

# Physics 102– Pledged Problem 7

Time allowed: 2 hours at a single sitting

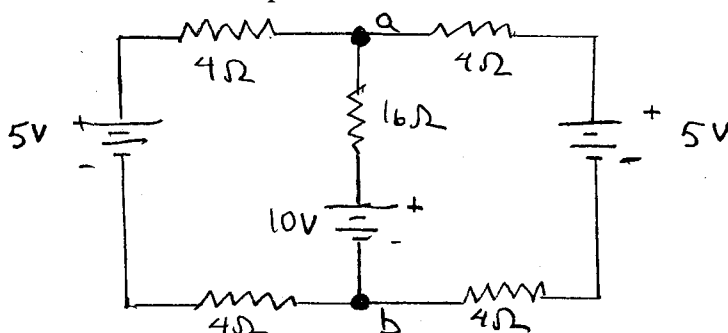
Due 5PM Tuesday, March 21, 2006, in the boxes marked Phys 101-102 in the physics lounge. You may use your own textbook, your notes, and a non-programmed calculator. You may also consult the on-line solutions to the corresponding suggested problems. You should consult no other help. Show how you arrived at your answer; the correct answer by itself may not be sufficient.

Further instructions:

- Write legibly on **one** side of 8.5" x 11" white or lightly tinted paper.
- Staple all sheets together, including this one, in the upper left corner and make one vertical fold.
- On the outside, staple side up, print your name in capital letters, your LAST NAME first followed by your FIRST NAME.
- Below your name, print the phrase "Pledged Problem 7", followed by the due date.
- Also indicate **start time and end time**.
- Write and sign the pledge, with the understanding that you may consult the materials noted above.

I. In the circuit below, determine the following:

- The current through each resistor, including the direction.
- The potential difference between points *a* and *b*.

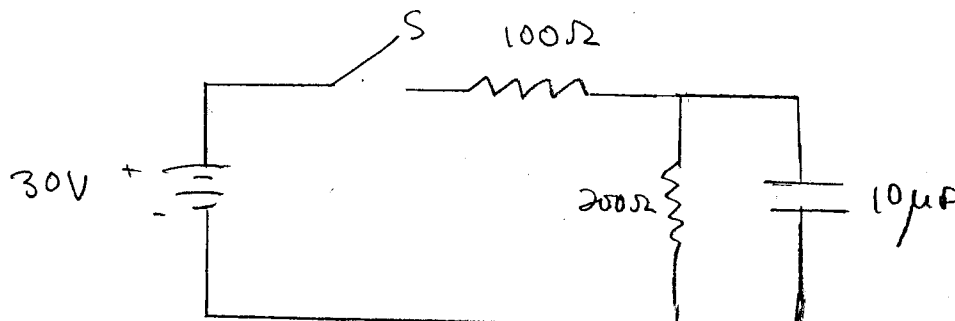


II. In the circuit below, the capacitors are initially uncharged. At  $t=0$  the switch is closed.

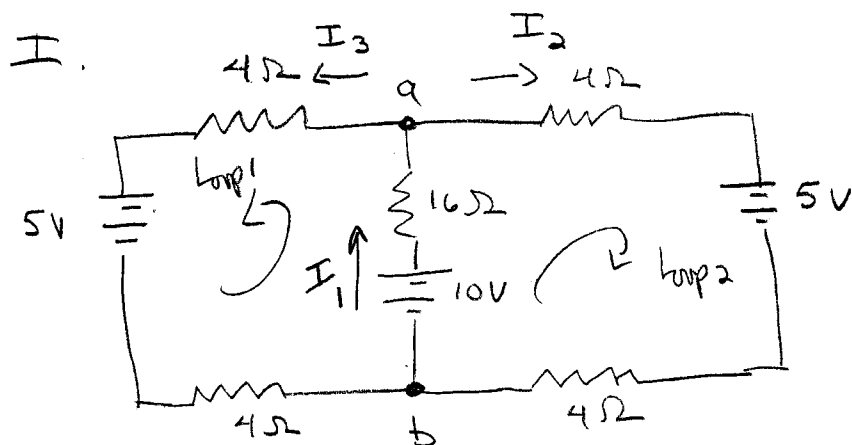
- Determine the current in each resistor immediately after the switch is closed.
- Determine the current in each resistor a long time after the switch is closed.
- Determine the voltage across the capacitor a long time after the switch is closed.

After the switch has been closed for a long time, it is opened again.

- Determine the initial current  $I_0$  through the  $200\Omega$  resistor immediately after the switch is opened.
- Determine  $I(t)$ , the current through the  $200\Omega$  resistor as a function of time after the switch is opened. Sketch  $I(t)$ .
- At what time after the switch is opened has the charge on the capacitor fallen to 0.1 of its initial value?



# Pledged Problem 7



$$I_1 = I_2 + I_3$$

(a) We could guess right away that  $I_2 = I_3$  by symmetry, but let's show it anyway.

Set up the Kirchhoff loop equations:

$$\textcircled{1} \quad 10 - 16I_1 - 4I_3 - 5 - 4I_3 = 0$$

$$5 - 16I_1 - 8I_3 = 0$$

$$\textcircled{2} \quad 10 - 16I_1 - 4I_2 - 5 - 4I_2 = 0$$

$$5 - 16I_1 - 8I_2 = 0$$

Subtracting  $\textcircled{2}$  from  $\textcircled{1}$  gives

$$-8I_3 + 8I_2 = 0$$

$$I_2 = I_3 = I_1/2$$

Now we can easily solve:

$$5 = 16I_1 + 8(I_1/2)$$

$$5 = 20I_1$$

$$I_1 = 1/4 \text{ A}$$

$$I_2 = I_3 = 1/8 \text{ A}$$

Current through  $16\Omega$  resistor =  $1/4 \text{ A}$

Current through all other resistors =  $1/8 \text{ A}$

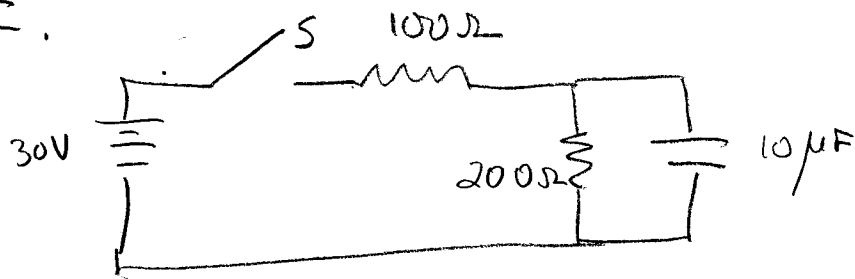
Directions are as shown on the sketch.

$$(b) \text{ DV } (b \rightarrow a) = 10 - 16I_1 = 10 - 4 = 6 \text{ V}$$

$$V_a - V_b = +6 \text{ V}$$

Potential at point a is higher.

II.



Switch is closed at  $t=0$ .

(a) Immediately after  $S$  is closed, the capacitor acts like a short, so no current flows through the  $200\Omega$  resistor.

$$I_{100} = \frac{30V}{100\Omega} = 0.3A$$

$$I_{100} = 0.3A$$

$$I_{200} = 0$$

(b) A long time after  $S$  is closed, the capacitor acts like an open circuit, so all the current flows through both resistors

$$I = \frac{30V}{300\Omega} = 0.1A$$

$$I_{100} = I_{200} = 0.1A$$

(c) After a long time, the voltage across the capacitor will be the same as the voltage across the  $200\Omega$  resistor, since they are in parallel,

$$V_C = I_{200} (200\Omega) = 20V = V_C$$

(d) When S is opened again, the  $100\Omega$  resistor & 30V battery are removed from the circuit. The capacitor discharges through the  $200\Omega$  resistor.

$$I_0 = \frac{V_C}{R} = \frac{20V}{200\Omega} = 0.1A$$

$$I_0 = 0.1A$$

Same as the current immediately before S is opened.

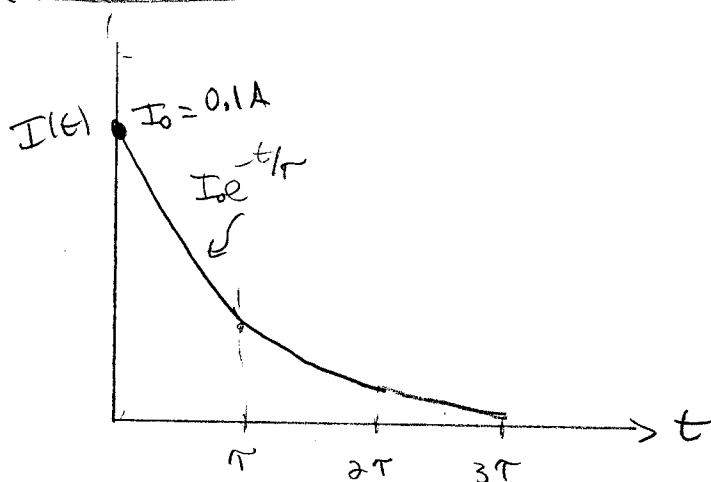
(e) The time constant for discharge is  $\tau = RC$  where only the  $200\Omega$  resistor contributes.

$$I(t) = I_0 e^{-t/\tau}$$

$$I(t) = 0.1 e^{-t/0.002}$$

$$\tau = (200\Omega)(10^{-5}F) = 2 \times 10^{-3}s$$

$$\tau = 0.002s = 2ms$$



$$(f) \frac{I}{I_0} = 0.1 = e^{-t/\tau}$$

$$\ln(0.1) = -t/\tau$$

$$t = 2.3\tau = 4.6ms = 0.0046s$$