Physics 102– Pledged Problem 2

Time allowed: **2 hours at a single sitting**

Due 5PM Monday, January 29, 2007, in the boxes marked Phys 101-102 in the physics lounge. You may use your own textbook, your notes, and a non-programmed calculator. You may also consult the on-line solutions to the corresponding suggested problems. You should consult no other help. Show how you arrived at your answer; the correct answer by itself may not be sufficient.

Further instructions:

(a) Write legibly on **one** side of 8.5" x 11" white or lightly tinted paper.
(b) Staple all sheets together, including this one, in the upper left corner.
(c)Make one vertical fold.
(d) On the outside, staple side up, print your name in capital letter, your LAST NAME first followed by your FIRST NAME.
(e) Below your name, print the phrase "Pledged Problem 2", followed by the due date.
(f) Also indicate **start time** and **end time.**
(g) Write and sign the pledge, with the understanding that you may consult the materials noted above.

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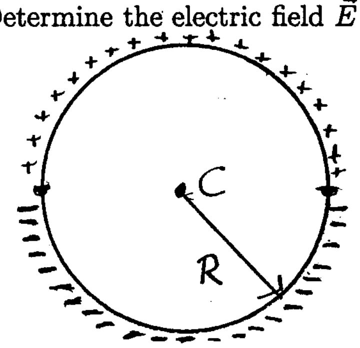
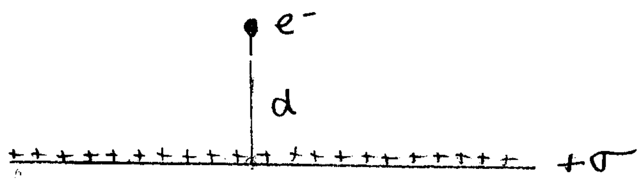
15 I. Two insulating rods, each of length L, carry uniform linear charge density along their length. One carries a total positive charge $+Q$, and the other carries a total negative charge $-Q$. The rods are each bent into semi-circles and joined to form a full circle of radius R, with an insulator between them so that no charge moves from one to the other. The upper half of the circle has positive charge, and the lower half has negative charge, as shown in the sketch below. Determine the electric field $\vec{E}$ at C, the center of the circle.



15 II. A very large, thin mesh is stretched flat so that it forms a planar surface. It carries a uniform positive charge density $+\sigma$. An electron moves near the surface of this material and is able to pass freely through the mesh without hitting it. The electron is released from rest a distance d above the surface, as shown below. Express your answers in terms of the charge of the electron $-e$, the mass of the electron m_e, the charge density on the mesh σ, the distance d, and possibly other constants.

3 (a) Determine the electric field at the location of the electron before it is released.
3 (b) Considering only the electric force (neglecting gravity) write down Newton's second law for the electron.
3 (c) Based on the form of your answer in (b), do you expect the electron to undergo simple harmonic motion?
3 (d) Taking $t = 0$ to be the time at which the electron is released from rest, determine the time at which the electron first reaches the mesh.
3 (e) When the electron reaches the mesh, it freely passes through. Describe the subsequent motion of the electron after it passes through the mesh.



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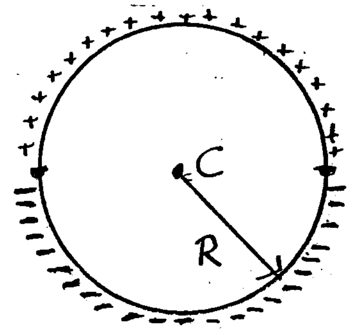
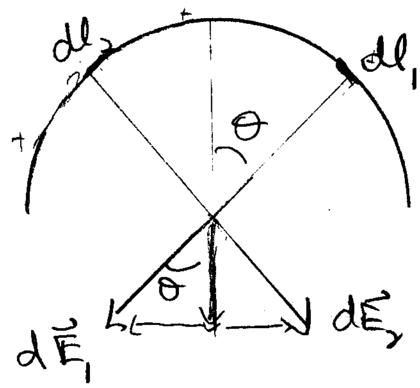
Phys 102

Pledged Problem 2.

I.



First consider just the top half, and consider two symmetrically-located line elements dl_1 and dl_2 giving rise to electric fields $d\vec{E}_1$ and $d\vec{E}_2$



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From the symmetry, it is clear that the x-components cancel and the y-components add.

$$dE_y = dE_{1y} + dE_{2y} = -2\frac{k\,dq}{R^2}\cos\theta$$

$dq = \lambda\,dl$ where λ is the linear charge density

$$\boxed{\lambda = \frac{Q}{\pi R}}$$

(2)

then

$$dE_y = \frac{-2k\lambda\, dl}{R^2}\cos\theta$$

$dl = R\,d\theta$
(polar coordinates)

Each line element dl_1 on the right side will have a matching element dl_2 on the left side. So we just need to integrate this expression for θ from $0 \to \pi/2$

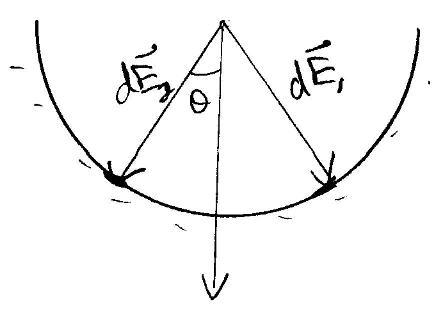
$$E_y = \int dE_y = -\frac{2k\lambda}{R^{\cancel{2}}}\int_0^{\pi/2} \cancel{R}\cos\theta\, d\theta$$

$$= -\frac{2k\lambda}{R}\underbrace{\sin\theta\Big|_0^{\pi/2}}_{=1} = -\frac{2k\lambda}{R}$$

Substitute back in for λ:

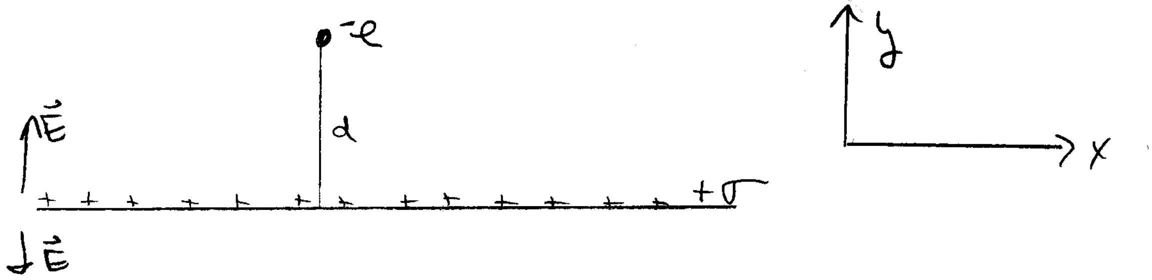
$$\underset{\text{(top half)}}{\vec{E}} = -\frac{2kQ}{\pi R^2}\hat{j} \quad (5)$$

Now consider the bottom half which is negatively charged. Exactly the same symmetry applies, and the direction is also the same! So the contribution from the bottom half is equal to the contribution from the top half!



$$\boxed{\vec{E}_{TOT} = -\frac{4kQ}{\pi R^2}\hat{j}} \quad (5)$$

II.



(a) Treat the mesh as an infinite plane of charge. As done in class & in the electric field above & below an infinite plane of charge is

$$\vec{E} = \frac{\sigma}{2\varepsilon_0}\hat{j} \qquad\qquad \vec{E} = -\frac{\sigma}{2\varepsilon_0}\hat{j}$$

(above) (below)

The electron is released from above, so

$$\boxed{\underset{(\text{electron})}{\vec{E}} = \frac{\sigma}{2\varepsilon_0}\hat{j}}$$

(b) $\vec{F} = m\vec{a} = q\vec{E} = -\dfrac{e\sigma}{2\varepsilon_0}\hat{j}$

$$\boxed{m_e\vec{a} = \frac{-e\sigma}{2\varepsilon_0}\hat{j}}$$

The force on the electron due to $\vec{E}$ is vertically downward.

(c) Simple harmonic motion is of the form

$\dfrac{d^2x}{dt^2} + \omega^2 x = 0$, or another way to denote the condition for SHM is that the restoring force is linear in the coordinate

$$F = -ky = m\frac{d^2y}{dt^2} \qquad k = \text{constant.}$$

The expression in (b) does not satisfy the condition for SHM. The restoring force is constant.

(d) This is a simple kinematic problem with constant acceleration

$$a_y = \frac{-e\sigma}{2\varepsilon_0 m_e}$$

$$y = \frac{1}{2}a_y t^2 = -d.$$

$$t^2 = \frac{2d\,(2\varepsilon_0 m_e)}{e\sigma}.$$

$$\boxed{t = \left(\frac{4d\,\varepsilon_0 m_e}{e\sigma}\right)^{1/2}}$$

Check units:

$$[t^2] \sim \frac{C^2}{N\text{-}\cancel{m^2}}\cdot\frac{m\,kg\,\cancel{m^2}}{C\,C}$$

$$\sim \frac{kg\text{-}m}{kg\text{-}m/s^2} \sim sec^2 \checkmark$$

$[t^2] \sim sec^2 \checkmark$

(e) When the electron passes through the mesh, $\vec{E}$ changes direction. The electron will continue to move in the $-y$ direction because of its velocity as it passes through the mesh. But the force will be in the $+y$ direction. The electron will come to a stop a distance d below the mesh, reverse direction & pass through the mesh again. The motion is periodic, but not simple harmonic