AC Circuits

"Look for knowledge not in books but in things themselves."

W. Gilbert (1540-1603)

OBJECTIVES

To study some circuit elements and a simple AC circuit.

THEORY

All useful circuits use varying voltages, changing magnitude or even completely reversing polarity. In the present exercise we will study the behavior of some basic components and two elementary electrical filters, of the sort that might be used in audio equipment, when stimulated with a varying voltage. To keep things simple, we will consider only a sinusoidal voltage which oscillates at a steady frequency.

For purposes of analysis, circuits are usually considered to be made from resistors, capacitors and inductors. The components themselves are frequently characterized by the current that flows through them in response to a sinusoidal voltage at angular frequency ω. As shown in your text, the current will then also be sinusoidal, with the ratio of peak voltage to peak current, the reactance, depending on frequency. There may also be a phase shift between the current and voltage, so that they peak at different times. The ideal relationships are given by

Component	Reactance	
resistor	$X_R = R$	(1)
capacitor	$X_C = 1/\omega C$	(2)
inductor	$X_L = \omega L$	(3)

Verifying the relations requires measuring the amplitudes of current and voltage to determine the reactance. A method for doing this will be described later.

As you might expect, real components are more complicated, but it is often possible to manufacture a reasonable approximation to these ideals. In fact, commercial resistors and capacitors are rather good, but inductors are not. The resistance of the coil turns out to be significant in most practical cases (superconductors are not practical), so we need to consider a more complex model for inductors. One approximation is to assume that the coil resistance is effectively in series with an ideal inductor, as shown in Fig. 1. Using the fact that the current flow through both components is the same, we can draw a phasor diagram showing the voltage

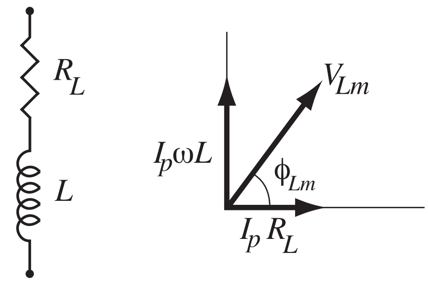


Fig. 1 Model of a real inductor and corresponding phasor diagram used to obtain reactance.

drop across the resistor and across the ideal inductor. The total voltage drop across the model inductor is then the vector sum

$$V_{Lm} = I_p(\omega^2 L^2 + R_L^2)^{1/2} \tag{4}$$

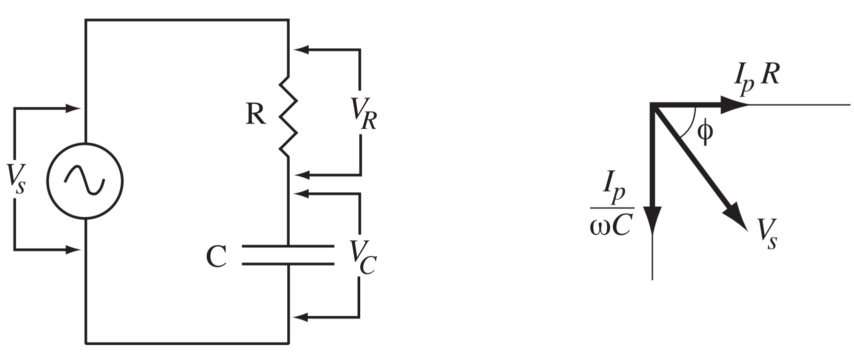
from which it is easy to predict the reactance for this model inductor

$$X_{Lm} = (\omega^2 L^2 + R_L^2)^{1/2} \tag{5}$$

At low frequencies, where $\omega L << R_L$, Eq. 5 suggests that the inductor behaves like a resistor, in that the reactance is almost equal to R_L. At high frequencies, $\omega >> R_L/L$ the reactance increases to ωL, as expected for an ideal inductance. Later we will see if this accounts for the properties of a real inductor.

The first circuit example is an RC filter, shown in Fig. 2. We are interested in finding the fraction of the input signal voltage V_s that appears across the capacitor as a function of frequency. Figure 2 provides the phasor diagram for the circuit, from which we see

$$V_s = I_p\left(R^2 + \frac{1}{\omega^2 C^2}\right)^{1/2} \tag{6}$$



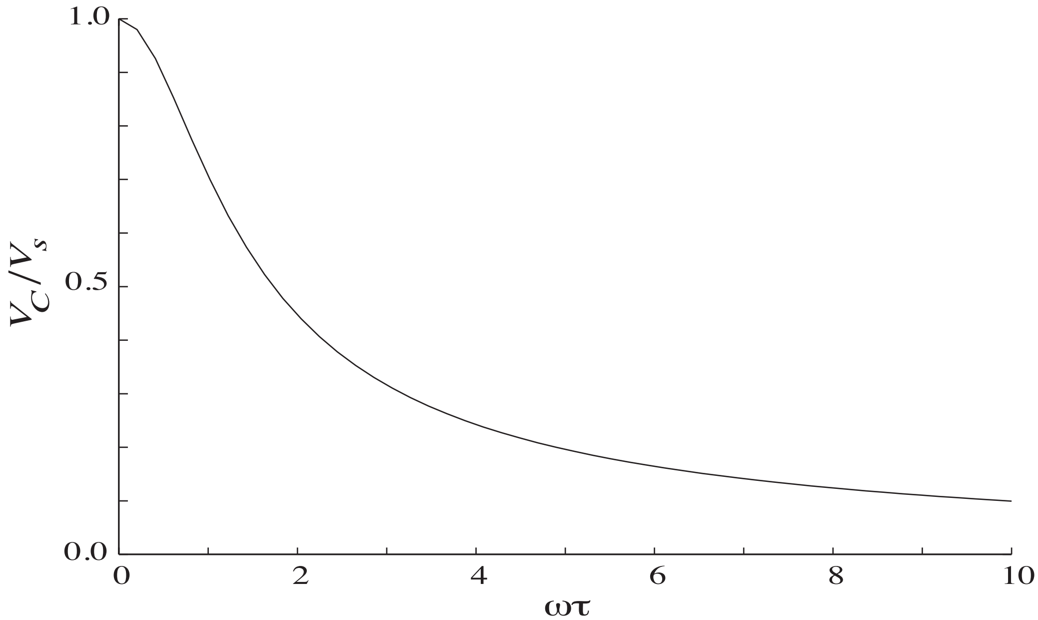


Fig. 3 Ratio of output to input voltage for a low-pass RC filter, as a function of scaled frequency.

Solving for the current and using Eq. 2 to relate V_C to I_p, we get

$$V_C = V_s \frac{1}{(1+\omega^2\tau^2)^{1/2}} \tag{7}$$

where $\tau = RC$ is the time constant of the circuit.

The circuit in Fig. 2 is called a low-pass filter because low input frequencies are passed to the output essentially unaffected but high frequencies are strongly attenuated, as graphed in Fig. 3. Low-pass filters are often used in audio devices to remove high-frequency digital artifacts from the desired audio signal.

Our other circuit example is a bandpass filter, which attenuates signals outside a certain frequency range. Bandpass filters are useful, for example, in tuning a radio receiver to a desired

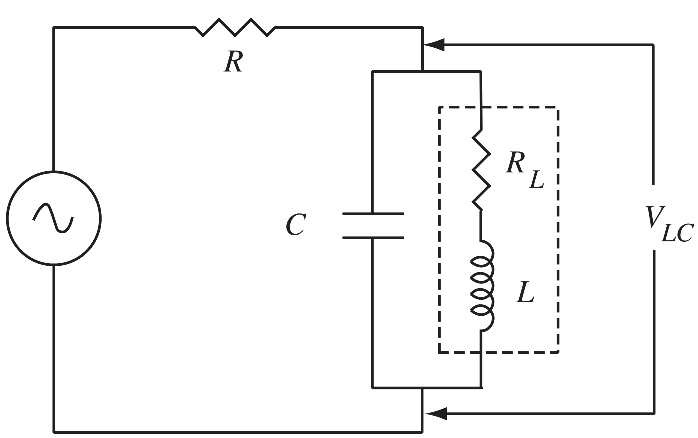


Fig. 4 Bandpass filter using a parallel LC resonant circuit.

station. This function can be accomplished with a parallel LC circuit, as shown in Fig. 4. A detailed analysis is complicated, but it can be shown that the reactance of the LC circuit has a maximum at the angular resonant frequency $\omega_0 = 1/\sqrt{LC}$. We therefore expect the voltage V_{LC} to have a maximum near that frequency.

EXPERIMENTAL PROCEDURE

The experimental work will consist of measuring the current-voltage characteristic of a typical commercial capacitor and an inductor, and observing the filtering action of two different circuits. The required procedures are presented separately, following a description of the AC signal generator.

Function generator operation

Figure 5 shows the control panel of the function generator most often used in this lab. The instrument is turned on with the power switch, item (1) in the figure. A sine, square or triangle waveform is selected with the buttons labeled FUNCTION (3). The frequency range is determined by pushbuttons (2), and the exact frequency is then set using the coarse and fine FREQUENCY knobs (12, 11). The output amplitude is determined by the AMPL knob (4). Pulling gently on the AMPL knob decreases the output amplitude by a factor of 10. The display (13) shows the output frequency numerically in Hz or kHz, according to the indicator (14) and decimal point on the display. The selected output appears as a voltage at OUTPUT (5). The other controls and outputs are more specialized, and will not be used here.

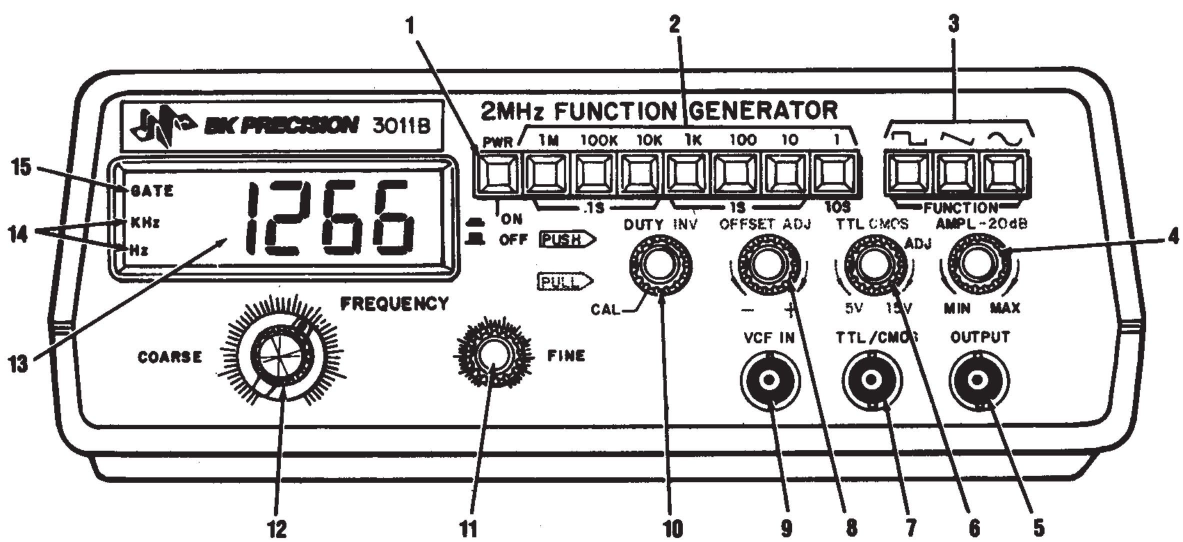


Fig. 5 The front panel of the function generator, with controls marked.

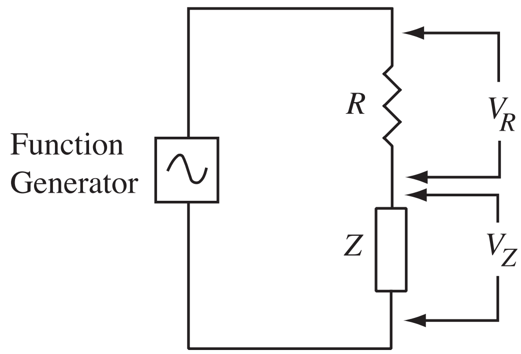


Fig. 6 Circuit for measuring I-V characteristic of component Z.

Component characteristics

Figure 6 specifies the circuit used to measure the current and voltage for the component, Z, to be tested. The voltage V_Z and V_R can be measured with the DMM set for AC voltage. The current is found by applying Ohm's Law with the known value of *R*. The voltage source is a function generator, set to a convenient amplitude and the desired frequency.

As a test, wire the circuit as shown, using $R = 150\ \Omega$, and another resistor for *Z*. Measure V_R and V_Z at any convenient frequency below 1000 Hz, and deduce the reactance of the resistor. You should get a value reasonably close to the marked resistance of the component.

When you are sure the circuit is correctly wired and the DMM is set properly, replace the test resistor at Z with a 0.47 μF capacitor. Measure the voltages across the resistor and capacitor at several frequencies from 30 Hz to 1 kHz. Use Ohm's law and the measured resistance (use the DMM) of the nominal 150 Ω resistor to get the current at each frequency. Use these data to estimate the reactance and plot it vs inverse frequency. This is most conveniently done by entering the raw data directly into the table in Reactance.cmbl. You will need to replace the number 150 in the column definition for Xz with the actual measured value of the resistor.

Is the graph of X_C vs $1/f$ well described by a proportional fit with slope $1/2\pi C$, as expected for an ideal capacitor? (The factor 2π comes from converting frequency to angular frequency.) The manufacturer claims the marked capacitance value is accurate to ±20%.

Replace the capacitor with the 100 mH inductor as Z, leaving the 150 Ω resistor, and repeat the measurements of reactance. Plot the measured reactance vs frequency, and use LoggerPro to fit the model expression, Eq. 5 to your data. The required function is labeled Inductive X in the Curve Fit menu. Are the fitted parameters reasonable? The inductance value should be within ±20%, and you can check the DC resistance of the inductor with the DMM.

RC Filter Circuit

The next exercise is to study the frequency response of an RC filter circuit, shown in Fig. 2. Component values of R = 2.5 kΩ and C = 0.47 μF will give a convenient time constant, and the function generator will supply V_s. Using the LoggerPro file RCfilter.cmbl, enter the measured V_C and V_s, and plot the amplitude ratio V_C/V_s as a function of frequency over the range 30 Hz – 1 kHz. Fit the data with the computer version of Eq. 9–7, labeled Low Pass in the Curve Fit menu. Does the model accurately describe this low-pass filter, with the expected value of τ? Evaluating the time constant with the measured values of C and R should improve the agreement.

LC Filter Circuit

The last exercise is to examine the frequency response of the LC filter circuit shown in Fig. 4. Component values of C = 0.47 μF and L = 100 mH with a series resistance R = 10 kΩ will give a resonant frequency within the 30 Hz - 1 KHz range that you can measure. Does a plot of V_{LC} vs frequency show a peak near the resonance frequency expected from the component values? The graph can be made very easily if you enter the data directly into Graph.cmbl and take enough points to define the curve.