Current, Resistance, and Direct Current (DC) Circuits

Conductivity

- A current density **J** and an electric field **E** are established in a conductor whenever a potential difference is maintained across the conductor
- $\mathbf{J} = \sigma \mathbf{E}$
- The constant of proportionality, σ, is called the **conductivity** of the conductor (describes the "ease" of an applied electric field forming a current).

Resistivity

- The inverse of the conductivity is the **resistivity**: (measures how "hard" it is for an electric field to establish a current in the conductor)
  - $\rho = 1 / \sigma$
- Resistivity has SI units of ohm-meters ($\Omega \cdot m$)

Ohm's Law

- **Ohm's law** states that for many materials, the ratio of the current density to the electric field is a constant σ that is independent of the electric field producing the current
  - Most metals obey Ohm's law
  - Mathematically, $\mathbf{E} = \rho \, \mathbf{J}$
  - Materials that obey Ohm's law are said to be *ohmic*

Ohm's Law, cont.

- Not all materials follow Ohm's law
  - Materials that do not obey Ohm's law are said to be *nonohmic*
- Ohm's law is ***not*** a fundamental law of nature
- Ohm's law is an empirical relationship valid only for certain materials

Ohm's Law

- Apply a potential difference across a conducting wire of length L and resistivity *ρ:*

$$\mathbf{J} = \mathbf{E} / \rho$$

$$J = I / A = (V/L)(1/\rho)$$

$$\rightarrow V = (\rho L / A)\, I$$

$$R = \rho \frac{\ell}{A}$$

Resistance

- In a conductor, the voltage applied across the ends of the conductor is proportional to the current through the conductor
- The constant of proportionality is called the **resistance** of the conductor

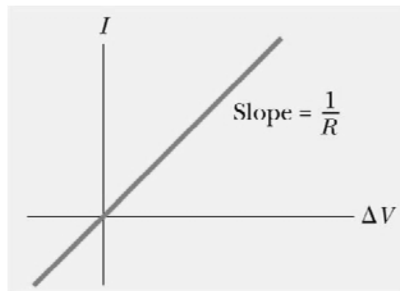
$$R = \frac{\Delta V}{I}$$

Resistance, cont.

- SI units of resistance are *ohms* (Ω)
  - 1 Ω = 1 V / A
- Resistance in a circuit arises due to collisions between the electrons carrying the current with the fixed atoms inside the conductor
- The circuit symbol for a resistor is:

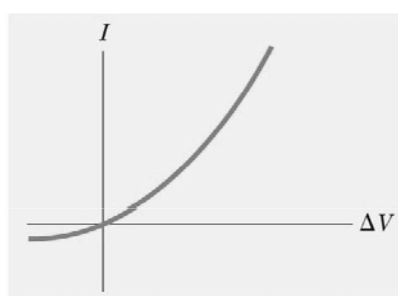
Ohmic Material, Graph

- The relationship between current and voltage is linear
- The slope is related to the resistance
- Engineers deal with I vs. V curves.



Nonohmic Material, Graph

- The current-voltage relationship is nonlinear
- A diode is a common example of a nonohmic device

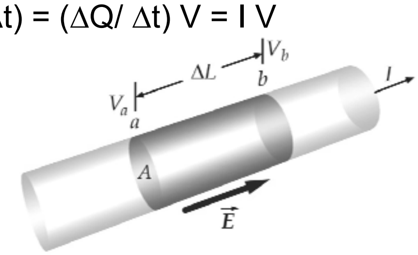


Electric Power

- The rate at which the system ***loses*** potential energy as a charge passes through the resistor is equal to the rate at which the system gains internal energy in the resistor.
  - Think of touching a light bulb after it has been on for a long time.
- The **power** is the rate at which the energy is delivered to the resistor

Electric Power

- The change in potential energy of a charge passing through the segment is
  - $\Delta U = \Delta Q(V_b – V_a) = \Delta Q(-V) = - \Delta QV$
- The **power** is the rate at which the energy is delivered to the resistor (Joule heating).
  - $-(\Delta U/ \Delta t) = (\Delta Q/ \Delta t) V = I V$



Electric Power

- The power is given by the equation:

$$P = I \Delta V$$

- Applying Ohm's Law, alternative expressions can be found:

$$P = I \Delta V = I^2 R = \frac{V^2}{R}$$

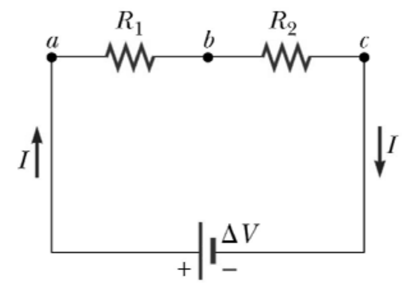
- Units: I is in A, R is in Ω, V is in V, and P is in W

Resistors in Series

- When two or more resistors are connected end-to-end, they are said to be in *series*

Resistors in Series, cont

- Potentials add
  - $\Delta V = IR_1 + IR_2$
    $= I(R_1+R_2)$
  - Consequence of Conservation of Energy
- The equivalent resistance has the same effect on the circuit as the original combination of resistors

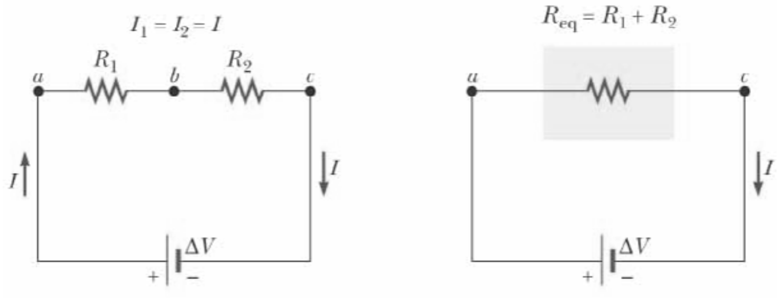


(b)

Equivalent Resistance – Series

- $R_{eq} = R_1 + R_2 + R_3 + \ldots$
- The equivalent resistance of a series combination of resistors is the algebraic sum of the individual resistances and is always greater than any individual resistance
- If one device in the series circuit creates an open circuit, all devices are inoperative

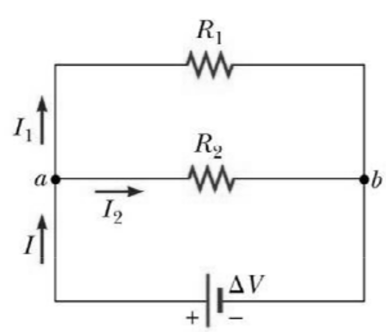
Equivalent Resistance – Series – An Example



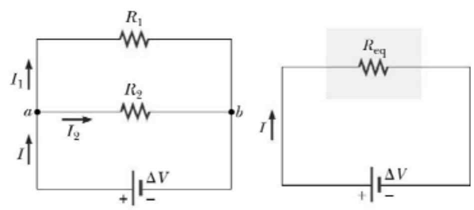
- Two resistors are replaced with their equivalent resistance

Resistors in Parallel

- The potential difference across each resistor in parallel is the same.
- The current, I, that enters a point must be equal to the total current leaving that point
  - $I = I_1 + I_2 = V_1/R_1 + V_2/R_2$
    $= V\,(1/R_1 + 1/R_2)$
  - The currents are generally not the same
  - Consequence of Conservation of Charge



Equivalent Resistance – Parallel, Examples



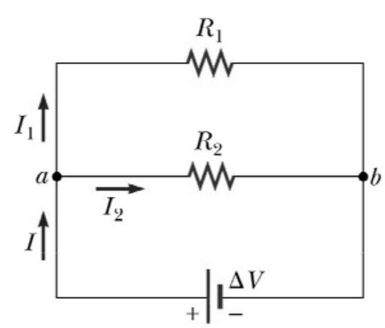
- Equivalent resistance replaces the two original resistances
- *Household circuits* are wired so that electrical devices are connected in parallel
  - Circuit breakers may be used in series with other circuit elements for safety purposes

Equivalent Resistance – Parallel

- Equivalent Resistance

$$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \ldots$$

- The inverse of the equivalent resistance of two or more resistors connected in parallel is the algebraic sum of the inverses of the individual resistance
  - The equivalent resistance is always less than the smallest resistor in the group



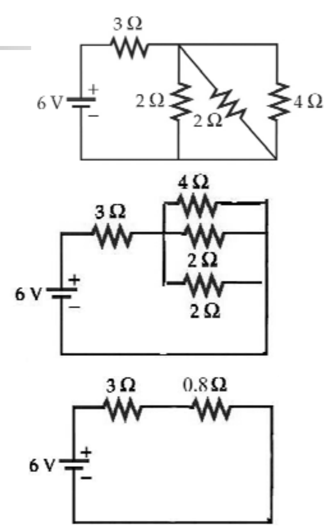
(b)

Resistors in Parallel, Final

- In parallel, each device operates independently of the others so that if one is switched off, the others remain on
- In parallel, all of the devices operate on the same voltage
- The current takes all the paths
  - The lower resistance will have higher currents
  - Even very high resistances will have some currents

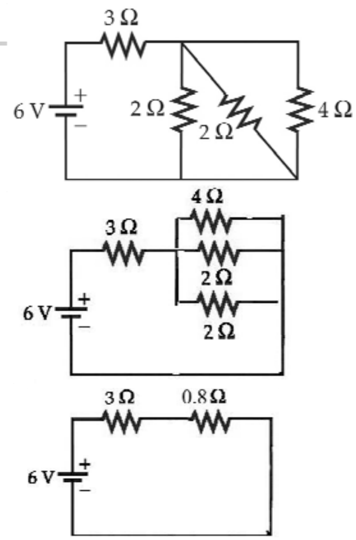
Combinations of Resistors

- The 2-Ω, 2-Ω and 4.0-Ω resistors are in parallel and can be replaced with their equivalent, 4/5 Ω = 0.8 Ω
- The 0.8-Ω and 3.0-Ω resistors are in series and can be replaced with their equivalent, 3.8 Ω
- The current through the circuit is:
  - 6V / 3.8 Ω = 1.58 A
  - This is the current through the 3 Ω resistor and the equivalent resistance 0.8 Ω.



Combinations of Resistors

- How would you find, the current through the other resistors?
  - Hint: The potential difference across the parallel components are the same. What is this potential difference?



Kirchhoff's Rules

- There are ways in which resistors can be connected so that the circuits formed cannot be reduced to a single equivalent resistor
- Two rules, called **Kirchhoff's rules**, can be used instead

Statement of Kirchhoff's Rules

- Junction Rule
  - The sum of the currents entering any junction must equal the sum of the currents leaving that junction
    - A statement of Conservation of Charge
- Loop Rule
  - The sum of the potential differences across all the elements around any closed circuit loop must be zero
    - A statement of Conservation of Energy

Mathematical Statement of Kirchhoff's Rules

- Junction Rule:

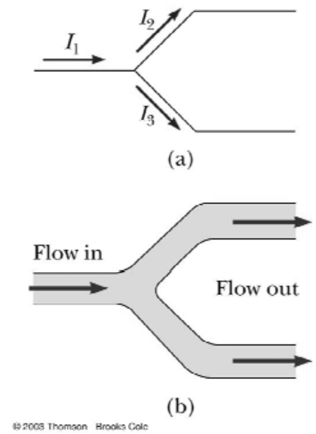
$$\Sigma I_{\text{in}} = \Sigma I_{\text{out}}$$

- Loop Rule:

$$\sum_{\substack{\text{closed}\\\text{loop}}} \Delta V = 0$$

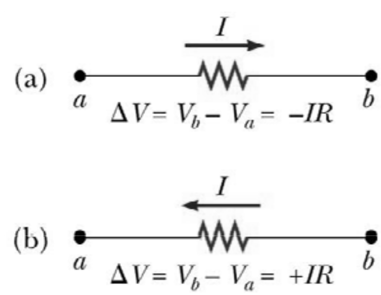
More about the Junction Rule

- $I_1 = I_2 + I_3$
- From Conservation of Charge
- Diagram (b) shows a mechanical analog



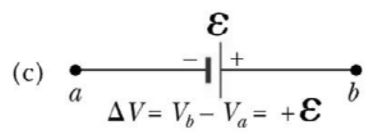
More about the Loop Rule

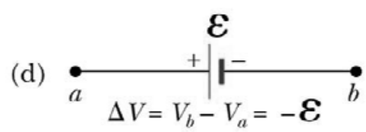
- The electric field (potential difference) yields direction of current flow.
- In (a), the potential difference across the resistor is – *IR.*
  - *V(b) – V(a) < 0.*
- In (b), the potential difference across the resistor is is + *IR.*
  - *V(b) - V(a) < 0.*



Loop Rule, final

- In (c), the source of (***e***lectro***m***otive ***f***orce) emf is traversed in the direction of the emf (from – to +), and the change in the electric potential is $+\varepsilon$
- In (d), the source of emf is traversed in the direction opposite of the emf (from + to -), and the change in the electric potential is $-\varepsilon$





Kirchhoff's Rules Equations

- In order to solve a particular circuit problem, the number of independent equations you need to obtain from the two rules equals the number of unknown currents
- Any fully charged capacitor acts as an open branch in a circuit
  - The current in the branch containing the capacitor is zero under steady-state conditions

Problem-Solving Hints – Kirchhoff's Rules

- Draw the circuit diagram and assign labels and symbols to all known and unknown quantities. Assign directions to the currents.
  - The direction is arbitrary, but you must adhere to the assigned directions when applying Kirchhoff's rules
- Apply the junction rule to any junction in the circuit that provides new relationships among the various currents

Problem-Solving Hints, cont

- Apply the loop rule to as many loops as are needed to solve for the unknowns
  - To apply the loop rule, you must correctly identify the potential difference as you cross various elements
- Solve the equations simultaneously for the unknown quantities
  - If a current turns out to be negative, the magnitude will be correct and the direction is opposite to that which you assigned