## 0.1 Implications

### Maxwell's Equations

$$\oint_S \vec{\mathbf{E}} \cdot d\vec{\mathbf{A}} = \frac{Q_{\text{enclosed}}}{\varepsilon_0}$$

$$\oint_C \vec{\mathbf{E}} \cdot d\vec{\mathbf{l}} = -\frac{d}{dt}\int \vec{\mathbf{B}} \cdot d\vec{\mathbf{A}}. \quad \text{(Changing } B\text{-flux creates } E\text{-field.)}$$

$$\oint_S \vec{\mathbf{B}} \cdot d\vec{\mathbf{A}} = 0.$$

$$\oint_C \vec{\mathbf{B}} \cdot d\vec{\mathbf{l}} = \mu_0 I_{\text{enclosed}} + \mu_0\,\varepsilon_0 \frac{d}{dt}\int \vec{\mathbf{E}} \cdot d\vec{\mathbf{A}}.$$

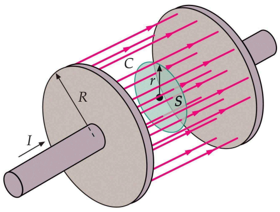


### Displacement Current

$$I_d = \varepsilon_0 \frac{d}{dt}\int \vec{\mathbf{E}} \cdot d\vec{\mathbf{A}}.$$

- Displacement current behaves like a real current.
  - That's right, a changing electric flux behaves like a conduction current.
  - A changing electric flux creates a magnetic field.
- Let's work out an example.

**Example - Displacement Current**




- A parallel-plate capacitor has closely spaced circular plates of radius $R$. Current $I$ is flowing onto the positive plate. Note: The surface $S$ is defined by a circle (radius $r < R$) centered along the axis of the plates. Find

  (a) the displacement current through the surface $S$ passing between the plates by directly computing $\frac{d\Phi_E}{dt}$ through $S$. Calculate $\Phi_E$.

$$\Phi_E = \int_S \vec{\mathbf{E}} \cdot d\vec{\mathbf{A}} = E\,\pi\,r^2.$$

$$\Phi_E = E\,\pi\,r^2. = \frac{q}{\pi\,R^2\,\varepsilon_0}\,\pi\,r^2$$

$$\Phi_E = \frac{q\,r^2}{R^2\,\varepsilon_0}$$

$$I_d = \varepsilon_0\,\frac{d}{dt}\Phi_E = \frac{r^2}{R^2}\,\frac{dq}{dt}$$

$$I_d = \varepsilon_0\,\frac{d}{dt}\Phi_E = \frac{r^2}{R^2}\,I$$

  (b) the magnetic field $B$ at a point $r$ from the axis of the plates when the current into the positive plate is $I$. We've already calculated $I_d$ through $S$. Apply Ampere's Law to surface $S$.

$$\int_{\partial S} \vec{\mathbf{B}} \cdot d\vec{\mathbf{l}} = \mu_0\,I_d$$

$$\int_{\partial S} \vec{\mathbf{B}} \cdot d\vec{\mathbf{l}} = B\,2\,\pi\,r$$

$$B\,2\,\pi\,r = \mu_0\,\frac{r^2}{R^2}\,I$$

$$B = \frac{\mu_0\,I\,r}{2\,\pi\,R^2}$$

## Implications of Maxwell's Equations

$$\oint_S \vec{\mathbf{E}} \cdot d\vec{\mathbf{A}} = \frac{Q_{\text{enclosed}}}{\varepsilon_0}$$

$$\oint_C \vec{\mathbf{E}} \cdot d\vec{\mathbf{l}} = -\frac{d}{dt} \int \vec{\mathbf{B}} \cdot d\vec{\mathbf{A}}. \quad \text{(Changing } B\text{-flux creates } E\text{-field.)}$$

$$\oint_S \vec{\mathbf{B}} \cdot d\vec{\mathbf{A}} = 0.$$

$$\oint_C \vec{\mathbf{B}} \cdot d\vec{\mathbf{l}} = \mu_0 \, I_{\text{enclosed}} + \mu_0 \, \varepsilon_0 \frac{d}{dt} \int \vec{\mathbf{E}} \cdot d\vec{\mathbf{A}}.$$

- Maxwell's equations drastically changed the way people viewed light.



## Changed Perceptions

- Light was once believed to need some medium in order to travel through space (think of sound waves).
- Maxwell's equations lead to a "wave equation" for electromagnetic waves. The waves travel with a speed given by $v = \frac{1}{\sqrt{\mu_0 \, \varepsilon_0}} \approx 3 \times 10^8 \, m/s$.
- Light is an electromagnetic wave - NO NEED FOR AN EXTERNAL MEDIUM.
- Awesome - Light is like a self-propelling virus.



## Thank You.

- I hope you leave this class with an appreciation of the underlying laws governing behaviors of charged particles.
- I will leave you with a quote from Isaac Asimov (*Inside the Atom*):
  "There is active pleasure in knowing, in understanding. If we do no more with our learning than look at the world about us with more understanding eyes, it will have paid for itself many times over. And there is always a chance that someone will start to learn with only the intention of being an appreciative spectator and end by finding himself part of the game."
  Good bye - Video Clip.