Physics 102 – Pledged Problem 4

Time allowed: **2 hours at a single sitting**

DUE 4PM MONDAY, February 11, 2008, in the boxes marked Phys 101-102 in the physics lounge. You may use your own textbook, your notes, and a non-programmed calculator. You may also consult the on-line solutions to the corresponding suggested problems. You should consult no other help. Show how you arrived at your answer; the correct answer by itself may not be sufficient.

Further instructions:

(a) Write legibly on **one** side of 8.5” x 11” white or lightly tinted paper.
(b) Staple all sheets together, including this one, in the upper left corner and make one vertical fold.
(c) On the outside, staple side up. Print your name in capital letters, your LAST NAME first followed by your FIRST NAME.
(d) Below your name, print the phrase “Pledged Problem 4”, followed by the due date.
(e) Also indicate **start time** and **end time.**
(f) Write and sign the pledge, with the understanding you may consult the materials noted above.

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1. (4 pts) A charge +4q is located at the origin and a charge –q is on the x axis at x=a.

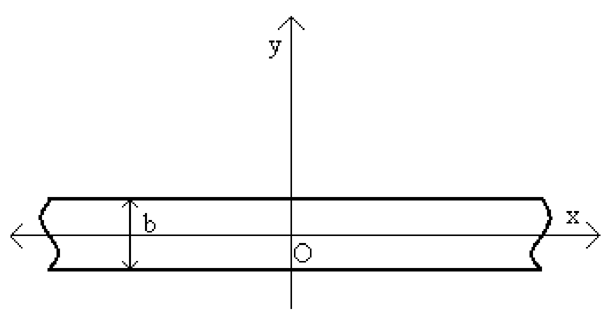
   (a) Write an expression for the potential on the x axis for $x > a$;

   (b) Find a point in this region where $V = 0$;

   (c) Use the result of (a) to find the electric field $\vec{E}$ on the x axis for $x > a$; and

   (d) Find a point where $E = 0$.

2. (6 pts) Consider an infinite non-conducting planar sheet of charge of uniform charge density ρ that lies in the *XZ* plane and has thickness b (see figure).



   (a) Use Gauss’ Law to calculate the electric field (vector) as a function of the y coordinate, i.e. the coordinate perpendicular to the plane of the charge sheet. (The origin is taken to be at the center of the sheet).

   (b) Calculate the resulting potential as a function of y. Set the potential at y = 0 to be zero. Sketch this potential.

Solution – Pledged Problems #4

1. (a) This problem requires applying the concept that the total potential at a point is the sum of the potentials associated with each charge element sourcing the potential - $V_P = \frac{1}{4\pi\varepsilon_0}\sum_{i=1}^{n}\frac{q_i}{r_i}$. So, for the charge distribution in the problem, V_P for $x > a$ is

$$V_P(x) = \frac{1}{4\pi\varepsilon_0}\frac{4q}{x} - \frac{1}{4\pi\varepsilon_0}\frac{q}{(x-a)}$$
$$= \frac{q}{4\pi\varepsilon_0}\left(\frac{4}{x} - \frac{1}{(x-a)}\right)$$

(b) $V_P(x)$ is zero when $\left(\frac{4}{x} - \frac{1}{(x-a)}\right) = 0$, or when $x = {}^4\!/_3\, a$. (Plugging this value for x back into the equation confirms this is the right answer).

(c) The electric field on the x-axis, E_x, is derivable from the potential through the relationship $E_x = -\frac{dV}{dx}$. Taking the derivative of $V_P(x)$ with respect to x yields –

$$E_x = -\frac{d}{dx}\left(\frac{q}{4\pi\varepsilon_0}\left(\frac{4}{x} - \frac{1}{(x-a)}\right)\right) = -\frac{q}{4\pi\varepsilon_0}\frac{d}{dx}\left(\frac{4}{x} - \frac{1}{(x-a)}\right)$$
$$= \frac{q}{4\pi\varepsilon_0}\left(\frac{4}{x^2} - \frac{1}{(x-a)^2}\right)$$

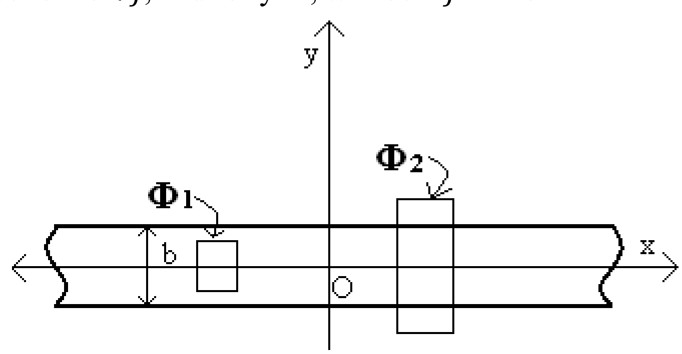
(d) $E_x = 0$ when $\left(\frac{4}{x^2} - \frac{1}{(x-a)^2}\right) = 0$,

$$4(x-a)^2 - x^2 = 3x^2 - 8ax + 4a^2 = 0$$

$$(x-2a)(3x-2a) = 0$$

The location where $E_x = 0$ for $x > a$ is, therefore, $x = 2a$. (Again, plugging this value for x into the equation for E_x confirms this is the right answer).

2. Symmetry of the charge distribution dictates the type of surface to be used in applying Gauss' Law. As we've discussed in class and in the book, a planar uniform charge distribution will produce electric fields that are perpendicular to the surface and of equal magnitude a given distance from the surface. In all regions, the direction of E will be away from the x-axis; so for y > 0, the direction is $+\hat{\jmath}$, and for y<0, will be $-\hat{\jmath}$. This symmetry is true inside and outside the planar sheet, so we will choose a structure whose sides are parallel to the electric fields, and whose top and bottom are equidistant from the y-axis. Φ_1 is flux through a representative surface inside the sheet; Φ_2 outside the sheet.



So, using Gauss' Law, $\Phi_s = \oint_s \vec{E} \cdot d\vec{A} = \frac{Q_{enc}}{\varepsilon_0}$, for $-{}^{b}/_{2} < y < {}^{b}/_{2}$,

$$\Phi_1 = \Phi_{top} + \Phi_{bot} + \Phi_{sides} = EA + EA + 0 = 2EA = \frac{Q_{enc}}{\varepsilon_0}$$

Where A is the area of the top and bottom of the surface and $Q_{enc} = \rho(2y)A$. Solving for E gives

$$E = \frac{\rho y}{\varepsilon_0} \ (-{}^{b}/_{2} < y < {}^{b}/_{2})$$

For |y| > b/2, the expression for Φ_2 will be substantially the same as for Φ_1, $\Phi_2 = \Phi_{top} + \Phi_{bot} + \Phi_{sides} = EA + EA + 0 = 2EA$, but now $Q_{enc} = \rho bA$, so that

$$E = \frac{\rho b}{2\varepsilon_0} \ (y < -{}^{b}/_{2} \ and \ y > {}^{b}/_{2})$$

(b) Obtaining potentials from electric fields involves the inverse process to that used in Problem 1, where we took the partial derivative of the potential in a certain direction to find the electric field in that direction. Now we will integrate over the electric field along a given path, (in this case, in the y direction) to find the potential difference between the endpoints of the integration, $\Delta V = V_{final} - V_{initial} = -\int_{y_i}^{y_f} \vec{E} \cdot d\vec{y}$.

Because we have designated the potential at y = 0 to be zero, this will be the initial point of integration. The ending point will be some distance in the y direction from y = 0. Because we are integrating in the *y* direction, and have determined that the electric field is pointing only in the *y* direction, the term being integrated will become $\vec{E} \cdot d\vec{y} = E(y)dy$. So, for $|y| < {}^{b}/_{2}$

$$\Delta V = V_{final} - V_{final} = V_y - 0 = -\int_0^y \frac{\rho y}{\varepsilon_0} dy$$

$$V_y = -\frac{\rho}{\varepsilon_0}\left(\frac{y^2}{2} - 0\right) = -\frac{\rho y^2}{2\varepsilon_0} \ (for \ |y| < {}^{b}/_{2})$$

For $y > {}^{b}/_{2}$, the term for the electric field changes at $y = {}^{b}/_{2}$, so the integration will have 2 components, one for 0 → b/2, the other from b/2 → y.

$$\Delta V = V_y - 0 = -\int_0^y E(y)dy = -\int_0^{{}^{b}/_{2}} \frac{\rho y}{\varepsilon_0} dy - \int_{{}^{b}/_{2}}^{y} \frac{\rho b}{2\varepsilon_0} dy$$

$$V_y = -\frac{\rho\left({}^{b}/_{2}\right)^2}{2\varepsilon_0} - \frac{\rho b}{2\varepsilon_0}\left(y - {}^{b}/_{2}\right) = -\frac{\rho b y}{2\varepsilon_0} + \frac{\rho b^2}{8\varepsilon_0}$$

Symmetry requires the potential for $y < -{}^{b}/_{2}$ to be the same as $y > {}^{b}/_{2}$. So, a sketch of y v. V(y) will look like