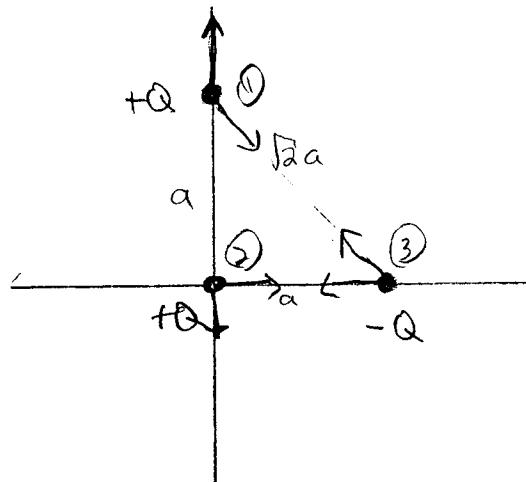


Phy 102  
Pledged Problem 1

I.



(a) Force on each due to the other two

Label charges as shown above

$$\vec{F}_1 = \underbrace{\frac{kQ^2}{a^2} \hat{j}}_{\text{due to } +Q \text{ at origin}} + \underbrace{\frac{kQ^2}{2a^2} (\hat{i} - \hat{j})}_{\text{due to } -Q \text{ at } x=a} \frac{1}{\sqrt{2}}$$

$$\boxed{\vec{F}_1 = \frac{kQ^2}{a^2} \left[ \frac{1}{\sqrt{2}} \hat{i} + \left(1 - \frac{1}{\sqrt{2}}\right) \hat{j} \right]} \quad \text{force on } ①$$

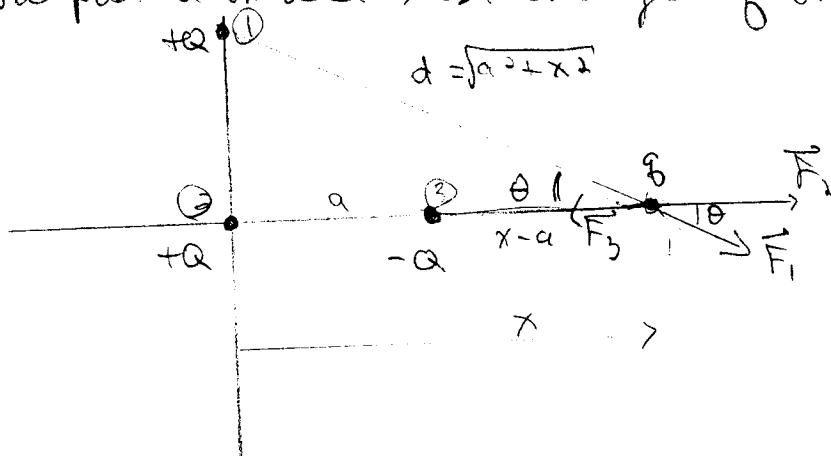
$$\vec{F}_2 = \underbrace{\frac{kQ^2}{a^2} (-\hat{i})}_{\text{due to } +Q \text{ at } y=a} + \underbrace{\frac{kQ^2}{a^2} \hat{i}}_{\text{due to } -Q \text{ at } x=a}$$

$$\boxed{\vec{F}_2 = \frac{kQ^2}{a^2} (\hat{i} - \hat{j})} \quad \text{force on } ②$$

$$\vec{F}_3 = \underbrace{\frac{kQ^2}{a^2}(-\hat{i})}_{\text{due to } +Q \text{ at origin}} + \underbrace{\frac{kQ^2}{2a^2}(-\hat{i} + \hat{j})}_{\text{due to } +Q \text{ at } y=a} \frac{1}{\sqrt{2}}$$

$$\boxed{\vec{F}_3 = \frac{kQ^2}{a^2} \left[ \left(1 + \frac{1}{2}\right)(-\hat{i}) + \frac{1}{2\sqrt{2}}\hat{j} \right]} \quad \text{force on ③}$$

(b) Now put a small test charge  $+q$  on the  $x$ -axis



(i) for  $x > a$  the three contributions to the force are:

$$\vec{F}_2 = \frac{kQq}{x^2} \hat{j} \quad \vec{F}_3 = \frac{kQq}{(x-a)^2} (-\hat{i})$$

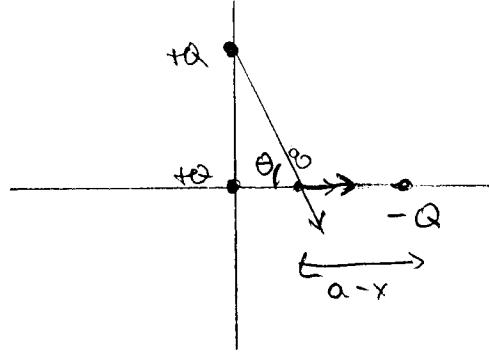
$$\vec{F}_1 = \frac{kQq}{a^2+x^2} \left[ \cos\theta \hat{i} - \sin\theta \hat{j} \right] \quad \text{with } \cos\theta = \frac{x}{\sqrt{a^2+x^2}} \quad \sin\theta = \frac{a}{\sqrt{a^2+x^2}}$$

$$\vec{F}_{tot}^b = \vec{F}_1 + \vec{F}_2 + \vec{F}_3$$

$$\boxed{\vec{F}_{tot}^b = kQq \left[ \left( \frac{1}{x^2} - \frac{1}{(x-a)^2} + \frac{x}{(a^2+x^2)^{3/2}} \right) \hat{i} + \frac{a}{(a^2+x^2)^{3/2}} \hat{j} \right]}$$

for  $x > a$

(ii) for  $0 < x < a$  the direction of  $\vec{F}_3$  changes,



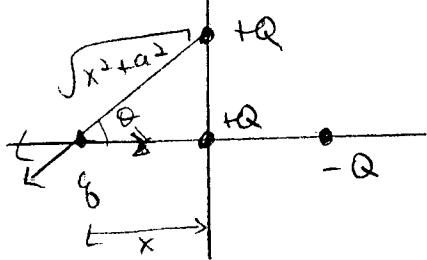
otherwise the expressions for the forces are the same

$$\vec{F}_1 = \frac{kQq}{x^2+a^2} (\cos\theta \hat{i} - \sin\theta \hat{j}) \quad \vec{F}_2 = \frac{kQq}{x^2} \hat{i}$$

$$\vec{F}_3 = \frac{kQq}{(a-x)^2} \hat{i}$$

$$\vec{F}_{\text{tot}} = kQq \left[ \left( \frac{x}{(a^2+x^2)^{3/2}} + \frac{1}{x^2} + \frac{1}{(a-x)^2} \right) \hat{i} - \frac{a}{(x^2+a^2)^{3/2}} \hat{j} \right] \quad | \text{ } 0 < x < a$$

(iii) for  $x < 0$  the directions of  $\vec{F}_1$  and  $\vec{F}_2$  change!



$$\vec{F}_1 = \frac{kQq}{x^2+a^2} (-\cos\theta \hat{i} - \sin\theta \hat{j})$$

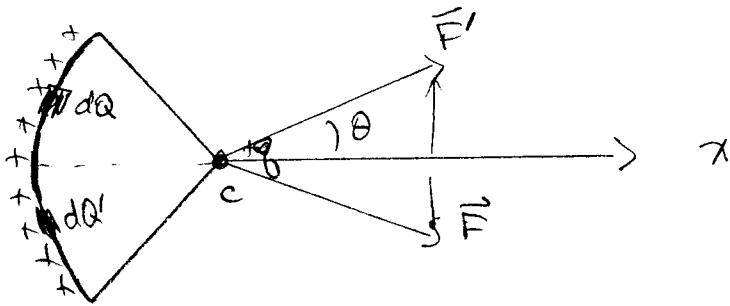
$$\vec{F}_2 = \frac{kQq}{x^2} (-\hat{i})$$

$$\vec{F}_3 = \frac{kQq}{(|x|+a)^2} \hat{i}$$

$$\vec{F}_{\text{tot}} = kQq \left[ \left( \frac{1}{(|x|+a)^2} - \frac{1}{(x^2+a^2)^{3/2}} - \frac{1}{x^2} \right) \hat{i} - \frac{a}{(x^2+a^2)^{3/2}} \hat{j} \right] \quad | \text{ } x < 0$$

Since  $x < 0$ , I use  $|x|$  for the distance.

For example, the distance between  $+Q$  and  $-Q$  is  $a + |x|$ .



$$(a) \lambda = \frac{Q}{L} = \frac{Q}{2\pi R/4} = \frac{2Q}{\pi R} \quad L = \frac{2\pi R}{4}$$

$$\boxed{\lambda = \frac{2Q}{\pi R} \text{ C/m}}$$

(b) For any charge element  $dQ$  there is a corresponding element  $dQ'$  such that the  $y$ -components of the forces cancel. So the net force is in the  $+x$  or horizontal direction only.

(c) Using (b), we need to consider only the  $x$ -component of the forces.

$$dF_x = \frac{kq dQ}{R^2} \cos\theta \quad dQ = \lambda ds = \lambda R d\theta$$

Integrate over  $\theta$ :

$$F_x = \int dF_x = \frac{kq}{R^2} \int_{-\pi/4}^{\pi/4} \lambda R \cos\theta d\theta$$

The range of integration is  $-\pi/4 \rightarrow +\pi/4$  ( $-45^\circ$  to  $+45^\circ$ )

$$F_x = \left. \frac{kq \lambda}{R} \sin\theta \right|_{-\pi/4}^{\pi/4} = \frac{kq \lambda}{R} \cdot \frac{2}{\sqrt{2}} \quad \text{use (a) for } \lambda$$

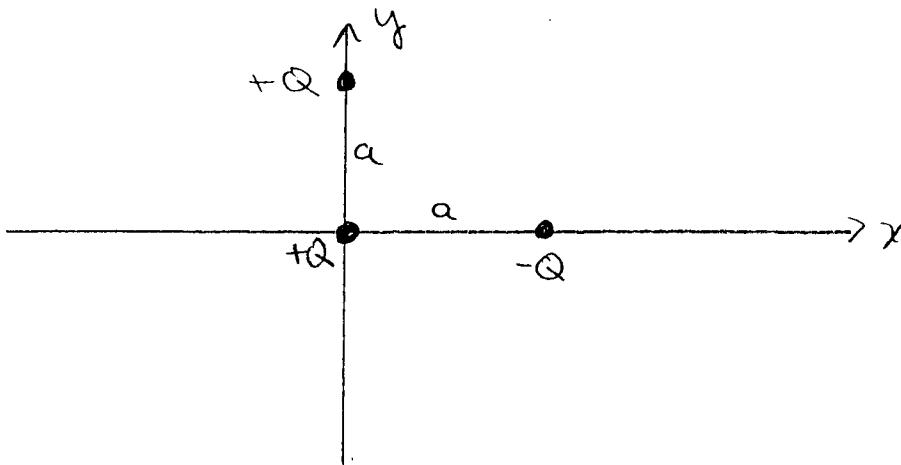
$$F_x = \frac{kq 2}{\sqrt{2} R} \cdot \frac{2Q}{\pi R} = \boxed{\frac{4kq Q}{\sqrt{2} \pi R^2}} = F_x$$

18 I. Three point charges are placed as shown in the sketch below. One has positive charge  $+Q$  and is placed at the origin, a second has positive charge  $+Q$  and is placed on the  $y$ -axis a distance  $a$  from the origin. The third charge, which is negative and has value  $-Q$ , is placed on the  $x$ -axis a distance  $a$  from the origin.

9 (a) Determine the net force on each of the three charges due to the other two.

(b) Determine the force  $\vec{F}(x)$  on a small positive test charge  $+q$  placed at an arbitrary point  $x$  on the  $x$ -axis in the three regions:

- 3 (i)  $x > a$
- 3 (ii)  $0 < x < a$
- 3 (iii)  $x < 0$ .



19 II. A thin wire is bent to form a quarter circle of radius  $R$ , as sketched below. The wire carries a total positive charge  $+Q$ , uniformly distributed over its length. A small test charge  $+q$  is located at the center of the circle  $C$ .

9 (a) Determine the linear charge density  $\lambda$  on the wire, in terms of  $Q$  and  $R$ .

9 (b) Using symmetry arguments, determine the *direction* of the force on  $+q$ .

9 (c) Determine the magnitude of the force on  $+q$  in terms of  $Q$ ,  $R$ , and other constants.

