Physics 102– Pledged Problem 11

Time allowed: **2 hours at a single sitting**

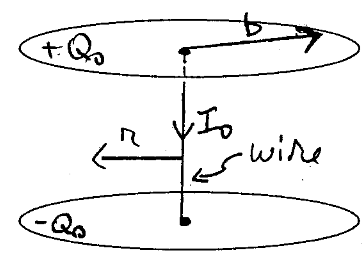
Due 5PM Thursday, April 27, 2006, in the boxes marked Phys 101-102 in the physics lounge. You may use your own textbook, your notes, and a non-programmed calculator. You may also consult the on-line solutions to the corresponding suggested problems. You should consult no other help. Show how you arrived at your answer; the correct answer by itself may not be sufficient.

Further instructions:

(a) Write legibly on **one** side of 8.5" x 11" white or lightly tinted paper.
(b) Staple all sheets together, including this one, in the upper left corner and make one vertical fold.
(c) On the outside, staple side up, print your name in capital letters, your LAST NAME first followed by your FIRST NAME.
(d) Below your name, print the phrase "Pledged Problem 11", followed by the due date.
(e) Also indicate **start time** and **end time.**
(f) Write and sign the pledge, with the understanding that you may consult the materials noted above.

I. A parallel plate capacitor has circular plates of radius b. At $t = 0$ it is charged with a total charge Q_o and then disconnected from the voltage source. But the capacitor has a leak! A very thin wire, right in the center of the plates, is slowly discharging the capacitor with a current I_o which we can take to be constant. Express your answers in terms of Q_o, b, I_o and possibly other constants. The coordinate r measures the distance from the axis of the capacitor.

(a) Determine the magnetic field $\vec{B}_W(r)$ due to the current through wire. Be sure to indicate the both the direction of the field and it's r-dependence.
(b) Determine the electric field in the capacitor as a function of time $\vec{E}(t)$ as the capacitor discharges.
(c) Determine the displacement current $I_D(r)$ in the capacitor, indicating both the direction and r-dependence.
(d) Determine the magnetic field $\vec{B}_D(r)$ due to the displacement current, indicating both the direction and r-dependence.

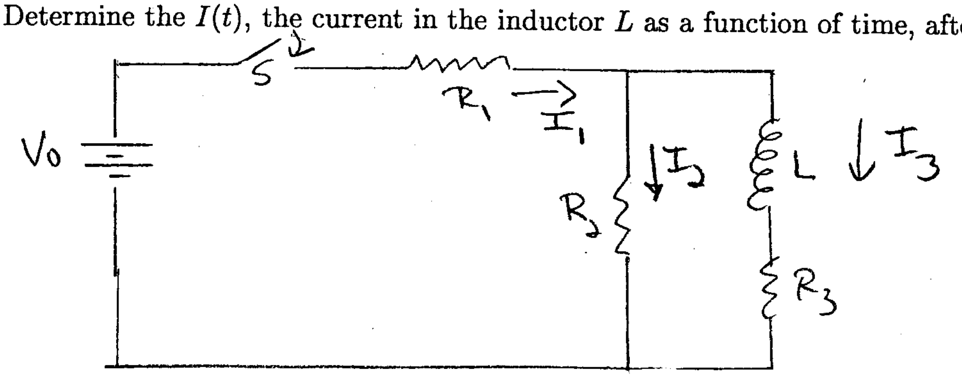


II. In the circuit shown below, the switch is initially opened, then at $t = 0$ it is closed.

(a) Determine the current through each resistor and the potential drop across the inductor $\mathcal{E}_L$ at $t = 0$.
(b) Determine the current through each resistor and $\mathcal{E}_L$ at $t \to \infty$.

After the switch has been closed for a long time, it is opened.

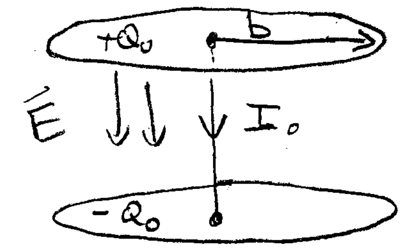
(c) Determine the three currents immediately after the switch is opened.
(d) As a numerical example, let V_0=10V, $R_1 = 10\Omega$, $R_3 = 5\Omega$, and $R_2 = 1000\Omega$. What is the potential drop across R_2 immediately after the switch is opened?
(e) Determine the $I(t)$, the current in the inductor L as a function of time, after the switch is opened.



Phys 102

Pledged Problem 11

I.



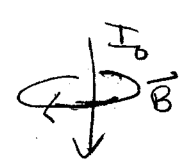
(a) Apply the extended form of Ampere's Law:

$$\oint \vec{B} \cdot d\vec{l} = \mu_0 I_{encl} + \mu_0 \varepsilon_0 \frac{d\Phi_E}{dt}$$

In this case we have both a real current plus a displacement current. Both contribute to the magnetic field $\vec{B}$.

First consider just the current through the wire:

$$\oint \vec{B}_w \cdot d\vec{l} = \mu_0 I_0$$



Our geometry has cylindrical symmetry, so we know $\vec{B}$ forms concentric loops around the wire. The direction will be clockwise as seen from above.

We can use Ampere's law in this case because, even though the wire segment is finite, there are no other currents nearby.

$$\oint \vec{B} \cdot d\vec{l} = 2\pi r B = \mu_0 I_0$$

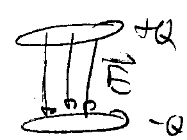
$$\boxed{\underset{\text{(due to wire)}}{\vec{B}_w} = \frac{\mu_0 I_0}{2\pi r}}$$

Direction is cw as viewed from above.

(b) $\vec{E}$ will be constant between the plates,

$$|\vec{E}| = \frac{\sigma}{\varepsilon_0} = \frac{Q}{\varepsilon_0 \pi b^2}$$

Direction is <u>down</u> from $+Q_0$ to $-Q_0$ plates.

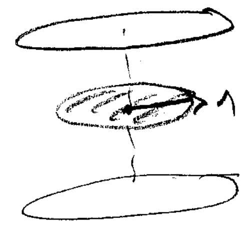


But Q is decreasing,

$$Q(t) = Q_0 - I_0 t$$

$$\boxed{\vec{E}(t) = \frac{Q_0 - I_0 t}{\varepsilon_0 \pi b^2} \quad \text{down}}$$

(c) $I_d = \varepsilon_0 \dfrac{d\phi_E}{dt}$



$\phi_E = \int \vec{E} \cdot d\vec{A}$

for $r<b$, $\vec{E}$ is constant in space & Area $= \pi r^2$

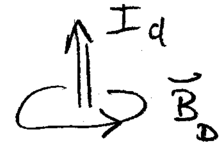
$$\phi_E = E(\pi r^2) = \frac{(Q_0 - I_0 t)\pi r^2}{\varepsilon_0 \pi b^2}$$

$$\varepsilon_0 \frac{d\phi_E}{dt} = \frac{-I_0 \cancel{\pi} r^2}{\cancel{\pi} b^2} = \boxed{\frac{-I_0 r^2}{b^2} = I_d(r) \text{ - direction is up}}$$

Note the (−) sign! Although $\vec{E}$ is down, the change in $\vec{E}$ is up & I_d is therefore up

(d) from Ampere's law, we can now find $\vec{B}_D$

$$\int \vec{B} \cdot d\vec{l} = \mu_0 I_d$$

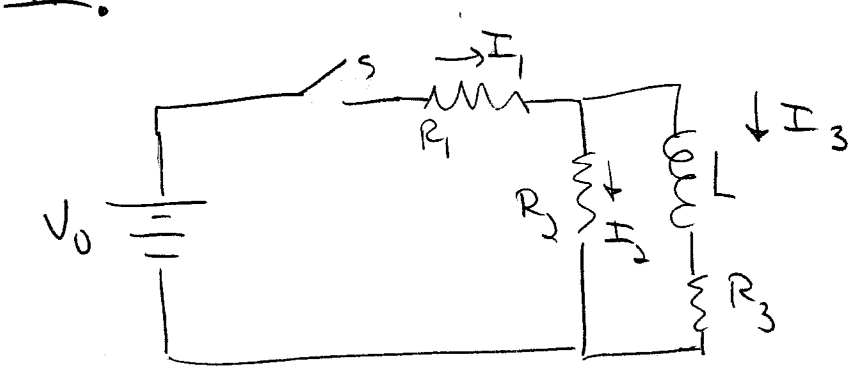


$$2\pi r B_d(r) = \mu_0 I_d$$

$$B_d = \frac{\mu_0 I_d}{2\pi r}$$

$$\boxed{|\vec{B}_d(r)| = \frac{\mu_0 I_0 r^{\cancel{2}}}{2\pi \cancel{r} b^2} = \frac{\mu_0 I_0 r}{2\pi b^2} \text{ - direction is counter clockwise loops}}$$

II.



(a) Switch is closed at $t=0$.

Immediately after the switch is closed, L acts like an open circuit, since current in an inductor cannot change quickly. Then

$\boxed{I_3 = 0,}$ $\boxed{I_1 = I_2 = \dfrac{V_0}{R_1 + R_2}}$

(b) for $t \to \infty$, L acts like a short - ignoring any internal resistance. R_2 & R_3 are then in parallel &

$$R_{23} = \frac{R_2 R_3}{R_2 + R_3} \qquad R_{TOT} = R_1 + R_{23} = \frac{R_1R_2 + R_1R_3 + R_2R_3}{R_2 + R_3}$$

$$\boxed{I_1 = \frac{V_0}{R_{TOT}} = \frac{V_0(R_2+R_3)}{R_1R_2 + R_1R_3 + R_2R_3}}$$

I_1 splits between R_2 & R_3 in such a way as to give the same voltage drop across them since they are in parallel.

$$I_2 R_2 = I_3 R_3 = R_3(I_1 - I_2)$$

$$I_2(R_2 + R_3) = R_3 I_1$$

$$R_2(I_1 - I_3) = R_3 I_3$$

$$R_2 I_1 = I_3(R_2 + R_3)$$

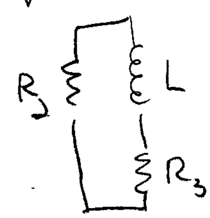
$$I_2(\cancel{R_2+R_3}) = \frac{R_3(\cancel{R_2+R_3})V_0}{R_1R_2+R_1R_3+R_2R_3} \qquad I_3 = \frac{R_2(R_2+R_3)V_0}{R_1R_2+R_1R_3+R_2R_3}$$

$$\boxed{I_2 = \frac{R_3V_0}{R_1R_2+R_1R_3+R_2R_3}} \qquad \boxed{I_3 = \frac{R_2V_0}{R_1R_2+R_1R_3+R_2R_3}}$$

(c) After a long time, the switch is opened again. Immediately after the switch is opened:

$$I_1 = 0 \qquad \boxed{I_2 = I_3 = \frac{R_2V_0}{R_1R_2+R_1R_3+R_2R_3}}$$

The current through the inductor can't change quickly, so the current through L is shunted through R_2 & R_3 in series.



(d) $V_2 = I_2R_2 = \frac{(V_0R_2)R_2}{R_1R_2+R_1R_3+R_2R_3}$

Let $V_0 = 10V$, $R_1 = 10\Omega$, $R_3 = 5\Omega$, $R_2 = 1000\Omega$

$$V_2 = \frac{10(10^3)(10^3)}{10^4+50+5000} = \frac{10^7}{1.5\times10^4} = .667\times10^3 = 667\,V$$

$\boxed{V_2 = 667\,V}$!

Even though the initial voltage in the battery was only 10 V, the voltage across R_2 is <u>much</u> <u>larger</u> when the switch is opened!

(e) The current in L decays with a time ~~constant~~

$$\tau = L/R_{eff} = \frac{L}{R_2+R_3}$$

The effective resistance that L discharges through is the series sum of R_2 & R_3

$$\boxed{I(t) = I_0\, e^{-t(R_2+R_3)/L} \quad \text{with } I_0 = \frac{V_0 R_2}{R_1R_2+R_1R_3+R_2R_3}}$$