Physics 102– Pledged Problem 6

Time allowed: **2 hours at a single sitting**
Due 5PM Monday, March 6, 2006, in the boxes marked Phys 101-102 in the physics lounge. You may use your own textbook, your notes, and a non-programmed calculator. You may also consult the on-line solutions to the corresponding suggested problems. You should consult no other help. Show how you arrived at your answer; the correct answer by itself may not be sufficient.
Further instructions:
(a) Write legibly on **one** side of 8.5" x 11" white or lightly tinted paper.
(b) Staple all sheets together, including this one, in the upper left corner and make one vertical fold.
(c) On the outside, staple side up, print your name in capital letters, your LAST NAME first followed by your FIRST NAME.
(d) Below your name, print the phrase "Pledged Problem 6", followed by the due date.
(e) Also indicate **start time** and **end time**.
(f) Write and sign the pledge, with the understanding that you may consult the materials noted above.

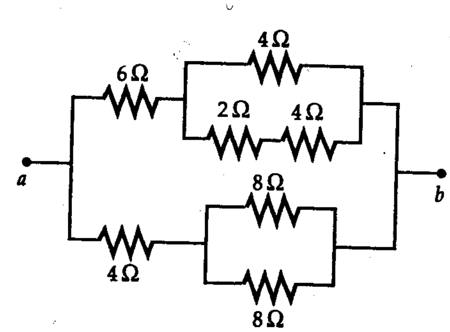
I. This problem consists of three questions about resistance, currents, and current densities.

(a) The wiring in a house must have low enough resistance so that it does not heat up too much while current is flowing. A particular copper wire needs to carry 20A of current, and it must not dissipate more than 2 Watts of power per meter of length. If the cross-section of this wire is circular, what is the minimum diameter that the wire must have so that it doesn't heat up too much? The resisitivity of copper is $\rho = 1.72 \times 10^{-8}\Omega - m$.

(b) The density of copper metal is $d = 9g/cm^3$, and the atomic mass (averaged over naturally occurring isotopes) is 63.5 g/mole. Avagadro's number is $N_A = 6.02 \times 10^{23}$ atoms/mole. Assume one charge carrier per atom and determine the density of charge carriers n (number of charge carriers/m^3) in copper.

(c) A cooper wire with a circular cross section with radius r = 1mm carries 1A of current. Determine the drift velocity of the electrons in the wire.

II. For the circuit shown below, a 12V battery is connected across the points a and b. Determine the following:
(a) The effective resistance R_{eff} of the circuit.
(b) The total current and the total power supplied by the battery.
(c) The current through each resistor and the power dissipated in each resistor.
(d) For the power dissipation, does the sum of all the contributions in (c) equal the result in (b)?



Phys102

Pledged Problem 6

I. (a) $\rho = 1.72 \times 10^{-8}\ \Omega\text{-m}$ $\qquad I = 20\ A$

Power $= I^2 R = 2$ Watts/m

$$R = \frac{2\ \text{Watts}}{400\ A} = .005\ \Omega$$

$$R = \frac{\rho l}{A} = \frac{(1.72 \times 10^{-8}\ \Omega\text{-m})(1\text{m})}{A} = .005\ \Omega$$

$$A = \frac{1.72 \times 10^{-8}}{5 \times 10^{-3}}\ m^2 = .344 \times 10^{-5}\ m^2 = \pi r^2$$

$$.109 \times 10^{-5}\ m^2 = r^2$$

$$1.046 \times 10^{-3}\ m = r = 1.046\ \text{mm}$$

diameter = 2.09 mm

(b) $d = 9\ \text{g/cm}^3$ $\quad A = 63.5\ \text{g/mole}$ $\quad N_A = 6.02 \times 10^{23}\ \text{atoms/mole}$

$$n = \frac{9\ \text{g/cm}^3\ (6.02 \times 10^{23}\ \text{atoms/mole})}{63.5\ \text{g/mole}}$$

$n = .85 \times 10^{23}\ \text{atoms/cm}^3$

watch units — need answer in atoms/m3

$n = 8.5 \times 10^{22}\ \text{atoms/cm}^3\ (10^6\ \text{cm}^3/\text{m}^3)$

$$\boxed{n = 8.5 \times 10^{28}\ /\text{m}^3}$$

(c)

$\Delta Q = n A v_d q \Delta t$

$\frac{\Delta Q}{\Delta t} = I = n A v_d q$

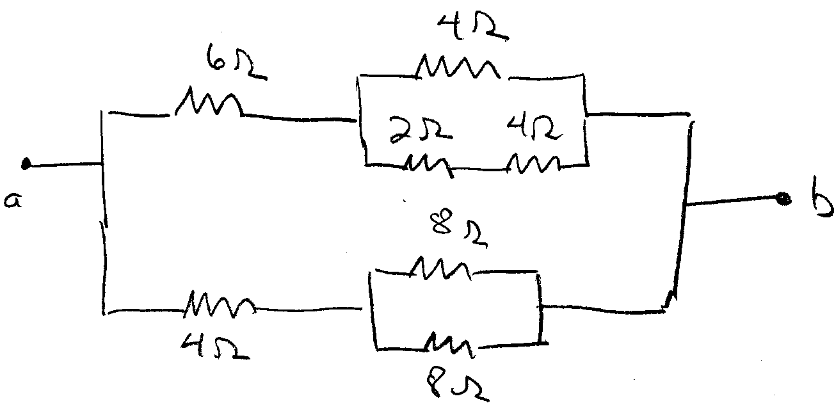
Use n from (b), $I = 1\ \text{Amp}$ $\quad r = 1\ \text{mm} = .1\ \text{cm}$

$q = 1.6 \times 10^{-19}\ \text{C}$ $\quad A = \pi r^2 = .0314\ \text{cm}^2$

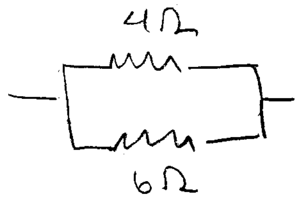
$$v_d = \frac{I}{nAq} = \frac{1\ \text{C/s}}{(8.5 \times 10^{22}/\text{cm}^3)(.0314\ \text{cm}^2)(1.6 \times 10^{-19}\ \text{C})}$$

$$\boxed{v_d = 2.34 \times 10^{-3}\ \text{cm/s} = 2.34 \times 10^{-5}\ \text{m/s}}$$

II.

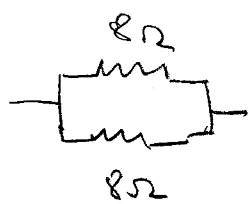


(a) Find R_{eff} – first combine the two sets of parallel resistors:



$$\Rightarrow \frac{1}{4}+\frac{1}{6} = \frac{1}{R_{ef1}} = \frac{3}{12}+\frac{2}{12} = \frac{5}{12}$$

$$R_{ef1} = \frac{12}{5} = 2.4\,\Omega$$



$$\Rightarrow \frac{1}{8}+\frac{1}{8} = \frac{2}{8} = \frac{1}{4} \qquad R_{ef2} = 4\,\Omega$$

R_{ef1} is in series with 6Ω $\Rightarrow R_{ef3} = 8.4\,\Omega$

R_{ef2} is in series with 4Ω $\Rightarrow R_{ef4} = 8\,\Omega$

R_{ef3} and R_{ef4} are in parallel with each other,

$$\frac{1}{R_{eff}} = \frac{1}{R_{ef3}} + \frac{1}{R_{ef4}} = \frac{1}{8.4} + \frac{1}{8}$$

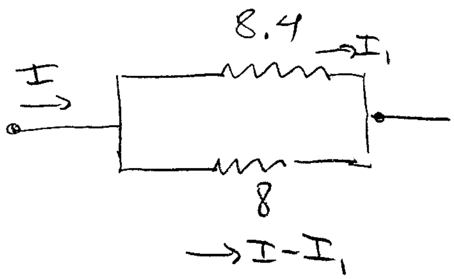
$$\boxed{R_{eff} = 4.1\,\Omega}$$

(b) $I = \frac{V}{R_{eq}} = \frac{12V}{4.1\Omega}$

$I = 2.93A$

Power $= IV = (2.93A)(12V) =$ 35.2 Watts $= P$

(c) To determine the current in each resistor, first see how it splits between R_{eq3} and R_{eq4}



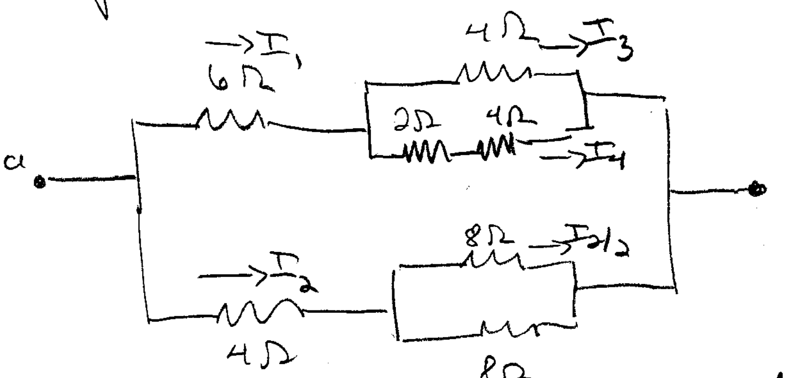
$8.4(I_1) = 8(I-I_1)$

$16.4 I_1 = 8I$

$I_1 = .49I = 1.436A$

$I_2 = I - I_1 = .51I = 1.494A$

These currents get split at the next set of parallel junctions



$I_2/2 = .747A$

In the lower section, the current must split evenly, since the two resistors are equal (8 Ω each).

In the upper section, the current will not split evenly, but the voltage drop must be the same:

$4I_3 = 6(I_1 - I_3)$

$10 I_3 = 6I_1$

$I_3 = .6 I_1$

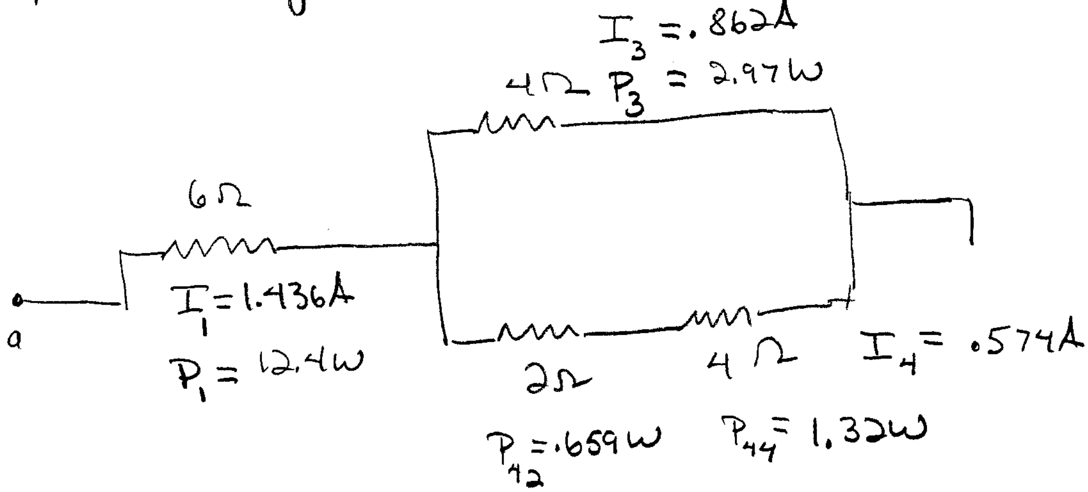
$I_4 = I_1 - I_3 = .4 I_1$

$I_3 = .862A$

$I_4 = .574A$

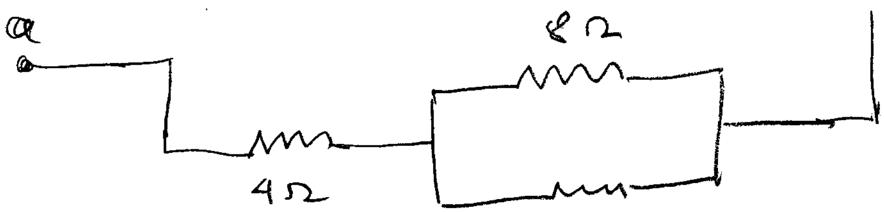
$P = I^2 R$ for each resistor

Upper section of circuit:



(current must be the same in 2Ω & 4Ω resistors)

Lower section:



$I_2 = 1.499A$

$P_2 = 8.93W$

$I_2/2 = .747A$ in each 8Ω resistor

$P_8 = 4.46$ W in each 8Ω resistor

(d) Add up power in all 7 resistors:

$$P_{TOT} = 12.4 + 2.97 + .659 + 1.32 + 8.93 + 2(4.46) \text{ W}$$

$$\boxed{P_{TOT} = 35.2 \text{ Watts}}$$

It works! Power delivered by the battery appears as heat in the resistors.