# Physics 102– Pledged Problem 4

Time allowed: **2 hours at a single sitting**
**Due 5PM Monday, February 13, 2006**, in the boxes marked Phys 101-102 in the physics lounge. You may use your own textbook, your notes, and a non-programmed calculator. You may also consult the on-line solutions to the corresponding suggested problems. You should consult no other help. Show how you arrived at your answer; the correct answer by itself may not be sufficient.
Further instructions:
(a) Write legibly on **one** side of 8.5" x 11" white or lightly tinted paper.
(b) Staple all sheets together, including this one, in the upper left corner and make one vertical fold.
(c) On the outside, staple side up, print your name in capital letters, your LAST NAME first followed by your FIRST NAME.
(d) Below your name, print the phrase "Pledged Problem 4", followed by the due date.
(e) Also indicate **start time** and **end time**.
(f) Write and sign the pledge, with the understanding that you may consult the materials noted above.

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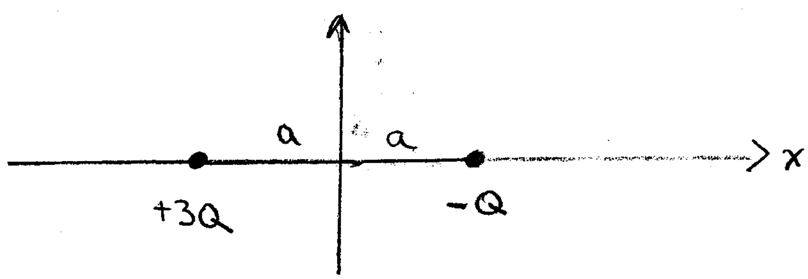
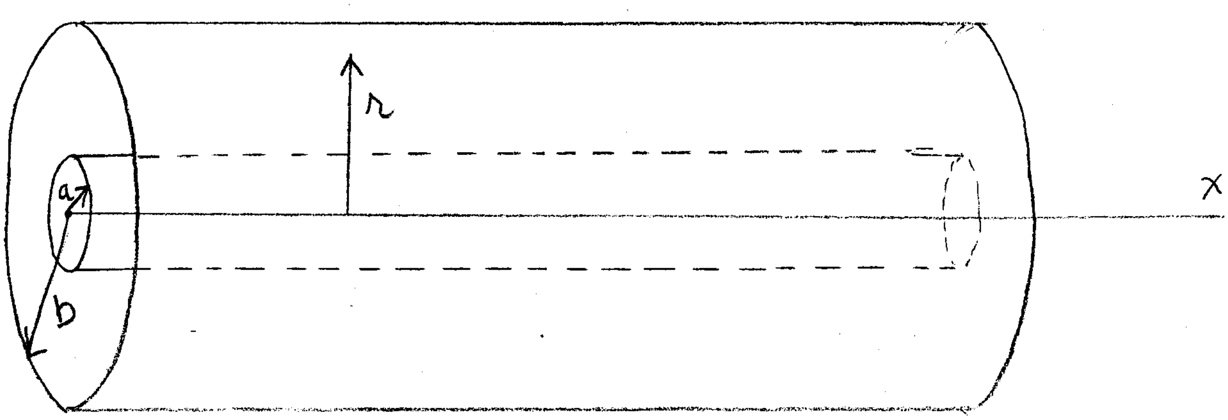
I. A positive point charge $+3Q$ is located on the $x$-axis at $x = -a$. A second negative point charge $-Q$ is located on the $x$-axis at $x = +a$.

(a) Determine the electorstatic potential $V(x)$ for every point on the $x$-axis. Take the zero of potential to be at infinity.
(b) Sketch $V(x)$ vs. $x$ for all points on the $x$-axis.
(c) At what values of $x$ is the potential zero? What is the electric field $\vec{E}$ where $V(x) = 0$?
(d) How much work must be done to move a positive test charge $+q$ from infinity to the point $x = 0$ on the $x$-axis?




II. Two long hollow conducting cylindrical shells are situated along the $x$-axis. The shells are concentric and have negligible thickness. The inner shell has radius $a$ and the outer shell has radius $b$. The inner shell has a positive linear charge density $+\lambda$, and the outer shell has a negative linear charge density $-\lambda$. Take the zero of the electrostatic potential to be at $r = 0$. The coordinate $r$ measures the distance from the common axis of the two cylinders.

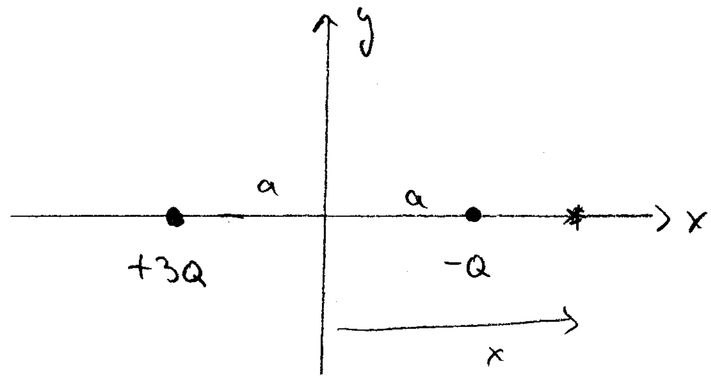
(a) Determine the electrostatic potential $V(r)$ for all values of $r$.
(b) Sketch $V(r)$ for all $r$.
(c) Determine the potential difference $\Delta V$ between $r = a$ and $r = b$.
(d) If a positive charge $+q$ is released from rest at $r = a$, what will be it's kinetic energy when it reaches the outer cylinder at $r = b$?



# Phys 102

## Pledged Problem 4

I.




(a) $$\boxed{V(x) = \frac{3kQ}{|x+a|} - \frac{kQ}{|x-a|}}$$

Note that this form works for all values of x.

If I don't use the absolute values, I have to break V(x) into three regions:

$$V(x) = \frac{3kQ}{x+a} - \frac{kQ}{x-a} \qquad x > a$$

Watch the signs!

$$V(x) = \frac{3kQ}{x+a} - \frac{kQ}{a-x} \qquad -a < x < a$$

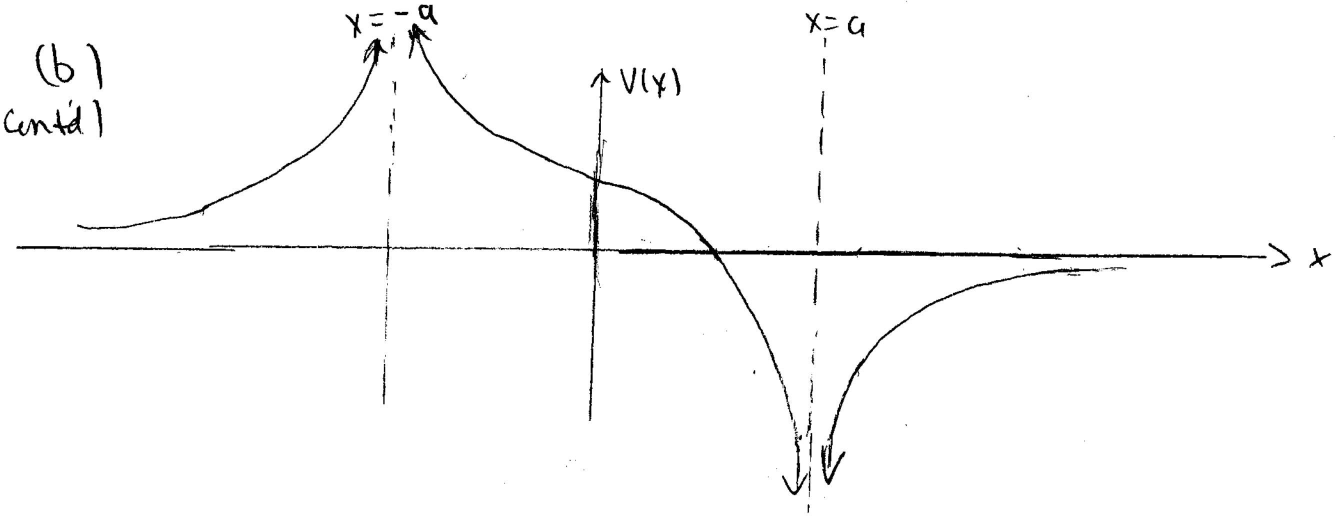
$$V(x) = \frac{3kQ}{-(x+a)} - \frac{kQ}{a-x} \qquad x < -a$$

(b) Plot this result for (b). Note $V \to +\infty$ at $x = -a$

$V \to -\infty$ at $x = a$

V must go through 0 between $x = -a$ and $x = a$

(b)
(cont'd)




(c) Exact location where $V(x) = 0$ can be found by setting $V = 0$
Choose the form appropriate for $-a < x < a$

$$V(x) = \frac{3kQ}{x+a} - \frac{kQ}{a-x} = 0$$

$$\boxed{V(x) = 0 \text{ at } x = a/2}$$

$$3(a-x) = x+a$$

$$\boxed{V(x) \to 0 \text{ at } \pm\infty \text{ as well}}$$

$$3a - 3x = x + a$$

$$2a = 4x$$

$$\boxed{x = a/2}$$

$V = 0$ at $x = a/2$, but the electric field is not zero there!

$$\vec{E}(x = a/2) = \underbrace{\left[\frac{3kQ}{(3/2\,a)^2} + \frac{kQ}{(a/2)^2}\right]}_{\text{both charges give } \vec{E} \text{ in the } +x \text{ direction}} \hat{\imath} = \left[\frac{12kQ}{9a^2} + \frac{4kQ}{a^2}\right]\hat{\imath}$$

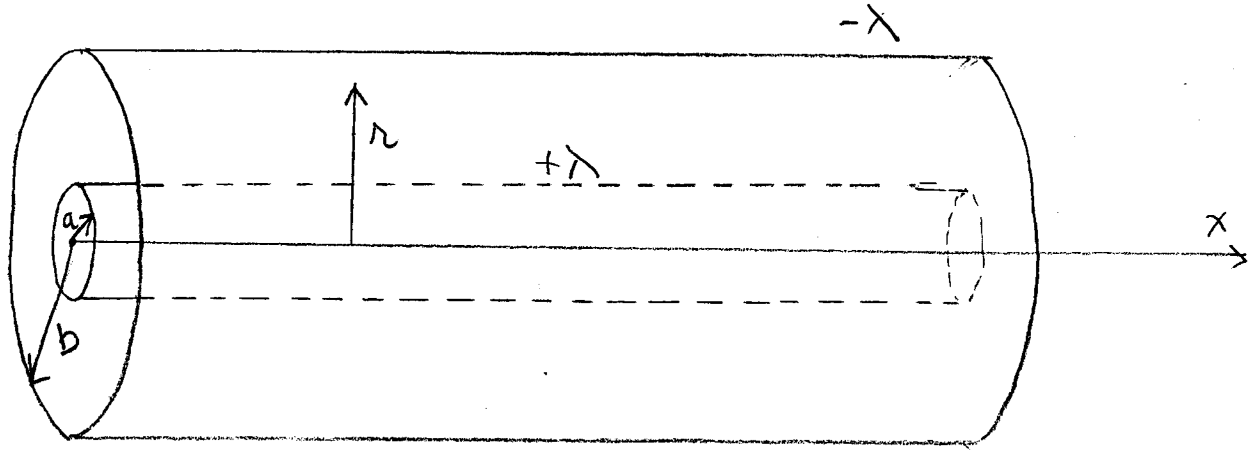
$$\boxed{\vec{E}(x = a/2) = \frac{16kQ}{3a^2}\,\hat{\imath}}$$

$$\boxed{\text{for } x \to \pm\infty,\ \vec{E} \to 0}$$

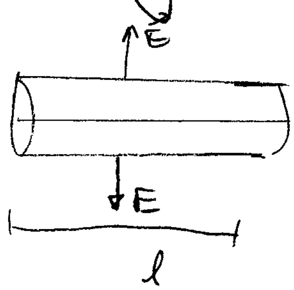
(d) $V(x=0) = \frac{3kQ}{a} - \frac{kQ}{a} = \frac{2kQ}{a}$

$$\boxed{\text{Work} = \Delta U = q\,\Delta V = \frac{2kQq}{a}}$$

II.




(a) To determine $V(r)$ we need to determine $\vec{E}$, which can be done easily from Gauss' Law: for cylindrical symmetry choose a cylindrical Gaussian surface




$$\oint \vec{E} \cdot d\vec{A} = 2\pi r l E = \frac{\lambda l}{\epsilon_0}$$

$$E = \frac{\lambda}{2\pi\epsilon_0 r}$$

For $r < a$, no charge is enclosed! The charge resides on the surface of the hollow cylinder. So $\vec{E} = 0$ for $r < a$

For $a < r < b$, $Q_{encl} = \lambda l$, so $E = \frac{\lambda}{2\pi\epsilon_0 r}$

For $r > b$, $Q_{encl} = 0$ again, so $E = 0$.

Note that $V = 0$ at $r = 0$ in this case!

$$\Delta V = -\int_0^r \vec{E} \cdot d\vec{r}$$

Start at $r = 0$ & integrate out
$\vec{E}$ and $d\vec{r}$ are parallel in the region where $\vec{E} \neq 0$.

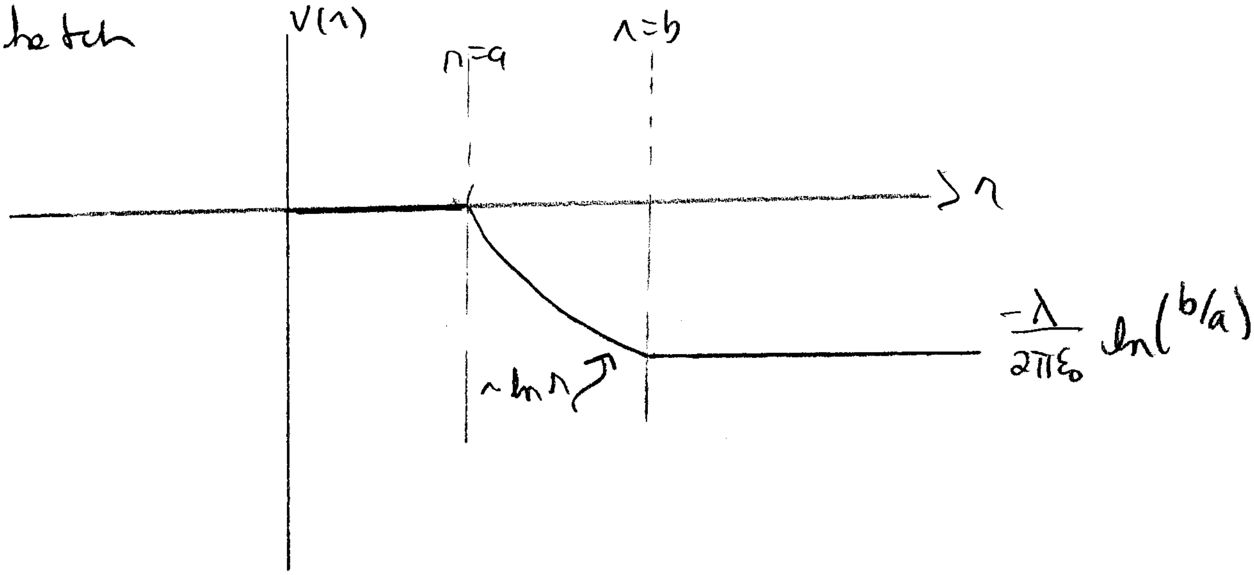
$$\Delta V\,(0\to a) = -\int_0^a \vec{E}\cdot d\vec{l} = 0$$ $$\boxed{V(r)=0 \text{ for } r<a}$$

$$\Delta V(a\to r) = -\int_a^r \frac{\lambda}{2\pi\varepsilon_0 r}\,dr = \boxed{-\frac{\lambda}{2\pi\varepsilon_0}\ln(r/a) = V(r) \text{ for } a\le r\le b}$$

$\Delta V(b\to\infty) = 0$ since $\vec{E}=0$ for $r>b$

$$\boxed{V(r>b) = \frac{-\lambda}{2\pi\varepsilon_0}\ln(b/a)}$$

(b) Sketch




$V(r)=0$ for $r<a$

$V(r)$ decreases logarithmically for $a<r<b$

$V(r)$ is constant at $\frac{-\lambda}{2\pi\varepsilon_0}\ln(b/a)$ for $r>b$.

(c) $\Delta V(r=a \text{ to } r=b) = V(b) - V(a)$

$$\boxed{\Delta V(a \to b) = \frac{-\lambda}{2\pi\varepsilon_0} \ln(b/a)}$$

Potential decreases from a → b.

(d) $\Delta KE = q(-\Delta V) = \frac{q\lambda}{2\pi\varepsilon_0} \ln(b/a)$

$$\boxed{\Delta KE = \frac{q\lambda}{2\pi\varepsilon_0} \ln(b/a)}$$

The loss of potential energy appears as an increase in kinetic energy. $\Delta KE = -\Delta U$