Physics 102– Pledged Problem 2

Time allowed: **2 hours at a single sitting**

Due 5PM Monday, January 30, 2006, in the boxes marked Phys 101-102 in the physics lounge. You may use your own textbook, your notes, and a non-programmed calculator. You may also consult the on-line solutions to the corresponding suggested problems. You should consult no other help. Show how you arrived at your answer; the correct answer by itself may not be sufficient.

Further instructions:

(a) Write legibly on **one** side of 8.5" x 11" white or lightly tinted paper.

(b) Staple all sheets together, including this one, in the upper left corner and make one vertical fold.

(c) On the outside, staple side up, print your name in capital letters, your LAST NAME first followed by your FIRST NAME.

(d) Below your name, print the phrase "Pledged Problem 1", followed by the due date.

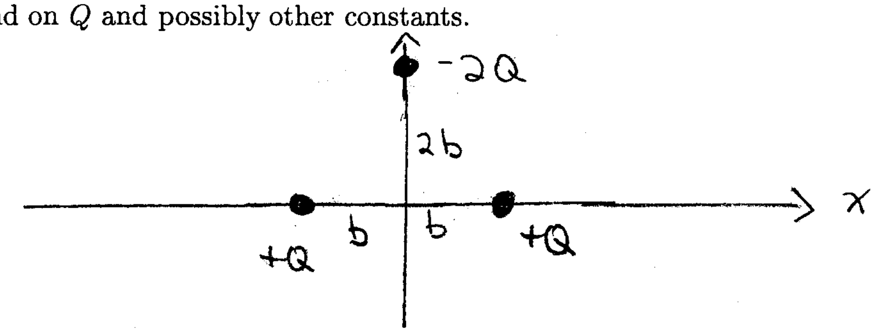
(e) Also indicate **start time** and **end time**.

(f) Write and sign the pledge, with the understanding that you may consult the materials noted above.

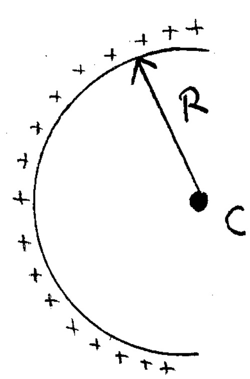
I. Three charges are arranged as shown below. Two identical positive charges $+Q$ are located on the x-axis at $x = \pm b$. A third negative charge $-2Q$ is located on the y-axis at $y = 2b$.

(a) Determine the electric field $\vec{E}$ at the origin.

(b) Determine the electric field $\vec{E}(y)$ for an arbitrary point on the y-axis. Your expression will be a function of y and will also depend on Q and possibly other constants.

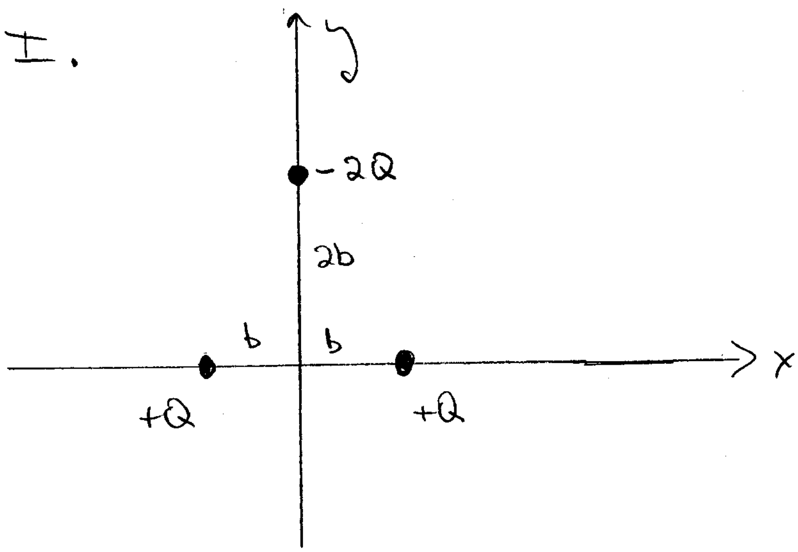


II. A thin insulating rod carries a total charge $+Q$ uniformly distributed over its length. The rod is bent to form a semicircle of radius R. Determine the electric field $\vec{E}$ at the center of the semicircle, at point C shown on the sketch.



Physics 102
Pledged Problem 2

I.

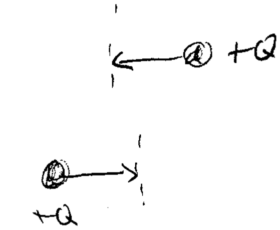


(a) Find $\vec{E}$ at the origin.

Each charge will contribute to $\vec{E}$. Find $\vec{E}$ for each charge & apply the superposition principle.

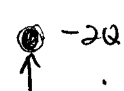
cancel

$\vec{E}_1$ (due to $+Q$ at $+b$) $= -\dfrac{kQ}{b^2}\,\hat{\imath}$



$\vec{E}_2$ (due to $+Q$ at $-b$) $= +\dfrac{kQ}{b^2}\,\hat{\imath}$

$\vec{E}_3$ (due to $-2Q$) $= \dfrac{2kQ}{4b^2}\,\hat{\jmath}$



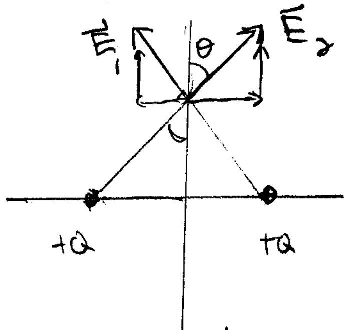
Note that $\vec{E}_3$ is in the $+y$ direction, toward the charge $-2Q$.

Also note that $\vec{E}_1$ and $\vec{E}_2$ cancel

$$\boxed{\vec{E}\,(x=y=0) = \frac{kQ}{2b^2}\,\hat{\jmath}}$$

(b) Find $\vec{E}(y)$ for any point on the y-axis.

First consider the two positive charges:



vertical components add
horizontal components cancel.

$$\vec{E}_1 + \vec{E}_2 = \frac{2kQ}{b^2+y^2}\cos\theta\,\hat{j} \quad \text{with } \cos\theta = \frac{y}{\sqrt{b^2+y^2}}$$

$$E_{12} = \frac{2kQy}{(b^2+y^2)^{3/2}}\,\hat{j}$$

Note that this expression changes sign when y changes sign, which is correct.

Now add $\vec{E}_3$ due to $-2Q$, being careful of the sign.

for $y > 2b$,

$$\vec{E}_3 = \frac{-2kQ}{(y-2b)^2}\,\hat{j} \qquad (\text{in } -y \text{ direction})$$


$-2Q$

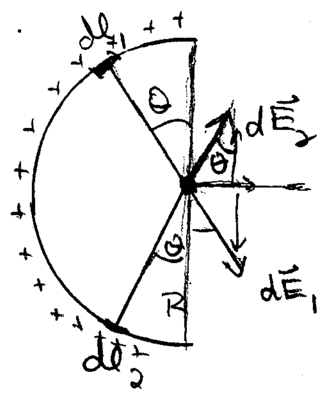
for $y < 2b$

$$\vec{E}_3 = \frac{2kQ}{(2b-y)^2}\,\hat{j} \qquad (\text{in } +y \text{ direction})$$

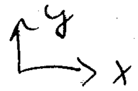
Combining all the contributions:

$$\boxed{\begin{aligned} \vec{E}(y) &= 2kQ\left[\frac{y}{(b^2+y^2)^{3/2}} - \frac{1}{(y-2b)^2}\right]\hat{j} \quad \text{for } y > 2b \\ \vec{E}(y) &= 2kQ\left[\frac{y}{(b^2+y^2)^{3/2}} + \frac{1}{(2b-y)^2}\right]\hat{j} \quad \text{for } y < 2b. \end{aligned}}$$

II.



$+Q$ uniformly distributed over wire gives $\lambda = \dfrac{Q}{\pi R}$



Choose two symmetrically located line elements dl_1 and dl_2. Each one contributes to the electric field

$$|dE| = \frac{k\,dq}{R^2} \quad \text{with } dq = \lambda\, dl = \lambda R\, d\theta$$

As can be seen from the sketch, the vertical components will ~~add~~ cancel, horizontal components add. Need $\sin\theta$ to get the horizontal components:

$$dE_x = \frac{2k\lambda \cancel{R} \sin\theta\, d\theta}{R^{\cancel{2}}}\ \hat{\imath}$$

This is the contribution from both dl_1 and dl_2.

Now integrate over θ from 0 to $\pi/2$. We only integrate to $\pi/2$ because we have already included the region from $\pi/2 \to \pi$ by adding dE_1 and dE_2.

$$\vec{E} = \int d\vec{E} = \frac{2k\lambda\hat{\imath}}{R}\int_0^{\pi/2} \sin\theta\, d\theta$$

$$-\cos\theta\Big|_0^{\pi/2} = 1$$

$$\boxed{\vec{E} = \frac{2k\lambda\hat{\imath}}{R} = \frac{2kQ}{\pi R^2}\ \hat{\imath}}$$

Direction is to the right.

Comment: If you only include dE_x due to dl_1 (i.e., no factor of 2), then you integrate from 0 to π.