Physics 102 – Pledged Problem 2

Time allowed: **2 hours at a single sitting**

DUE 4PM MONDAY, January 28, 2008, in the boxes marked Phys 101-102 in the physics lounge. You may use your own textbook, your notes, and a non-programmed calculator. You may also consult the on-line solutions to the corresponding suggested problems. You should consult no other help. Show how you arrived at your answer; the correct answer by itself may not be sufficient.

Further instructions:

(a) Write legibly on **one** side of 8.5” x 11” white or lightly tinted paper.
(b) Staple all sheets together, including this one, in the upper left corner and make one vertical fold.
(c) On the outside, staple side up. Print your name in capital letters, your LAST NAME first followed by your FIRST NAME.
(d) Below your name, print the phrase “Pledged Problem 1”, followed by the due date.
(e) Also indicate **start time** and **end time.**
(f) Write and sign the pledge, with the understanding you may consult the materials noted above.

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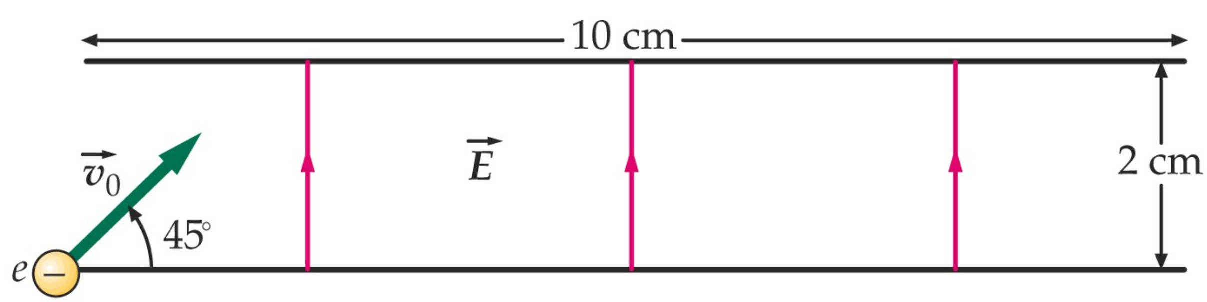
1. An electron starts at the position shown in the figure below with an initial speed $v_0 = 5 \times 10^6$ m/s at 45° to the x axis. The electric field is in the positive y direction and has a magnitude of 3.5×10^3 N/C.

(a) On which plate will the electron strike?

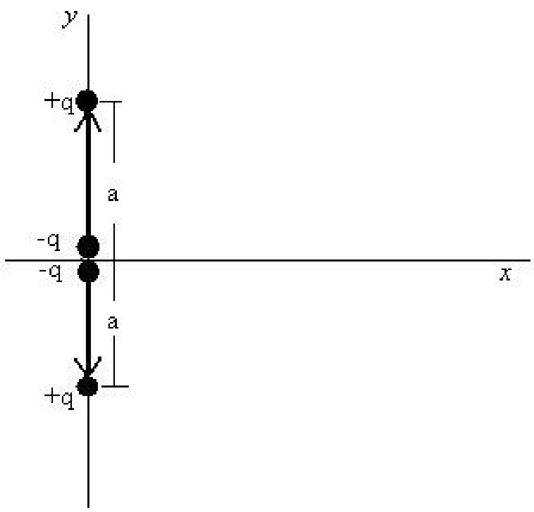
(a) Where will it strike the plate?



2. A quadrupole consists of two dipoles that are close together as shown in the figure below. The effective charge at the origin is -2q and the other charges on the *y* axis at y = a and y = -a are each +q.

(a) Find the electric field at a point on the *x* axis far away so that x >> a.

(b) Find the electric field on the *y* axis far away so that y >> a.



Solution – Pledged Problems #2

1. This should be recognized as an equation of motion problem, with the constant electric field, **E**, exerting a constant force and, therefore, causing the electron to undergo a constant acceleration in the negative y direction ($a = F/m_e = -eE/m_e$).

Several approaches can be taken to solve this problem. The equations of motion in the x and y directions are –

$x = v_x t = (v_0 \cos\theta)t$ and

$y = v_y t + 1/2at^2 = (v_0 \sin\theta)t - 1/2\ eE/m_e\ t^2$

By eliminating t from these equations, one obtains an expression for the position of y in terms of x, and then, by taking the derivative with respect to x, can determine where the electron will reach its highest point.

$y(x) = (\tan\theta)x - (eE/2m_e v_0^2 \cos^2\theta)x^2$.

$dy/dx = \tan\theta - (eE/m_e v_0^2 \cos^2\theta)x$

This equals 0 when $x = m_e v_0^2 \sin 2\theta/2eE$

And occurs at $y = m_e v_0^2 \sin^2\theta/2eE = \dfrac{(9.11\times10^{-31}\text{ kg})(5\times10^{6}\text{ m/s})^2 \sin^2 45°}{2(1.6\times10^{-19}\text{ C})(3.5\times10^{3}\text{ N/C})} = 1.02\,\text{cm}$.

Because the plates are separated by 2 cm, the electron doesn't hit the upper plate, but instead returns to the lower plate, hitting it when $y = 0$.

Another approach is to use the equation for vertical velocity as a function of distance –

$$v^2 = v_y{}^2 + a(y - y_0)$$

The maximum height occurs when $v = 0$, so that

$$y_{max} = -\frac{v_y{}^2}{a}$$

Plugging in the appropriate values produces the same y found above.

The position the electron strikes the lower plate can be solved using the range equation above, setting $y = 0$. This occurs at $x = m_e v_0^2 \sin 2\theta/eE$ (not coincidentally twice the horizontal distance at which the electron reaches its highest point). Substituting in numerical values gives

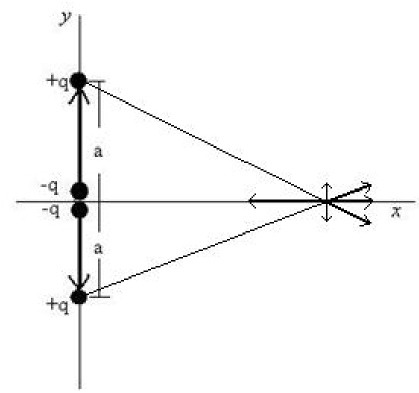
$$x = \frac{(9.11\times10^{-31}\text{ kg})(5\times10^{6}\text{ m/s})^2 \sin 90°}{(1.6\times10^{-19}\text{ C})(3.5\times10^{3}\text{ N/C})} = \boxed{4.07\,\text{cm}}$$

An alternative approach would be to find the time it took to reach y_{max}, using

$$y_{max} = y_0 + v_{0y}t + \frac{1}{2}at^2$$

The total flight time will be twice that value. The final x position can then be found by plugging this *t* into the equation of motion in the *x* direction, getting the same result as above.

2. (a) The law of superposition states that the total electric field, **E**, along the *x* axis will equal the sum of the fields associated with each charge. Symmetry considerations indicate that the vertical components of the electric fields produced by the 2 positive charges will cancel, so that only their horizontal components need be included.



$$\sum \vec{\boldsymbol{E}}_x = -\frac{2q}{4\pi\epsilon_0 x^2} + \frac{2q}{4\pi\epsilon_0(x^2 + a^2)^2}\frac{x}{\sqrt{x^2 + a^2}}$$

$$= -\frac{2q}{4\pi\epsilon_0 x^2} + \frac{2qx}{4\pi\epsilon_0 x^3(1 + (a/x)^2)^{3/2}}$$

Now, we can apply a generalized form of the binomial theorem we used in Suggested Problem 22-43, in which $(1+b)^n = 1 + nb + \frac{n(n-1)}{2}b^2 + \ldots$, to the last part of the denominator of the second term, keeping only the first, non-vanishing term (which will be the term proportional to b in this case).

$$(1 + (a/x)^2)^{-3/2} = 1 - \frac{3}{2}(a/x)^2$$

Inserting this into the equation above produces an electric field, $\overrightarrow{E_x}$, in the *x* direction –

$$= -\frac{2q}{4\pi\epsilon_0 x^2} + \frac{2qx}{4\pi\epsilon_0 x^3}(1 - \frac{3}{2}(a/x)^2)$$

$$= \frac{-3qa^2}{4\pi\epsilon_0 x^4}\hat{\imath}$$

(b) Along the *y* axis, the electric fields associated with the 3 charges will be –

$$\sum \vec{\boldsymbol{E}}_y = -\frac{2q}{4\pi\epsilon_0 y^2} + \frac{q}{4\pi\epsilon_0(y + a)^2} + \frac{q}{4\pi\epsilon_0(y - a)^2}$$

Again applying the binomial theorem, where n = -2 and b = a (similar to what we did with problem 22-43), we get

$$= -\frac{2q}{4\pi\epsilon_0 y^2} + \frac{q}{4\pi\epsilon_0 y^2(1 + \frac{a}{y})^2} + \frac{q}{4\pi\epsilon_0 y^2(1 - \frac{a}{y})^2}$$

$$= -\frac{2q}{4\pi\epsilon_0 y^2} + \frac{q}{4\pi\epsilon_0 y^2}\left[1 - 2\frac{a}{y} + 3\left(\frac{a}{y}\right)^2\right] + \frac{q}{4\pi\epsilon_0 y^2}\left[(1 - 2\left(-\frac{a}{y}\right) + 3\left(-\frac{a}{y}\right)^2\right]$$

$$= \frac{q}{4\pi\epsilon_0 y^2}\left(-2+1\ -2\frac{a}{y}+\ 3\left(\frac{a}{y}\right)^2+1+2\left(\frac{a}{y}\right)+\ 3\left(\frac{a}{y}\right)^2\right)$$

$$= \frac{6qa^2}{4\pi\epsilon_0 y^4}\hat{j}$$