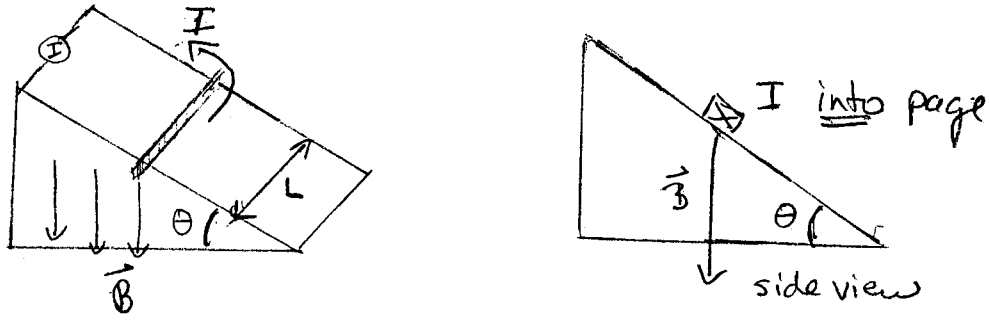
3 (b) If the bar starts from rest at $t = 0$, determine its velocity at a time t later. Express your answer in terms of M, L, I, B, and t.



Now suppose the rails are tilted upward so that they make an angle θ with the horizontal, as shown below. The magnetic field B is still in the vertical direction. [NOTE: In the original diagram, the current was in the wrong direction. An email to all students pointed out the error. Graders, give students the benefit of the doubt, if they are confused by the direction of the current give them credit]

5 (c) For this configuration, what magnitude of magnetic field B is needed to keep the bar from sliding down the rails?

3 (d) If B has twice the value found in (c), what is the acceleration of the bar?



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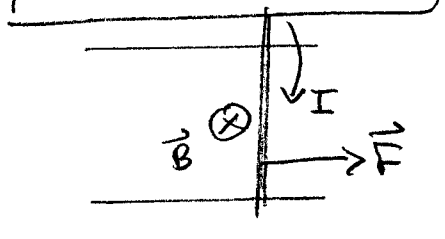
(a) The force on the bar is given by

$$\vec{F} = I\vec{L} \times \vec{B}.$$

In the sketch the magnetic field is into the page, and the current at the location of the bar is down. The cross product gives a force to the right in the figure.



(b) The magnitude of the force is ILB, since I and B are perpendicular. As long as the current is constant, the

force will be constant, and the acceleration a will be constant.

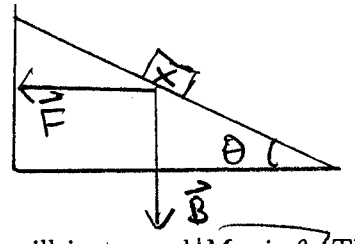
$$F = ILB = Ma$$

$$\boxed{v = at = \frac{ILBt}{M}}$$

(c) With the bars tilted up, as shown below, the current into the page, and the magnetic field down, the force $I\vec{L} \times \vec{B}$ will be horizontal to the left. Break down $\vec{F}$ into components up the incline ($F\cos\theta$) and a component perpendicular to the incline ($F\sin\theta$) The component perpendicular to the incline doesn't matter, since the bars are frictionless. (If there were friction, this component would increase the friction). To keep the bar from sliding down the ramp, the upward component of the magnetic force must balance the downward force of gravity. The component of gravity down the ramp is $Mg\sin\theta$, so

$$Fcos\theta = ILBcos\theta = Mg\sin\theta$$

$$\boxed{B = \frac{Mg\sin\theta}{IL\cos\theta} = \frac{Mg\tan\theta}{IL}}$$



(d) If the magnitude of B is doubled, the force up the ramp will just equal $Mg\sin\theta$. The acceleration would be $g\sin\theta$.

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II. Protons and deuterons (each with charge $+e$) and alpha (α) particles (with charge $+2e$) of the same kinetic energy enter a uniform magnetic field $\vec{B}$ that is perpendicular to their velocities. Make the approximation that $m_\alpha = 2m_d = 4m_p$.

(a) Let r_p, r_d and r_α be the radii of their circular orbits. Determine expressions for the ratios r_d/r_p and r_α/r_p.
(b) Let T_p, T_d, and T_α be the periods of rotation for the particles. Determine expressions for the ratios T_d/T_p and T_α/T_p.

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(a) For a charged particle in a uniform magnetic field, the radius of the circular path can be determined by balancing the magnetic force with the centripetal force needed to keep the particles in circular orbit:

$$\frac{mv^2}{r} = qvB \qquad \text{or } r = \frac{mv}{qB}$$

Since the kinetic energies of all the particles are the same, we can get the ratios of the velocities:

$$m_p v_p^2 = m_d v_d^2 = m_\alpha v_\alpha^2 \qquad \text{which gives} \qquad \frac{v_d}{v_p} = \sqrt{\frac{m_p}{m_d}} \text{ and } \frac{v_\alpha}{v_p} = \sqrt{\frac{m_p}{m_\alpha}}$$

Then (recall that the charge on the α particle is twice the charge on the proton or deuteron):

$$r_p = \frac{m_p v_p}{eB} \qquad r_d = \frac{m_d v_d}{eB} \qquad \text{and } r_\alpha = \frac{m_\alpha v_\alpha}{2eB}$$

When taking the ratios of the radii, eB will cancel.

$$\boxed{\frac{r_d}{r_p} = \frac{m_d v_d}{eB} \cdot \frac{eB}{m_p v_p} = \frac{m_d}{m_p} \cdot \frac{v_d}{v_p} = \frac{m_d}{m_p}\sqrt{\frac{m_p}{m_d}} = \sqrt{2}}$$ 4

Likewise for r_α / r_p

$$\boxed{\frac{r_\alpha}{r_p} = \frac{m_\alpha}{2m_p}\sqrt{\frac{m_p}{m_\alpha}} = 2/2 = 1.}$$ 4

(b) The periods of rotation through a complete circle are given by

$$T = \frac{2\pi r}{v}$$

Taking the ratios,

$$\boxed{\frac{T_d}{T_p} = \frac{r_d}{v_d}\frac{v_p}{r_p} = \frac{r_d}{r_p}\frac{v_p}{v_d} = 2}$$ 4

For $\frac{T_\alpha}{T_p}$:

$$\boxed{\frac{T_\alpha}{T_p} = \frac{r_\alpha}{r_p}\frac{v_p}{v_\alpha} = 2}$$ 4

Physics 102– Pledged Problem 7

Time allowed: **2 hours at a single sitting**

Due 5PM Monday, March 19, 2007, in the boxes marked Phys 101-102 in the physics lounge. You may use your own textbook, your notes, and a non-programmed calculator. You may also consult the on-line solutions to the corresponding suggested problems. You should consult no other help. Show how you arrived at your answer; the correct answer by itself may not be sufficient.

Further instructions:

(a) Write legibly on **one** side of 8.5" x 11" white or lightly tinted paper.
(b) Staple all sheets together, including this one, in the upper left corner. Make one vertical fold.
(c) On the outside, print your name in capital letters, your LAST NAME followed by your FIRST NAME.
(d) Below your name, print the phrase "Pledged Problem 7", followed by the due date.
(e) Write and sign the pledge, with the understanding that you may consult the materials noted above.
(f) Indicate your **start time** and **end time**.

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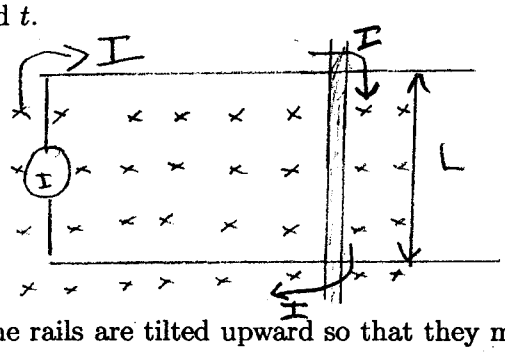

$\vec{B}$ into page

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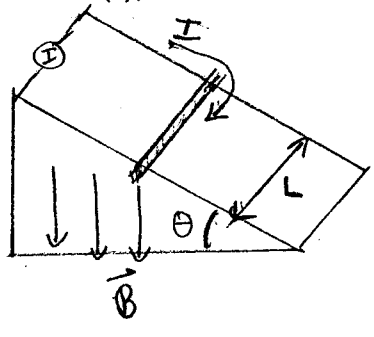

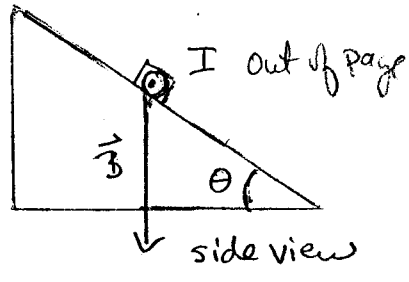

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(original)