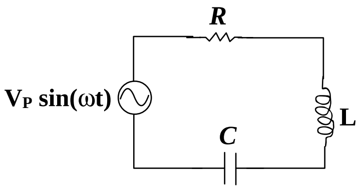
0.1 Series RLC Circuit

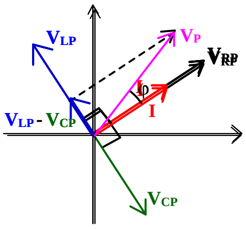
Series RLC Circuit - Phasors

- Let's analyze the following circuit.



- Calculate the peak current through the circuit
  - ○ We need to keep track of the phase differences PHASORS!

Phasor Analysis (A Single Moment in Time)



- The voltage across the resistor is represented by the phasor above since the driving voltage is sinusoidal. The current is in phase with the voltage across the resistor. The voltage across an inductor leads the current by $\pi/2$. The voltage across a capacitor lags behind the current by $\pi/2$. Apply Kirchhoff's Loop rule to find a relationship between all the voltages. Summing the phasors for the voltage across the capacitor and inductor. Summing the phasors for the voltage across the resistor and capacitor/inductor. The length of the resultant phasor represents the peak voltage supplied by the AC Voltage source. Finding relationships between the peak current in the circuit and the peak voltages is now a trigonometry problem. NOTE: The driving peak voltage is out of phase with the peak current through the circuit.
  $\tan\varphi = \frac{V_{LP}-V_{CP}}{V_{RP}} = \frac{\chi_L-\chi_c}{R}$

$$V_P = \sqrt{V_{RP}^2 + (V_{LP} - V_{CP})^2}$$

Finding Peak Current in RLC - Circuit

$$V_P = \sqrt{V_{RP}^2 + (V_{LP} - V_{CP})^2}$$

$$V_{RP} = I_P\, R$$

$$V_{LP} = I_P\, \chi_L$$

$$V_{CP} = I_P\, \chi_c$$

$$V_P = I_P \sqrt{R^2 + (\chi_L - \chi_C)^2}$$

$$\Rightarrow I_P = \frac{V_P}{\sqrt{R^2 + (\chi_L - \chi_C)^2}}$$ (Resembles Ohm's Law)

$$Z \equiv \sqrt{R^2 + (\chi_L - \chi_C)^2} \Rightarrow I_P = \frac{V_P}{Z}$$

Impedance

- The quantity Z is called the impedance of this series circuit.
- Impedance is a generalization of resistance to include the frequency-dependent effects of capacitance and inductance.

Current and Driving Voltage

- In an AC circuit containing resistors, inductors, and capacitors, the current through the circuit will not be in phase with the driving voltage source.

$$\tan\varphi = \frac{\chi_L - \chi_c}{R}$$

- A purely resistive circuit will have $\tan\varphi = 0 \Rightarrow \varphi = 0$.
- The current in a purely resistive circuit will be in phase with the driving voltage.

0.2 Time Averages

Power in AC Circuits

- Can we talk about power in AC circuits?
  - It is more difficult than DC Circuits because of the phase shifts.
  - Remember, without phases $P = I^2 R$.
  - There is a standard engineering technique that allows one to discuss the average power.
  - What is the average of a sinusoidally varying function over one period of oscillation? ZERO.
  - Does it make sense to talk about averages for sinusoidally varying functions? Yes, because the wall socket is a type of average.

0.3 Definition

Definition of Root-Mean-Square

- The average of a sine function (or cosine) is zero over one time period.
- If we square a sine (or cosine) function, then its average is 1/2 over one time period.
- Defining the root-mean-square (engineering practice) as:

$$V = V_P \sin \omega t$$

$$V_{RMS} = \sqrt{\left\langle V_P^2 \sin^2 \omega t \right\rangle} \quad \text{where } \langle\rangle \text{ denotes time-average}$$

$$\left\langle \sin^2 \omega t \right\rangle = \frac{1}{T} \int_0^T \sin^2 \omega t \, dt \quad \text{where } T \text{ is one period}$$

$$\left\langle \sin^2 \omega t \right\rangle = \frac{1}{2}$$

$$V_{RMS} = V_P \frac{1}{\sqrt{2}}$$

Time-Averaged Power

- The time-average product of voltage and current with an arbitrary phase difference φ is given by

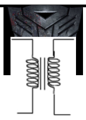
$$\begin{aligned}
&\langle P\rangle = \langle I_P\,\sin(\omega t+\varphi)\,V_P\,\sin\omega t\rangle \\
&= I_P\,V_P\,\big\langle(\sin^2\omega t)\,(\cos\varphi) + (\sin\omega t)(\cos\omega t)(\sin\varphi)\big\rangle \\
&\langle P\rangle = \frac{1}{2}\,I_P\,V_P\,\cos\varphi \\
&V_P = \sqrt{2}\,V_{RMS} \quad \text{and } I_P = \sqrt{2}\,I_{RMS} \\
&\langle P\rangle = I_{RMS}\,V_{RMS}\,\cos\varphi
\end{aligned}$$

$\cos\varphi$ (is called the power factor.)

Transforming Voltage Amplitudes - AC - Circuits

slide 9

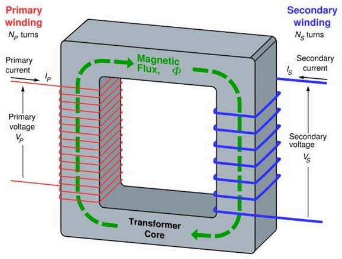
Transformers



- Now that we have power dissipated through an RLC series circuit, let’s address an important issue.
- Not all devices require 120-V AC. Some devices require only 12-V AC.
    - How do we “transform” the amplitude of the voltage provided by the power company to another amplitude?
    - We go back to Faraday’s Law of Induction.
- If we strategically place two *different* solenoids near each other in an AC circuit, then the EMF through the solenoids will have different values.
- A device which uses an arrangement of coils to vary the amplitude of the primary voltage source is called a transformer and one of its circuit symbol is shown above in the title.

Transformers - Picture

- The artist rendition below is that of a typical transformer.



- Iron core used to concentrate magnetic flux which ensures
  - the magnetic flux through primary and secondary coils is the same.

Transformers - Voltage

- Since the magnetic flux is the same through both coils, the rate of change of magnetic flux is the same through the two coils.

$$V_P = N_P \frac{d\Phi_B}{dt}$$

$$V_S = N_S \frac{d\Phi_B}{dt}$$

$$\Rightarrow \frac{V_P}{N_P} = \frac{V_S}{N_S}$$

$$\Rightarrow V_S = V_P \frac{N_S}{N_P}$$

Transformers - Power

- It seems that the secondary voltage can be arbitrarily large.
- Does this violate conservation of energy?
  - No.
  - A transformer can not increase power.
- Ideal transformers transfer all the power supplied by the primary source to the secondary.

$$I_P\,V_P = I_S\,V_S \quad \text{(Statement of Conservation of Energy)}$$