Physics 102 – Pledged Problem 1

Time allowed: **2 hours at a single sitting**

Due 4PM Tuesday, January 22, 2008, in the boxes marked Phys 101-102 in the physics lounge. You may use your own textbook, your notes, and a non-programmed calculator. You may also consult the on-line solutions to the corresponding suggested problems. You should consult no other help. Show how you arrived at your answer; the correct answer by itself may not be sufficient.

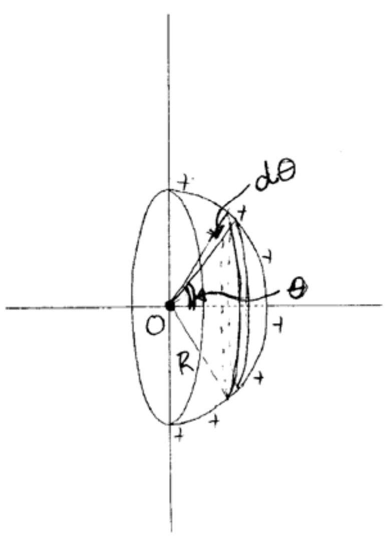
Further instructions:

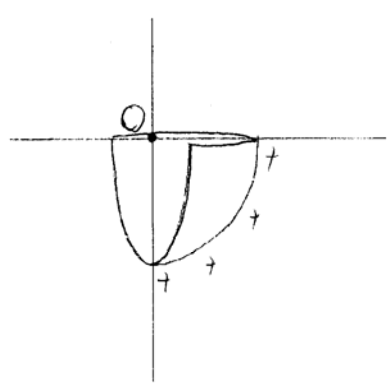
(a) Write legibly on **one** side of 8.5" x 11" white or lightly tinted paper.
(b) Staple all sheets together, including this one, in the upper left corner and make one vertical fold.
(c) On the outside, staple side up. Print your name in capital letters, your LAST NAME first followed by your FIRST NAME.
(d) Below your name, print the phrase "Pledged Problem 1", followed by the due date.
(e) Also indicate **start time** and **end time.**
(f) Write and sign the pledge, with the understanding you may consult the materials noted above.

I. Consider a uniform hemispherical charge distribution comprised of positive charge distributed uniformly over a hemisphere of radius R with charge density (charge per unit area) σ.

(a) What is the direction of the electric field at O, the center of curvature of the hemisphere?
(b) If the electric field at O is 10 N/C, what is the charge density σ? (Hint: in calculating the field produced by the charge distribution, consider the charge distribution as a series of concentric rings of charge subtending angles $d\theta$ at O)
(c) If the upper half of the charge distribution is removed, as shown below, what is now the electric field (magnitude and direction) at O? (Hint: superposition).

Note: $\int \sin\theta \cos\theta d\theta = \frac{1}{2}\sin^{-2}\theta$





Solution – Pledged Problem #1

(a) By symmetry, the resultant total field will be along the – x axis.

(b) Consider initially the electric field due to the ring of charge of angle width $d\theta$ at θ. The incremental amount of charge associated with that ring, dQ, is the charge density σ times its area – $2\pi r \times R d\theta$ (where R is the radius of the hemisphere and $r(= \sin\theta\, R)$ is the radius of the circle of charge), or

$$dQ = \sigma 2\pi R^2 \sin\theta d\theta.$$

The incremental electric field in the x direction associated with that ring of charge will be –

$$dE_x = dE_{tot} = \frac{dQ}{4\pi\varepsilon_0 R^2}\cos\theta = \frac{\sigma 2\pi R^2 \sin\theta d\theta}{4\pi\varepsilon_0 R^2}\cos\theta = \frac{\sigma}{2\varepsilon_0}\sin\theta\cos\theta\, d\theta$$

Integration will take place from $\theta = 0\ to\ {}^{\pi}/_{2}$, so the total field will be

$$E_{tot} = \frac{\sigma}{2\varepsilon_0}\int_0^{\pi/2}\sin\theta\cos\theta\, d\theta = \frac{\sigma}{2\varepsilon_0}\left({}^{1}/_{2}\left(\sin^2{}^{\pi}/_{2} - \sin^2 0\right)\right) = \frac{\sigma}{4\varepsilon_0}$$

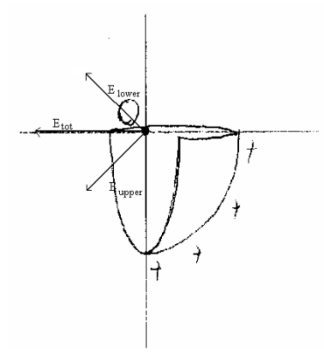
in the negative x direction. Note, the integration hint, $\int \sin\theta\cos\theta d\theta = \frac{1}{2}\sin^{-2}\theta$

should not have had the negative sign; it should have been $\int \sin\theta\cos\theta d\theta = \frac{1}{2}\sin^{2}\theta$. No points will be deducted for using the incorrect integrated term.

With E = 10N/C, this provides a charge density of

$$\sigma = E_{tot}4\varepsilon_0 = 10\ {}^{N}/_{C} \times 4 \times 8.85 \times 10^{-12}\ {}^{C^2}/_{Nm^2} = 3.54 \times 10^{-10}\ {}^{C}/_{m^2}$$

(c) The superposition principal states that the total electric field of a charge distribution is equal to the vector sum of the fields associated with the component parts of the charge distribution. In this case, we can treat the components as the upper and lower halves of the hemisphere, and we're concerned with the field associated with only the lower half. Symmetry considerations indicate that the fields will be at ±45° to the *x* axis, will have the same magnitude and will sum to equal the total field calculated in part (b) (E_{tot} in the figure to the right). Therefore, the electric field component associated with the lower half of the hemisphere will have a value equal to



$$E = E_{tot} \times \cos 45 = 10\ {}^{N}/_{C} \times \frac{\sqrt{2}}{2} = 7.1 N/C$$

pointing at 45° above the horizontal negative *x* axis (labeled E_{lower} in the figure to the right).