PHYS102 - Gauss's Law.

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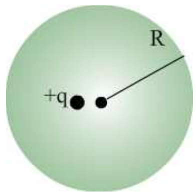
0.1 Flux - General

Question #1

A charge +q is located inside a sphere of radius R. The charge is NOT at the center of the sphere. According to Gauss's Law, which of the following statement(s) is (are) true

I. The magnitude of the electric field is constant over the surface of the sphere.
II. The electric flux varies over the surface of the sphere.
III. The electric flux is constant.
IV. The electric flux is directly proportional to +q.



1. Only I is correct.
2. Only II is correct.
3. Only III is correct.
4. Only II and IV are correct.
5. Only III and IV are correct.

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Answer to Question #1

- The electric flux is given by $\Phi = \frac{Q_{enclosed}}{\varepsilon_0}$.
- Q is the amount of charge contained inside the closed surface (in this case $Q_{enclosed} = +q$).
- Electric flux is constant.
- The answer is 5.
  Note: The magnitude of the electric fieldmagnitude of the electric field over the spherical surface is not constantnot constant since the chargecharge is NOT centered with the sphere.NOT centered with the sphere.

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0.2 General Equation

Gauss's Law - General

$$\Phi = \oint_S \vec{E} \cdot d\vec{A} = \frac{Q_{enclosed}}{\varepsilon_0} \qquad \text{(GAUSS'S LAW)}$$

- The above equation is a very general equation and holds true for any surface.
- This is an electric flux law - NOT AN ELECTRIC FIELD LAW.
  - Gauss's Law is always true, but the law is NOT always useful in determining electric fields from charge distributions.
  - We will examine the only THREE cases where the law is useful in determining the electric field.

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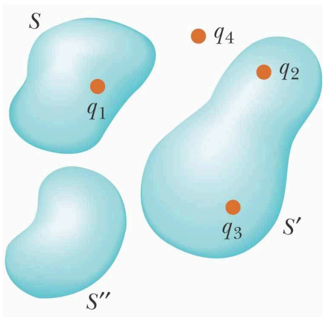
Applying Gauss's Law
Consider the figure on the right:

- For the arbitrarily shaped surfaces:

$$\Phi_S = \frac{Q_{enclosed}}{\varepsilon_0} = \frac{q_1}{\varepsilon_0}$$

$$\Phi_{S'} = \frac{q_2 + q_3}{\varepsilon_0}$$

$$\Phi_{S''} = 0$$



- One CANNOT use Gauss's Law to find the electric field due to the charge configuration.

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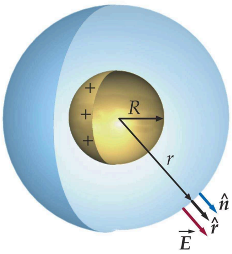
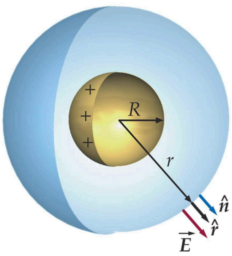
Spherical Symmetry



1. Spherical Symmetry.

- A charge distribution has *spherical symmetry* if the views of it from all points on the spherical surface are the same.
  - Choose a spherical surface of radius r, centered at the charge distribution - such surfaces are called "Gaussian surfaces" "Gaussian surfaces"

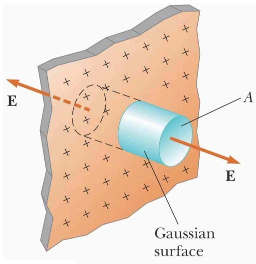
Spherical Symmetry



1. Spherical Symmetry.

- By symmetry, the electric field is directed radiallyradiallyradially (inwardinward if charge distribution is negativenegative or outwardoutward if charge distribution is positivepositive).

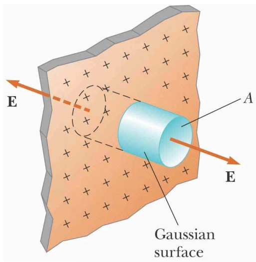
Plane Symmetry



2. Plane Symmetry.

- A charge distribution has *plane symmetry* if the views of it from all points on an infinite (or very long) plain surface are the same.
  - Choose a soup-can shaped cylinder, with the charged plane bisecting the cylinder.
  - The only contributing flux is that due to the flat ends.

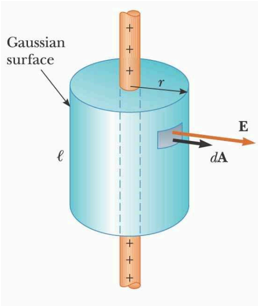
Plane Symmetry



2. Plane Symmetry.

- By symmetry, the electric field is directed perpendicular (away for positive and toward for negative) to the plane.

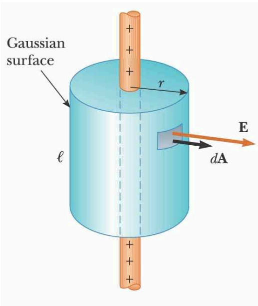
Cylindrical Symmetry



3. Cylindrical Symmetry.

- A charge distribution has *cylindrical symmetry* if the views of it from all points on a cylindrical surface of infinite (or very long) length are the same.
  - ○ Choose a cylindrical Gaussian surface with the center of the Gaussian cylinder coincident with the cylindrical charge distribution.

Cylindrical Symmetry

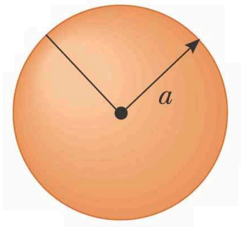


3. Cylindrical Symmetry.

- The only contributing flux is along the curved piece of the cylinder.
- By symmetry, the electric field is directed (away for positive or toward for negative) from the line charge.
- The magnitude of E depends only on the radial distance from the line charge.

0.3 Spherical Symmetry

Spherical Symmetry - Problem

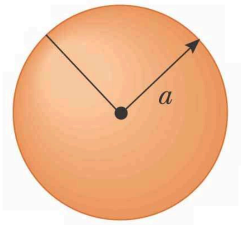


Problem: The volume charge density inside a solid sphere of radius a is given by $\rho = \rho_0\, r/a$, where ρ_0 is a constant. Find

(a). the total charge.

(b). the electric field strength for $r > a$ and $r < a$.

Spherical Symmetry - Problem II



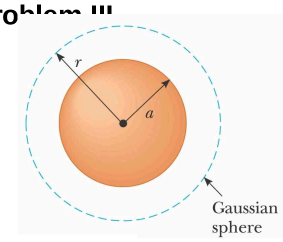
(a). to find the total charge:

$$dq = \rho\, dV$$

$$Q = \int_0^a \rho\, dV = \int_0^a \frac{\rho_0\, r}{a}\, 4\,\pi\, r^2\, dr$$

$$Q = \rho_0\, \pi\, a^3$$

Spherical Symmetry - Problem III

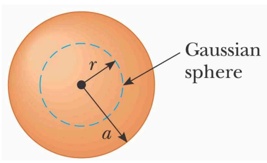


(b). To find the electric field ($r > a$) first construct a Gaussian surface as shown above and note that the magnitude of the electric field is constant over the sphere.

$$\Phi = \oint \vec{E} \cdot d\vec{A} = E \oint dA = \frac{Q_{\text{enclosed}}}{\varepsilon_0} = \frac{\rho_0 \pi a^3}{\varepsilon_0}$$

$$\Rightarrow E\, 4\pi r^2 = \frac{\rho_0 \pi a^3}{\varepsilon_0} \Rightarrow E = \frac{\rho_0 a^3}{4 \varepsilon_0 r^2}$$ (Note: the $1/r^2$ dependance)

Spherical Symmetry - Problem IV



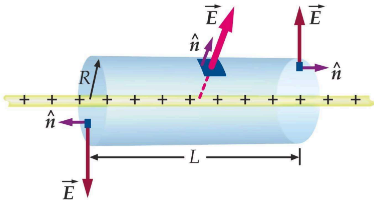
(b). To find the electric field ($r < a$) first construct a Gaussian surface as shown above and note that the magnitude of the electric field is constant over the sphere.

$$\Phi = \oint \vec{E} \cdot d\vec{A} = E \oint dA = \frac{Q_{\text{enclosed}}}{\varepsilon_0} = \frac{\rho_0 \pi r^4}{a \varepsilon_0}$$

$$E\, 4\pi r^2 = \frac{\rho_0 \pi r^4}{a \varepsilon_0} \Rightarrow E = \frac{\rho_0 r^2}{4 \varepsilon_0 a}$$ (Note: NO $1/r^2$ dependance)

0.4 Cylindrical Symmetry

Cylindrical Symmetry - Problem



Problem: Find the electric field strength for a very long wire carrying uniform charge density ($+\lambda$) as a function of the distance away from wire.

To find the electric field first construct a Gaussian surface as shown above (in blue) and note that the magnitude of the electric field is constant over the cylinder. The electric flux through the Gaussian surface is

$$\Phi = \oint \vec{E} \cdot d\vec{A} = E \oint dA = \frac{Q_{\text{enclosed}}}{\varepsilon_0} = \frac{\lambda L}{\varepsilon_0}$$

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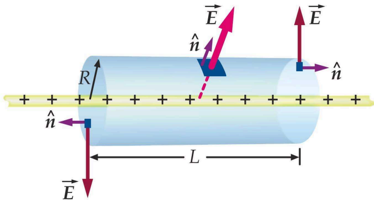
Cylindrical Symmetry - Problem II



Problem: Find the electric field strength for a very long wire carrying uniform charge density ($+\lambda$) as a function of the distance away from wire.

$$\Phi = \oint \vec{E} \cdot d\vec{A} = E \oint dA \oint dA = \frac{Q_{\text{enclosed}}}{\varepsilon_0} = \frac{\lambda L}{\varepsilon_0}$$

$$\Rightarrow E\, 2\pi R L \; 2\pi R L = \frac{\lambda L}{\varepsilon_0} \Rightarrow E = \frac{\lambda}{2\pi R \varepsilon_0}$$

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Conductors

Let's move to the chalkboard.

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