Physics 102– Pledged Problem 1

Time allowed: **2 hours at a single sitting**

Due 4PM Monday, January 23, 2006, in the boxes marked Phys 101-102 in the physics lounge. You may use your own textbook, your notes, and a non-programmed calculator. You may also consult the on-line solutions to the corresponding suggested problems. You should consult no other help. Show how you arrived at your answer; the correct answer by itself may not be sufficient.

Further instructions:

(a) Write legibly on **one** side of 8.5" x 11" white or lightly tinted paper.

(b) Staple all sheets together, including this one, in the upper left corner and make one vertical fold.

(c) On the outside, staple side up, print your name in capital letters, your LAST NAME first followed by your FIRST NAME.

(d) Below your name, print the phrase "Pledged Problem 1", followed by the due date.

(e) Also indicate **start time** and **end time**.

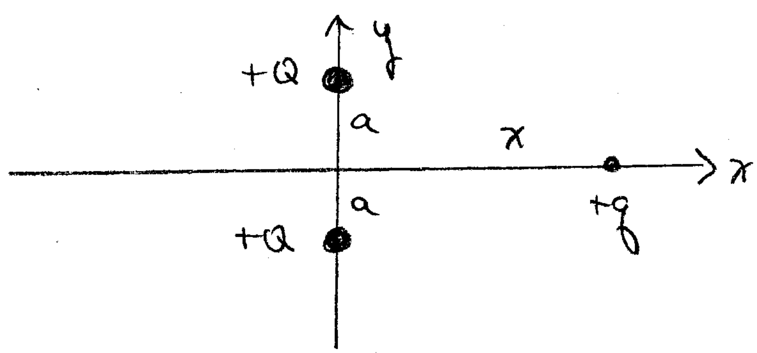
(f) Write and sign the pledge, with the understanding that you may consult the materials noted above.

I. Two identical positive charges $+Q$ are fixed on the y-axis at $\pm a$ as shown in the sketch below. A third positive charge $+q$ is moved along the x-axis toward the origin.

(a) Determine the force $\vec{F}_E$ on q as a function of x, the distance from the origin for all points on the x-axis. Express your answer in terms of Q, q, a, x and possibly other constants.

(b) Sketch the x-component of the force F_x as a function of x.

(c) Sketch the y-component of the force F_y as a function of x.

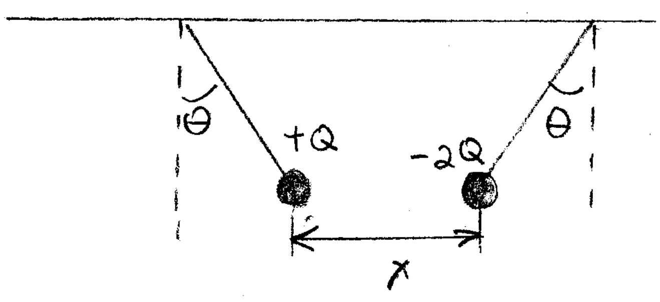


II. Two styrofoam spheres of equal mass M are each suspended from threads of equal length as shown below. One sphere has a positive charge $+Q$ and the other has negative charge $-2Q$. When in equilbrium, the centers of the spheres are a distance x apart, and they make an angle θ with the vertical.

(a) Draw a free body diagram for one of the masses showing all the forces acting on it.

(b) Even though the magnitudes of the charges on the spheres are different, the angle that they make with the vertical, θ, is the same for both. Explain why.

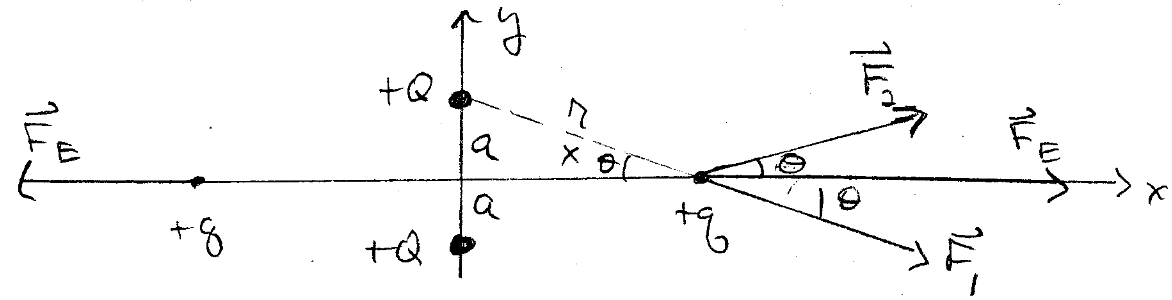
(c) Determine the distance between the sphere centers, x, in terms of the mass M, the charge Q, the angle θ and possibly other constants.



Phys 102

Pledged Problem 1.

I.



(a) $|\vec{F}_1| = |\vec{F}_2| = \dfrac{kQq}{r^2}$ where $r^2 = a^2 + x^2$

By symmetry, the y-components of $\vec{F}_1$ & $\vec{F}_2$ cancel, and the x-components add.

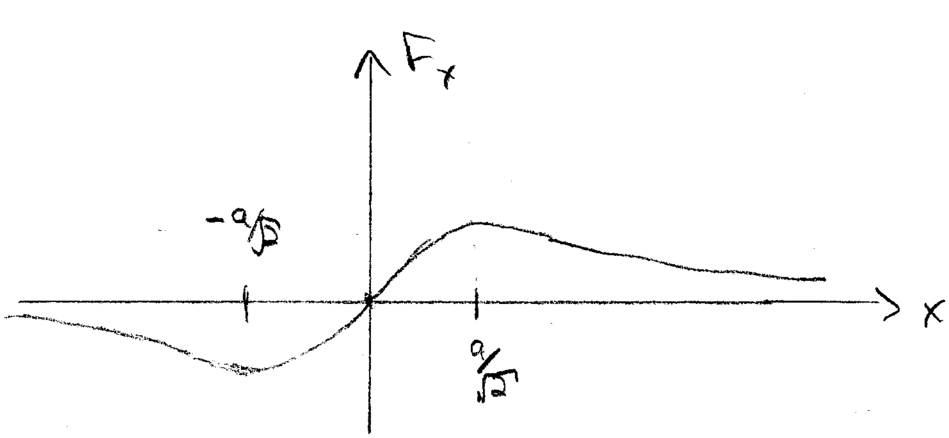
$F_{1x} = F_1 \cos\theta$ with $\cos\theta = \dfrac{x}{r} = \dfrac{x}{\sqrt{x^2+a^2}}$

$$\vec{F}_E = \frac{2kQq}{x^2+a^2} \cdot \cos\theta\, \hat{\imath} = \boxed{\frac{2kQq\,x}{(x^2+a^2)^{3/2}}\, \hat{\imath} = \vec{F}_E}$$

(mag)

Note that $\vec{F}_E$ changes sign for $x<0$, but since the x in the numerator also changes sign, this expression is correct for all x.

(b)



Note that $F_x \to 0$ at $x = 0$ as well as $x \to \pm\infty$.

F_x will have a maximum somewhere in between, which we can easily find by taking the derivitive:

$$F_x = \frac{2kQq\,x}{(x^2+a^2)^{3/2}}$$

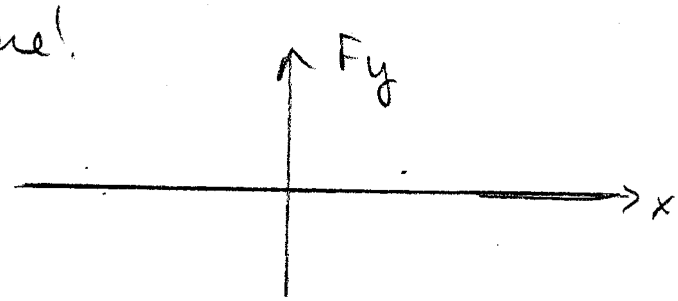
$$\frac{dF_x}{dx} = 2kQq\left[\frac{1}{(x^2+a^2)^{3/2}} - \frac{3/2(2x)(x)}{(x^2+a^2)^{5/2}}\right] = 0$$

$$\frac{3x^2}{(x^2+a^2)^{5/2}} = \frac{1}{(x^2+a^2)^{3/2}}$$

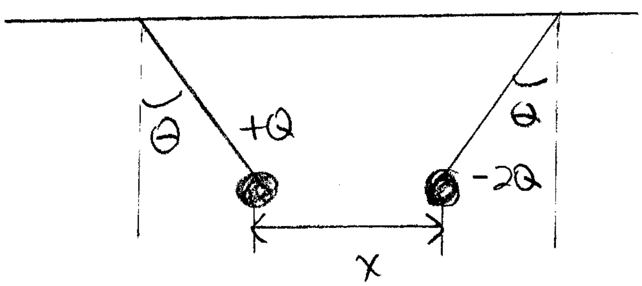
$$3x^2 = x^2 + a^2$$

$$2x^2 = a^2 \qquad x = \pm\frac{a}{\sqrt{2}}$$

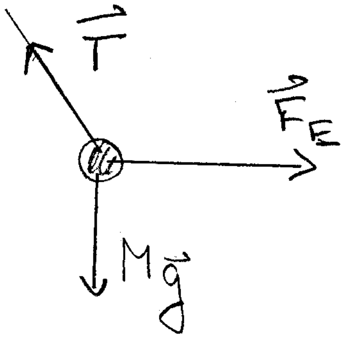
(c) $F_y = 0$ everywhere!



II.



(a)

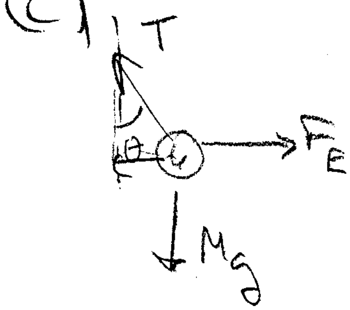


(b) Since the masses are the same, the gravitational forces will be the same. Even though the charges are different, the magnitude of $\vec{F}_E$ is the same. This fact can be seen either from the form of Coulomb's Law.

$$|F_E| = \frac{k q_1 q_2}{r^2} \quad (\text{same for } q_1 \,\&\, q_2)$$

It also follows directly from Newton's 3rd law. Since the forces are the same, by symmetry the angle Θ must be the same for both masses.

(c)



Forces in x: $T\sin\theta = F_E = \frac{2kQ^2}{x^2}$

Forces in y: $T\cos\theta = Mg$

Take the ratio to cancel out T

$$\tan\theta = \frac{2kQ^2}{x^2 Mg}$$ solve for x

$$x^2 = \frac{2kQ^2}{Mg\tan\theta}$$

$$x = \sqrt{\frac{2kQ^2}{Mg\tan\theta}}$$