Physics 102– Pledged Problem 8

Time allowed: **2 hours at a single sitting**

Due 5PM Monday, March 28, 2006, in the boxes marked Phys 101-102 in the physics lounge. You may use your own textbook, your notes, and a non-programmed calculator. You may also consult the on-line solutions to the corresponding suggested problems. You should consult no other help. Show how you arrived at your answer; the correct answer by itself may not be sufficient.

Further instructions:

(a) Write legibly on **one** side of 8.5" x 11" white or lightly tinted paper.

(b) Staple all sheets together, including this one, in the upper left corner and make one vertical fold.

(c) On the outside, staple side up, print your name in capital letters, your LAST NAME first followed by your FIRST NAME.

(d) Below your name, print the phrase "Pledged Problem 8", followed by the due date.

(e) Also indicate **start time** and **end time.**

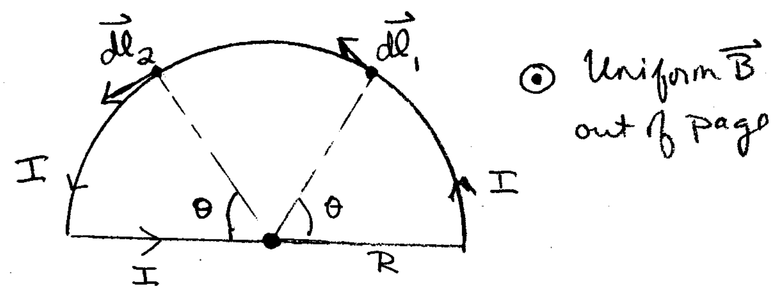
(f) Write and sign the pledge, with the understanding that you may consult the materials noted above.

I. The current loop shown below lies in the horizontal plane and consists of a straight segment of length $2R$ and a semicircular segment of radius R. A current I flows through the loop in the direction shown. A uniform magnetic field $\vec{B}$ points out of the page as shown.

(a) Determine the force on the horizontal section of wire due to the magnetic field.

(b) Determine the net force on two small, symmetric elements of current $\vec{dl}_1$ and $\vec{dl}_2$ on the semicircle, as shown in the figure below. What symmetry arguements can you use to simplify the result?

(c) Based on the result in (b), determine the total force on the semicircular part of the current loop and compare to your answer in (a).



II. Protons and deuterons (each with charge $+e$) and alpha (α) particles (with charge $+2e$) of the same kinetic energy enter a uniform magnetic field $\vec{B}$ that is perpendicular to their velocities. Make the approximation that $m_\alpha = 2m_d = 4m_p$.

(a) Let r_p, r_d and r_α be the radii of their circular orbits. Determine the ratios r_d/r_p and r_α/r_p.

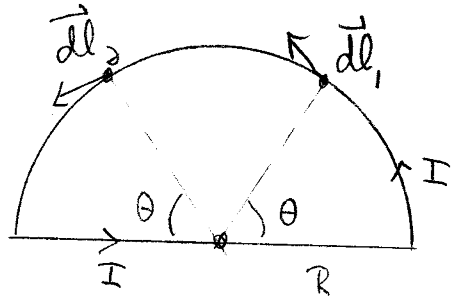
(b) Let T_p, T_d, and T_α be the periods of rotation for the particles. Determine the ratios T_d/T_p and T_α/T_p.

(c) As a numerical example, calculate the radius of the orbit for a proton if the magnetic field is 0.5T and the velocity of the proton is 1% of the speed of light.

Phys 102

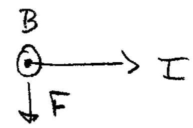
Pledged Problem 8

I.



$\odot \vec{B}$ out of page

(a) $\vec{F} = I\vec{L} \times \vec{B}$

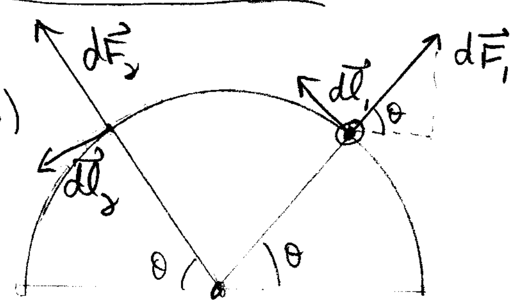


$L = 2R$

$|\vec{F}| = 2IRB$ Direction is <u>downward</u>, or $-\hat{j}$

$$\boxed{\vec{F} = -2IRB\,\hat{j}}$$

(b)



from symmetry we see that the horizontal components of $d\vec{F}_1$ and $d\vec{F}_2$ cancel and vertical components add.

$|dF_1| = |dF_2| = I\,\vec{dl} \times B$

$(dF_1 + dF_2)_y = 2I\,dl \sin\theta\, B$ $\qquad dl = R\,d\theta$

$$\boxed{(dF_1 + dF_2)_y = 2IRB\sin\theta\, d\theta}$$

(c) Now just integrate over $d\theta$. Be careful of the limits! Since I have added $d\vec{F}_1$ and $d\vec{F}_2$, I take θ from $0 \rightarrow \pi/2$ only.

$$|F_{upper}| = 2IRB \int_0^{\pi/2} \sin\theta \, d\theta = 2IR \underbrace{(-\cos\theta)\Big|_0^{\pi/2}}_{=1}$$

$$\boxed{\vec{F}_{upper} = 2IRB \, \hat{j}}$$

Note this is the same magnitude as in (a) but opposite direction!

II.

Proton: charge $+e$ deuteron charge $+2e$
mass m_p mass $2m_p$

alpha: charge $+2e$
mass $4m_p$

If they have the same kinetic energy K, the velocities will be:

$$v_p = \sqrt{\frac{2K}{m_p}} \qquad v_d = \sqrt{\frac{2K}{2m_p}} \qquad v_\alpha = \sqrt{\frac{2K}{4m_p}} = \sqrt{\frac{K}{2m_p}}$$

(a) Radius of curvature:

$$\frac{mv^2}{r} = qvB \qquad r_p = \frac{m_p\left(\frac{2K}{m_p}\right)^{1/2}}{eB} \qquad r_d = \frac{2m_p\left(\frac{K}{m_p}\right)^{1/2}}{eB}$$

$$r = \frac{mv}{qB} \qquad r_\alpha = \frac{4m_p\left(\frac{K}{2m_p}\right)^{1/2}}{2eB}$$

$$\frac{r_d}{r_p} = \frac{2\cancel{m_p}\left(\frac{K}{\cancel{m_p}}\right)^{1/2}}{\cancel{m_p}\left(2\cancel{K}/\cancel{m_p}\right)^{1/2}} = \frac{2}{\sqrt{2}} = \boxed{\sqrt{2} = \frac{r_d}{r_p}}$$

$$\frac{r_\alpha}{r_p} = \frac{4\cancel{m_p}\left(\frac{K}{2\cancel{m_p}}\right)^{1/2}}{2\cancel{m_p}\left(2\cancel{K}/\cancel{m_p}\right)^{1/2}} = 2\left(\frac{1}{2}\right) = \boxed{1 = \frac{r_\alpha}{r_p}}$$

(b) $T = \frac{2\pi r}{v} = \frac{2\pi m \cancel{v}}{qB\cancel{v}} = \frac{2\pi m}{qB}$

Note that the period T is independent of v!

$$\frac{T_d}{T_p} = \frac{\cancel{2\pi}(2m_p)}{\cancel{eB}} \cdot \frac{\cancel{eB}}{\cancel{2\pi} m_p} = \boxed{2 = \frac{T_d}{T_p}}$$

$$\frac{T_\alpha}{T_p} = \frac{\cancel{2\pi}(4m_p)}{2\cancel{eB}} \cdot \frac{\cancel{eB}}{\cancel{2\pi} m_p} = \frac{4}{2} = \boxed{2 = \frac{T_\alpha}{T_p}}$$

(c) $B = 0.5\,T$

$v = 3\times10^{6}\,m/s$ (1% speed of light)

$m_p = 1.67\times10^{-27}\,kg$

$e = 1.6\times10^{-19}\,C$

$$r = \frac{mv}{eB} = \frac{(1.67\times10^{-27}\,kg)(3\times10^{6}\,m/s)}{(1.6\times10^{-19}\,C)(.5\,T)}$$

$$r = 6.26\times10^{-2}\,\frac{kg\ m/s}{\cancel{C}-N-s/\cancel{C}-m} = 6.26\times10^{-2}\,\frac{kg\,m/s^2-m}{N}$$

$$\boxed{r = 6.26\times10^{-2}\,m = 6.26\,cm}$$