Physics 102– Pledged Problem 3

Time allowed: **2 hours at a single sitting**
Due 5PM Monday, Februray 5, 2007, in the boxes marked Phys 101-102 in the physics lounge. You may use your own textbook, your notes, and a non-programmed calculator. You may also consult the on-line solutions to the corresponding suggested problems. You should consult no other help. Show how you arrived at your answer; the correct answer by itself may not be sufficient.
Further instructions:
(a) Write legibly on **one** side of 8.5" x 11" white or lightly tinted paper.
(b) Staple all sheets together, including this one, in the upper left corner. Make one vertical fold.
(c) On the outside, print your name in capital letters, your LAST NAME followed by your FIRST NAME.
(d) Below your name, print the phrase "Pledged Problem 3", followed by the due date.
(e) Write and sign the pledge, with the understanding that you may consult the materials noted above.
(f) Indicate your **start time** and **end time**.

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16 I. A hollow conducting spherical shell has an inner radius of a and outer radius b. The shell carries a total charge of $+2Q$. The coordinate r measures the distance from the center of the spherical shell.

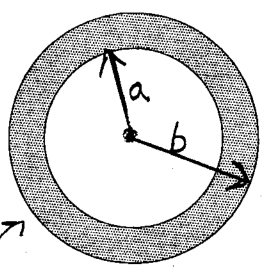
3 (a) Determine the total charge on the inner surface of the shell ($r = a$) and on the outer surface ($r = b$).
4 (b) Determine the electric field $\vec{E}(r)$ for all r and sketch $\vec{E}(r)$ vs. r.

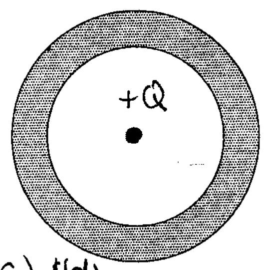
Now suppose a positive point charge of $+Q$ is suspended from a thin string and placed at the center of the shell, at $r = 0$.

3 (c) For this situation, determine the total charge on the inner surface ($r = a$) and the outer surface ($r = b$).
4 (d) Determine the electric field $\vec{E}(r)$ for all r and sketch $\vec{E}(r)$ vs. r.
2 (e)Suppose that the point charge $+Q$ is inside the shell but not located exactly at the center. Would the total charge on the inner and outer surfaces of the shell change? Would the distribution of the charges on the inner and outer surfaces change?

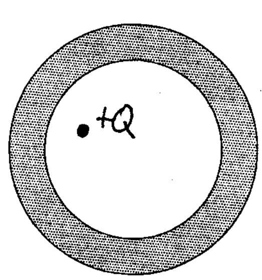
(a) & (b)

Total charge $+2Q$





(c) & (d)



(e)

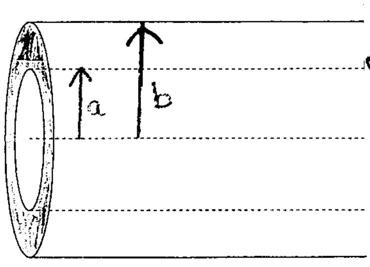
14 II. A very long, hollow, conducting cylindrical shell carries a uniform, linear charge density $+\lambda$. The shell has inner radius a and outer radius b. The coordiate r measures the distance from the axis of the cylinder.

3 (a) Determine the linear charge density on the inner surface $r = a$, and on the outer surface $r = b$.
4 (b) Determine the electric field $\vec{E}(r)$ for all r and sketch $\vec{E}(r)$ vs. r.

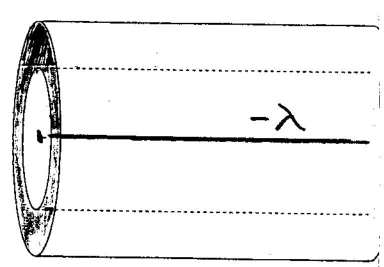
Now suppose a thin wire with negative linear charge density $-\lambda$ is placed inside the hollow cylinder, exactly along the axis of the cylinder, $r = 0$.

3 (c) For this situation, determine the charge density on the inner and outer surfaces of the cylindrical shell.
4 (d) Determine the electric field $\vec{E}(r)$ for all r and sketch $\vec{E}(r)$ vs. r for all r.

(a) & (b)



Total linear charge density $+\lambda$



(c) & (d)

Physics 102– Pledged Problem 3 Solution

I. The conducting spherical shell has total charge $+2Q$. Since the electric field inside the conductor must be zero, Gauss Law tells us that all the charge must reside on the outer surface. Otherwise, a Gaussian surface drawn *inside* the conductor would enclose charge, and we would have an electric field inside the conductor.

(a) The total charge on the inner surface of the conductor is zero. The total charge on the outer surface of the conductor is $+2Q$.

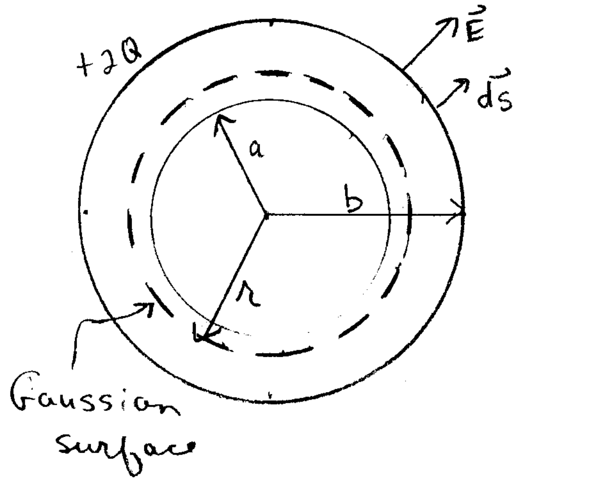
(b) Use Gauss Law and the spherical symmetry of the problem to determine the electric field. Draw a spherical Gaussian surface with the same center as the spherical shell.

$$\int \vec{E} \cdot d\vec{S} = \frac{Q_{enc}}{\epsilon_o}$$

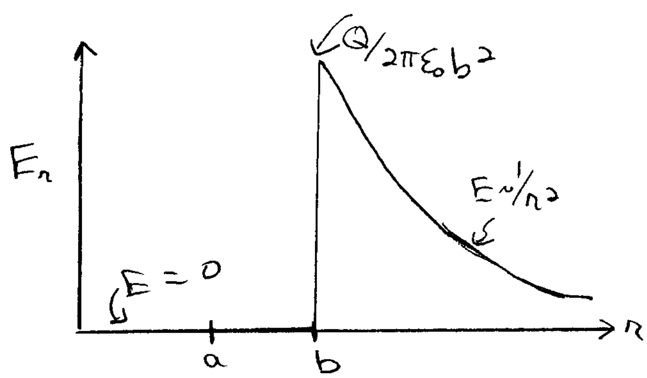
From the symmetry we know that the electric field $\vec{E}$ must be radially outward if it is non-zero. Then $\vec{E}$ and $d\vec{S}$ are parallel on the Gaussian surface. We also know that the magnitude of $\vec{E}$ is constant on the Gaussian surface, so we can call it E_r and pull it out of the integral. We get

$$4\pi r^2 E_r = Q_{enc}/\epsilon_o$$

For $r < b$ no charge is enclosed, so $\vec{E}$=0.
For $r > b$, the full charge of $+2Q$ is enclosed, so we have



$$E_r = \frac{2Q}{4\pi\epsilon_o r^2} = \frac{Q}{2\pi\epsilon_o r^2}$$



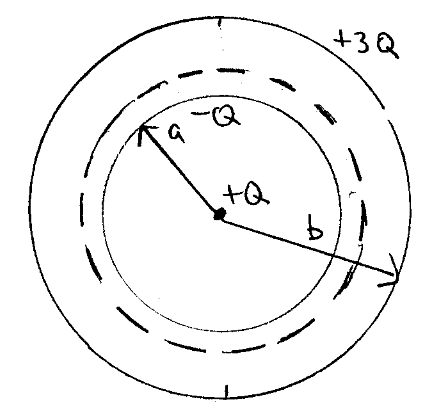
(c) Now suppose a charge $+Q$ is placed in the center of the hollow spherical shell. Draw a spherical Gaussian surface inside the conductor. The field must be zero everywhere inside the conductor, so the net charge enclosed must be zero. Therefore, there must be a charge of $-Q$ in the inner surface of the conductor, at $r = a$. Since the conductor still has a total charge $+2Q$, the outer surface $r = b$ must now have a charge of $+3Q$.

(d) Use Gauss Law as above.
For $r < a$, the total charge enclosed is $+Q$, so

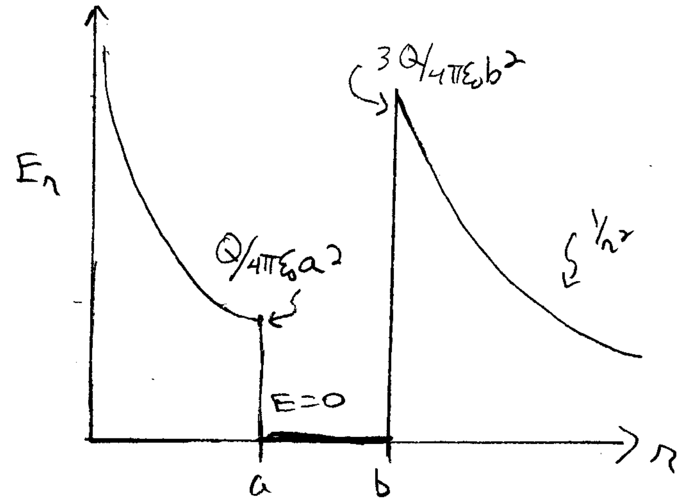
$$E_r = \frac{Q}{4\pi\epsilon_o r^2} \text{ for } r < a.$$

For $a < r < b$, we are inside the conductor and $E = 0$.

For $r > b$, the total charge enclosed is $3Q$, so



$E_r = \frac{3Q}{4\pi\epsilon_o r^2}$ for $r > b$.



(e) If the charge $+Q$ were not exactly in the center of the sphere, the *total* charges on the inner and outer surfaces would not change. The field inside the conductor is still zero, so the total charge on the inner surface of the conductor must still be $-Q$. And the total charge on the spherical shell is still $+2Q$, so the charge on the outer surface must still be $+3Q$. But these charges will no longer be uniformly distributed over the surfaces.

II. This problem is similar to I, but now we must use cylindrical symmetry. The Gaussian surfaces we will use will be cylinders rather than spheres. From symmetry arguements, the electric field from a long, uniformly charged cylinder is radially outward from the axis of the cylinder for positive charge and radially inward for negative charge.

(a) Since the electric field inside a conductor must be zero, the charge must all reside on the outer surface. The linear charge density on the inner surface is zero and on the outer surface it is $+\lambda$.

(b) To determine the electric field, use Gauss Law with a cylindrical Gaussian surface.

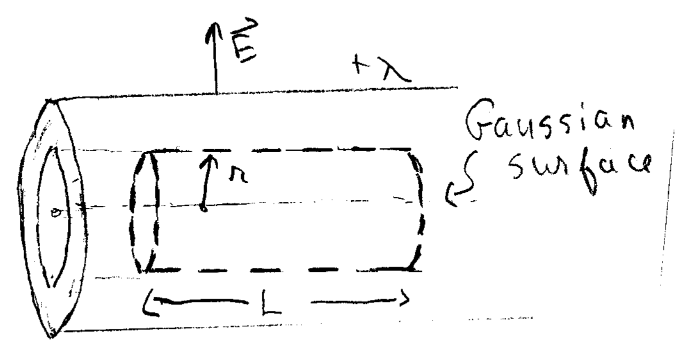
$$\int \vec{E} \cdot d\vec{S} = \frac{Q_{enc}}{\epsilon_o}$$

In this case $\vec{E}$ and $d\vec{S}$ will be perpendicular on the end caps of the cylinder, so the dot product makes that contribution zero. On the curved part of the Gaussian cylinder $\vec{E}$ and $d\vec{S}$ are parallel and the magnitude of $\vec{E}$ is constant, so we can pull E_r out of the integral. The charge enclosed will by λL, where λ is the linear charge density and L is the length of the Gaussian cylinder. Gauss' Law them becomes

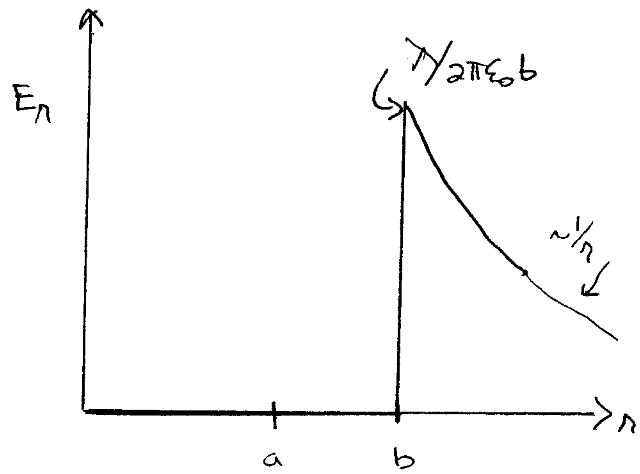
$$2\pi r L E_r = \lambda L/\epsilon_o.$$

Note that L cancels out, which it must. The length of the Gaussian cylinder is arbitrary, and our answer cannot depend on it!

For $r < b$, no charge is enclosed so the electric field is zero.
For $r > b$, the charge enclosed is λL, so we get



$E_r = \frac{\lambda}{2\pi\epsilon_o r}$ $(r>b)$



(c) Now suppose a wire with linear charge density $-\lambda$ is placed on the axis of the cylinder. Since the field inside the conductor must be zero, a positive charge density $+\lambda$ will move to the inner surface of the cylindrical shell. Since the shell carries a total charge density of $+\lambda$, the outer surface will have no charge in this case.

(d) For $r < a$, the charge from the wire is enclosed, so from the result above we have

$$E_r = \frac{-\lambda}{2\pi\epsilon_o r} \text{ for } r < a$$ field points radially inward.

For $r > a$, the total charge enclosed is zero, so the field is zero.