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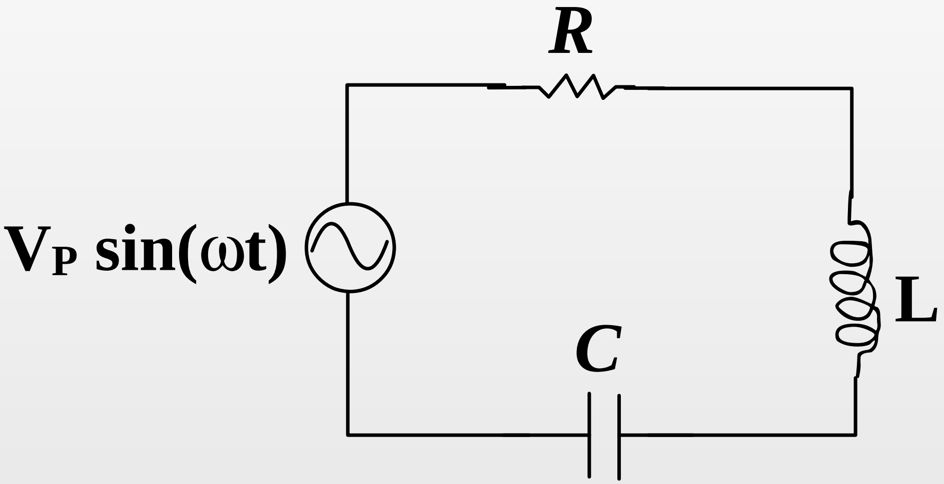

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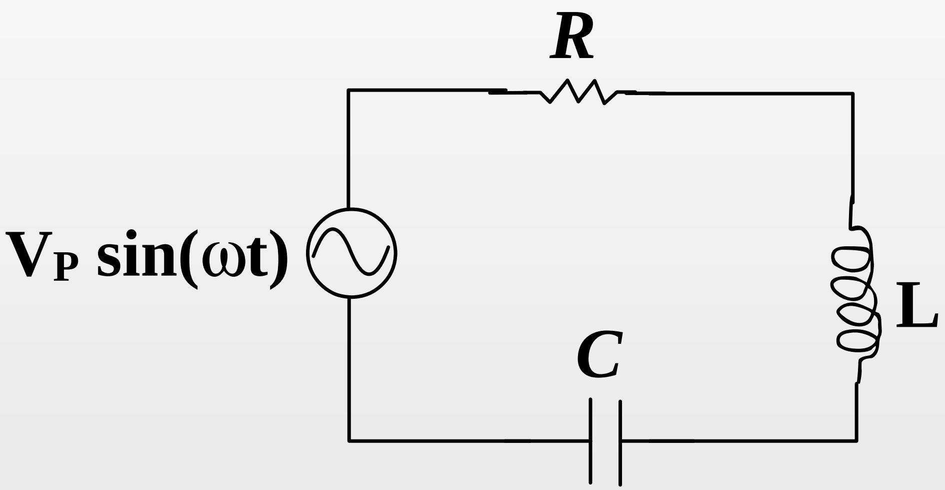

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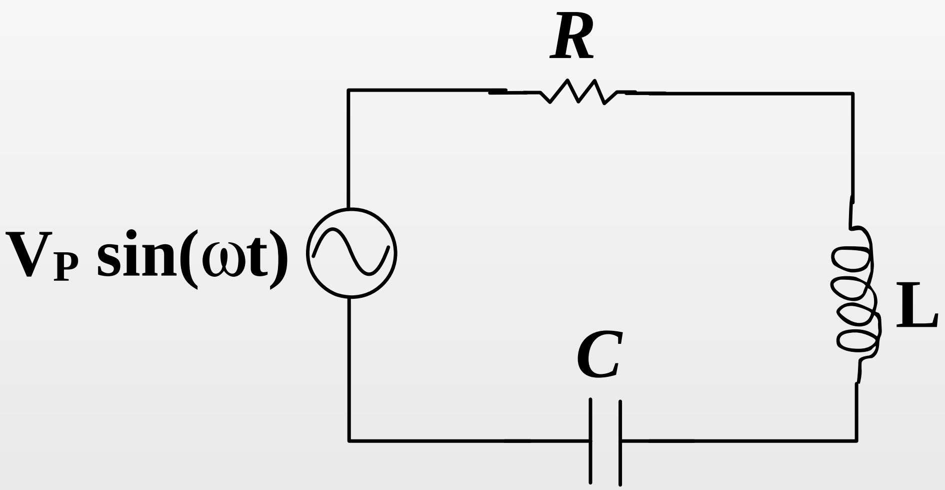

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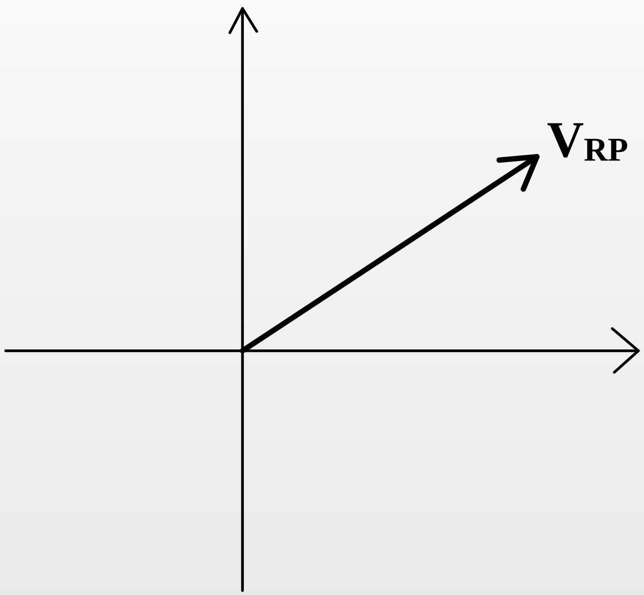
Phasor Analysis (A Single Moment in Time)



- The voltage across the resistor is represented by the phasor above since the driving voltage is sinusoidal.

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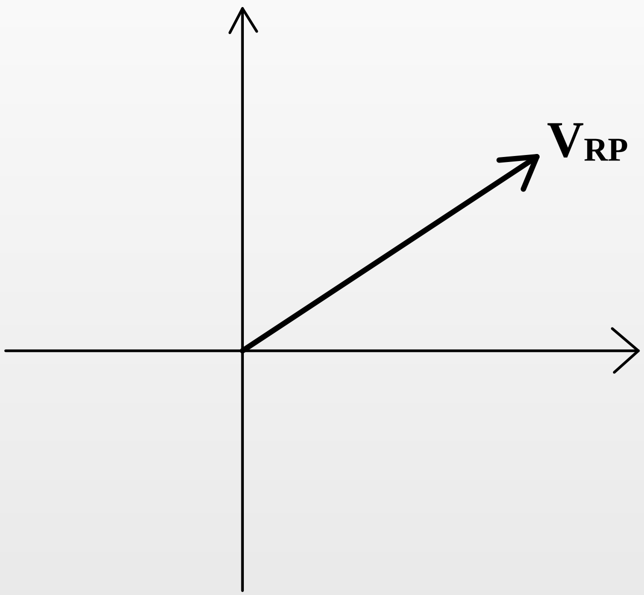

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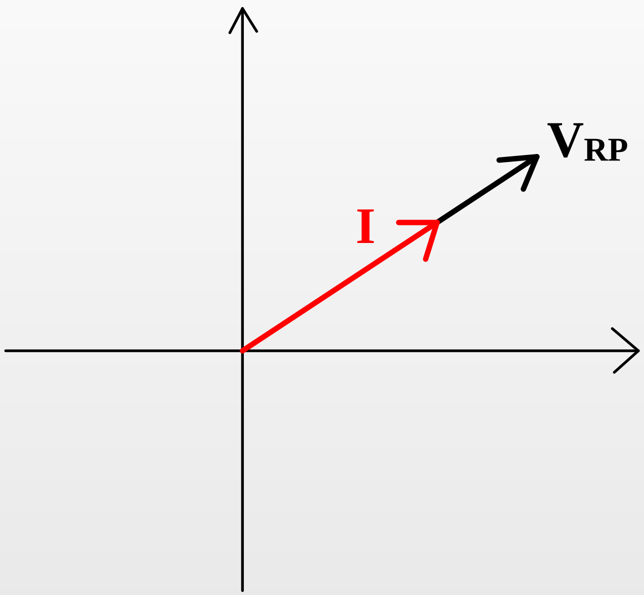

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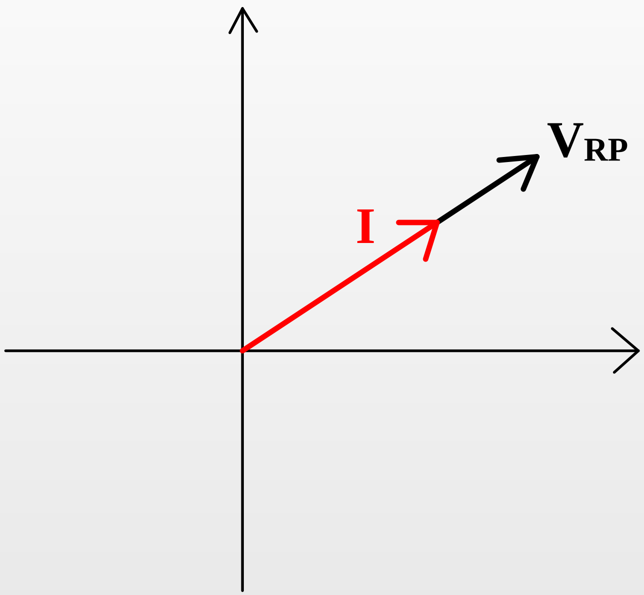

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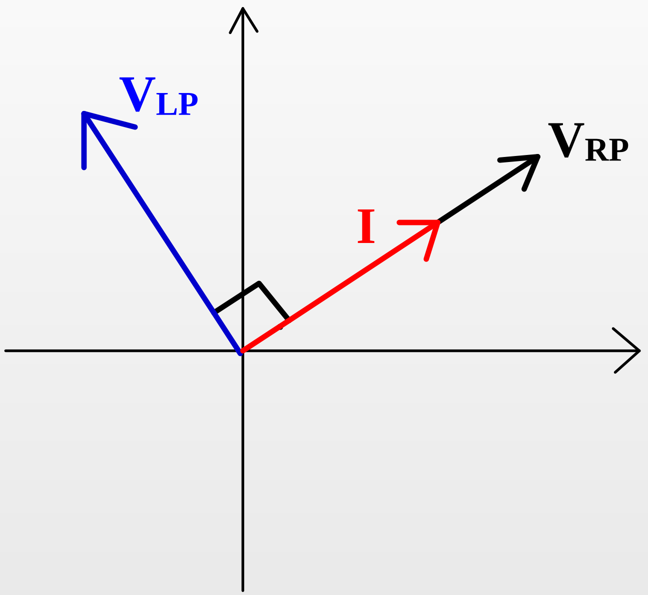

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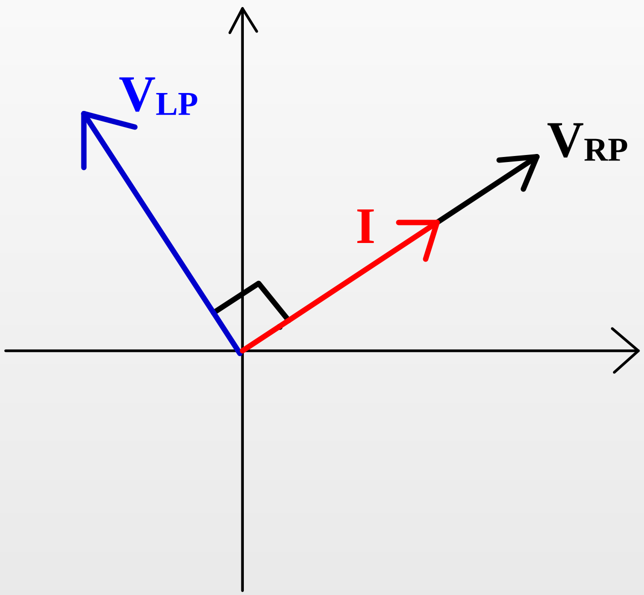

- The voltage across a capacitor lags behind the current by $\pi/2$.

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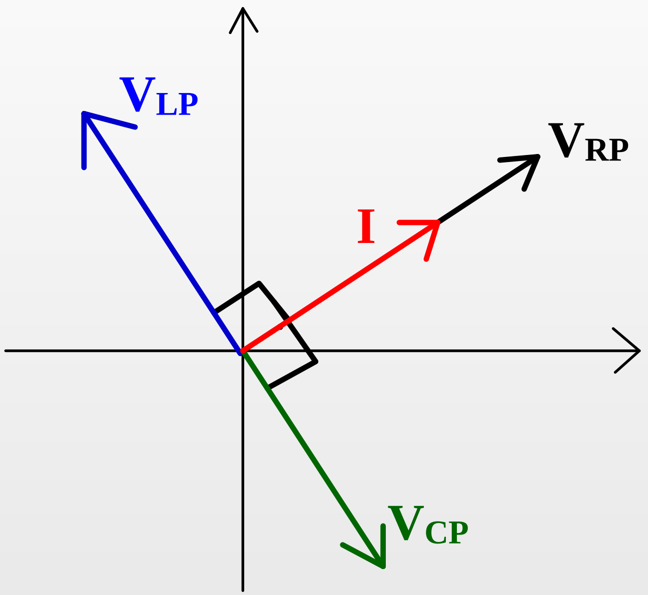

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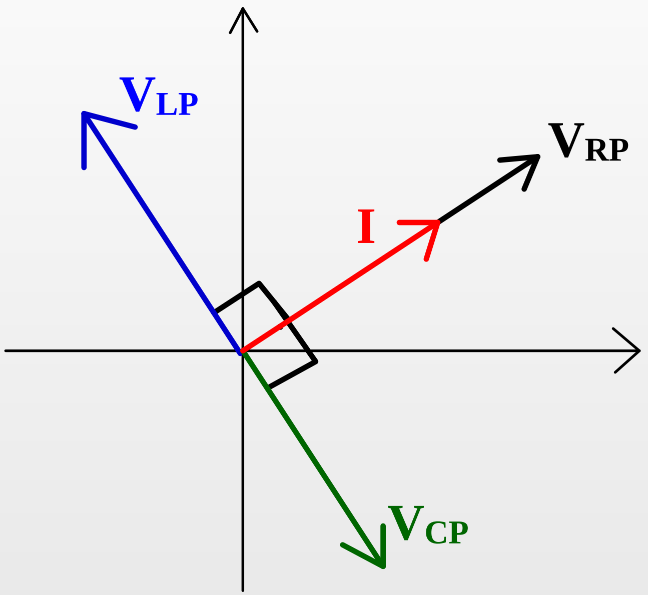

- Apply Kirchhoff's Loop rule to find a relationship between all the voltages.

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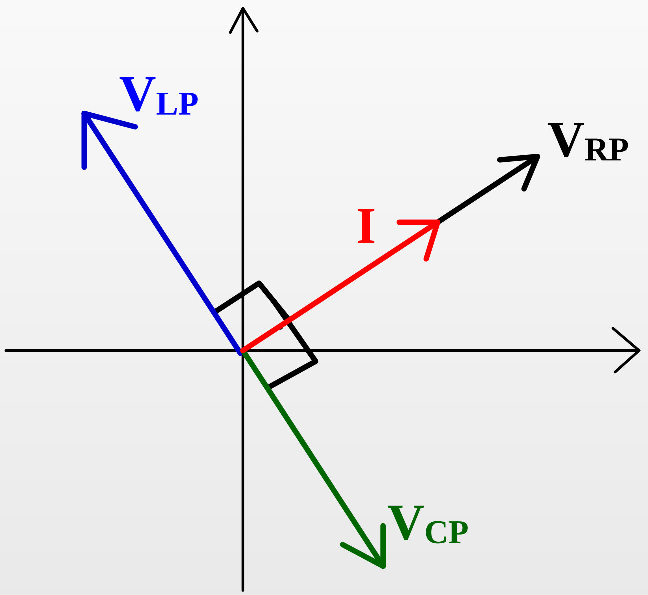

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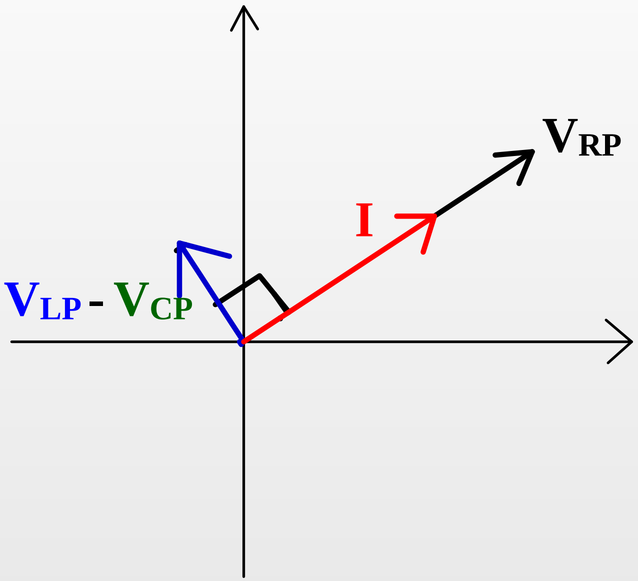

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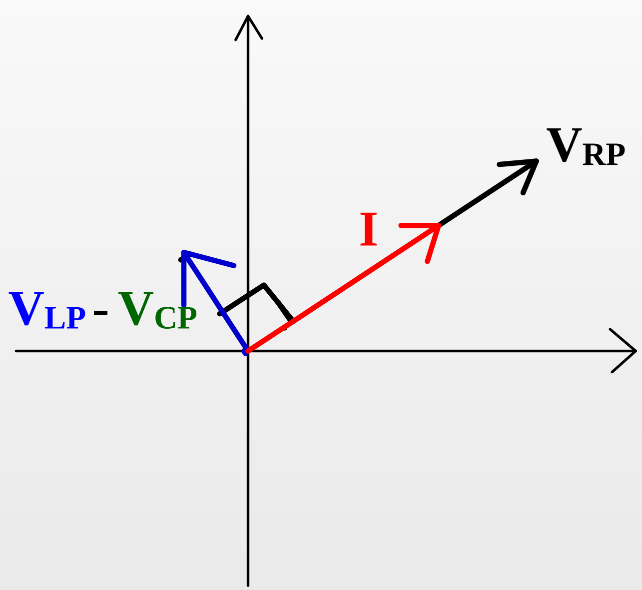

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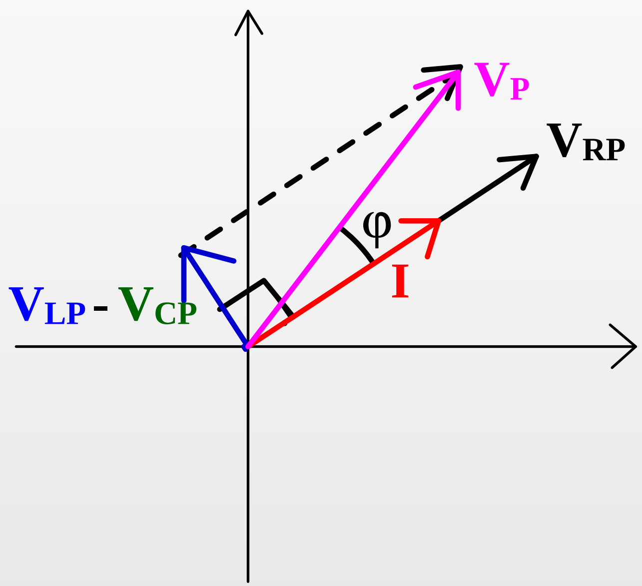

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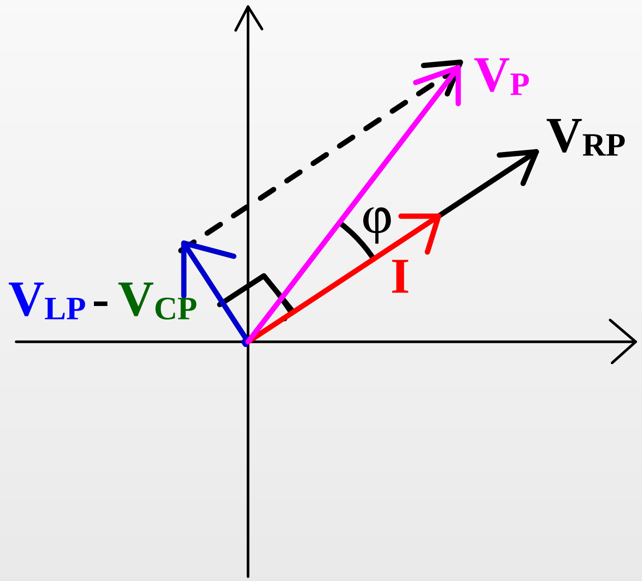

- The length of the resultant phasor represents the peak voltage supplied by the AC Voltage source.

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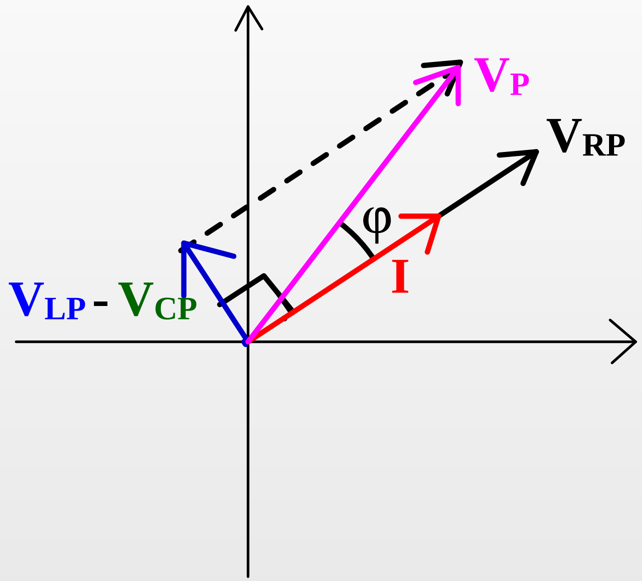

- Finding relationships between the peak current in the circuit and the peak voltages is now a trigonometry problem.

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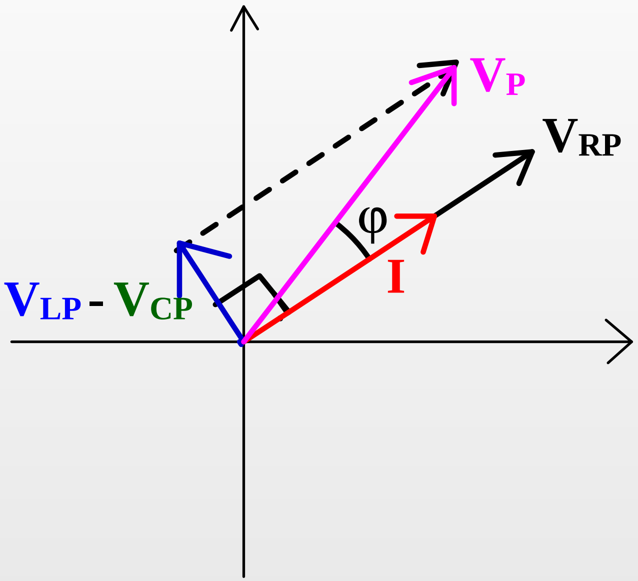

- NOTE: The driving peak voltage is out of phase with the peak current through the circuit.

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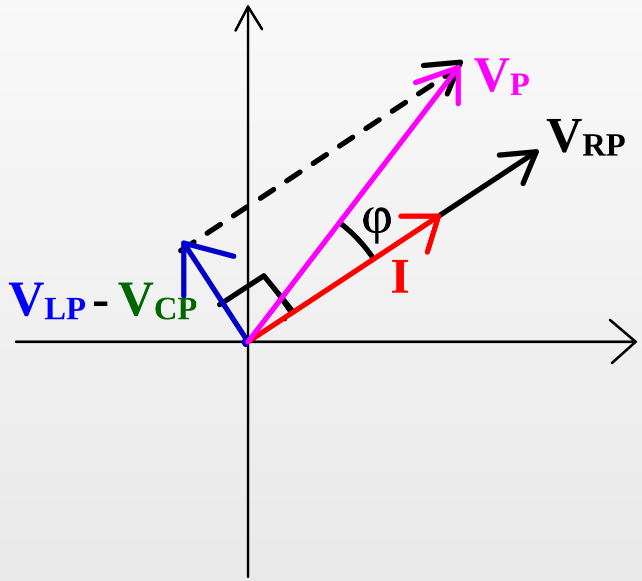

- $\tan\varphi = \frac{V_{LP} - V_{CP}}{V_{RP}} = \frac{\chi_L - \chi_c}{R}$

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$\cos \varphi$ (is called the power factor.)

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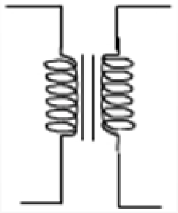

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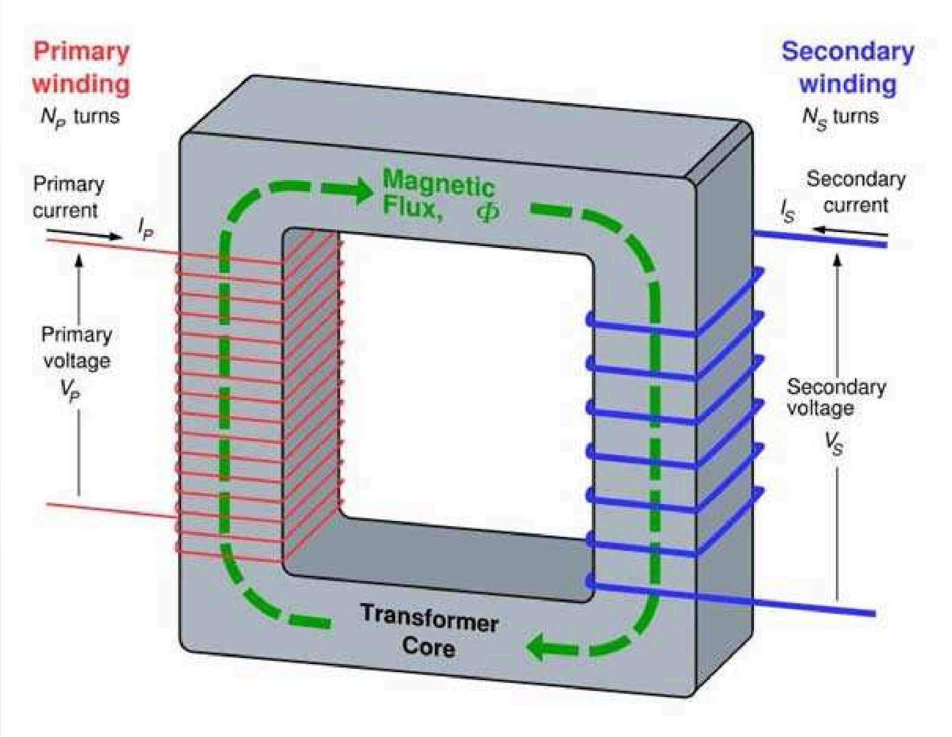

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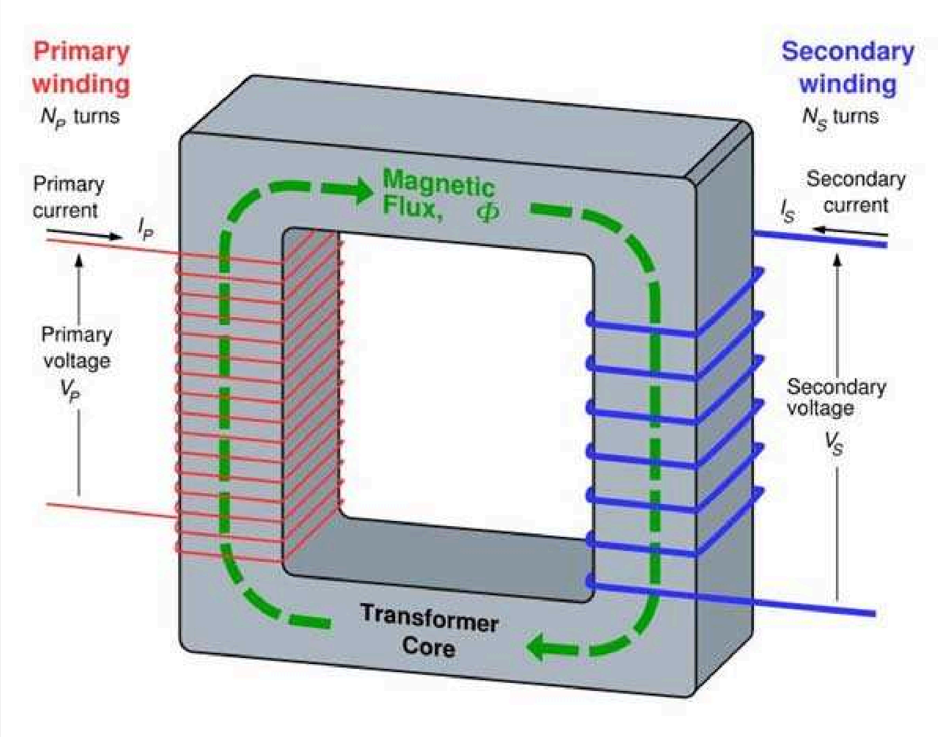

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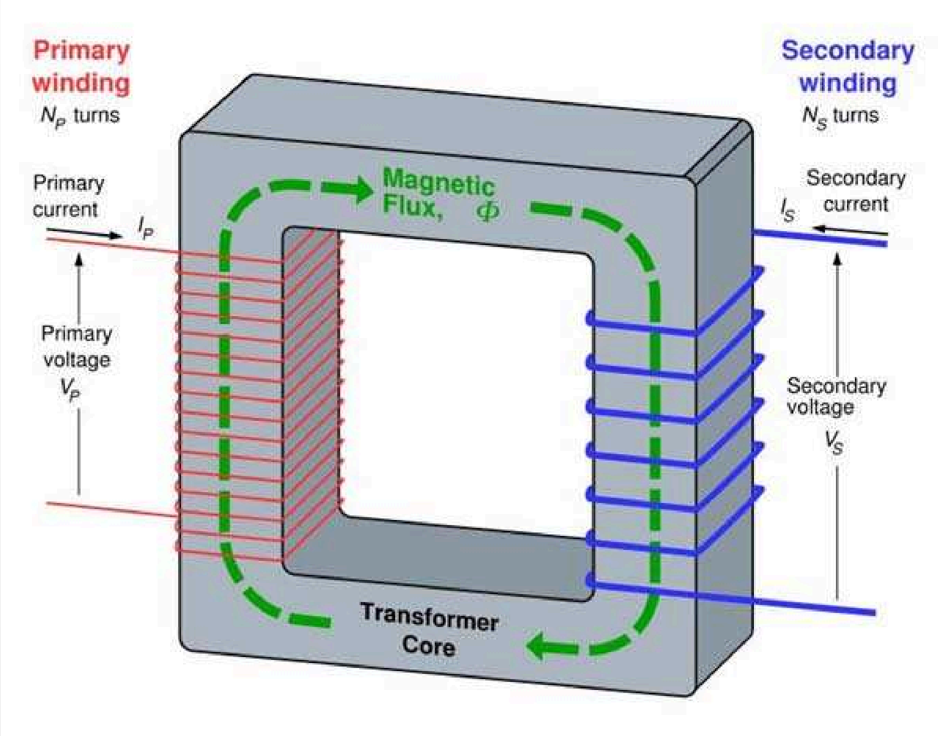

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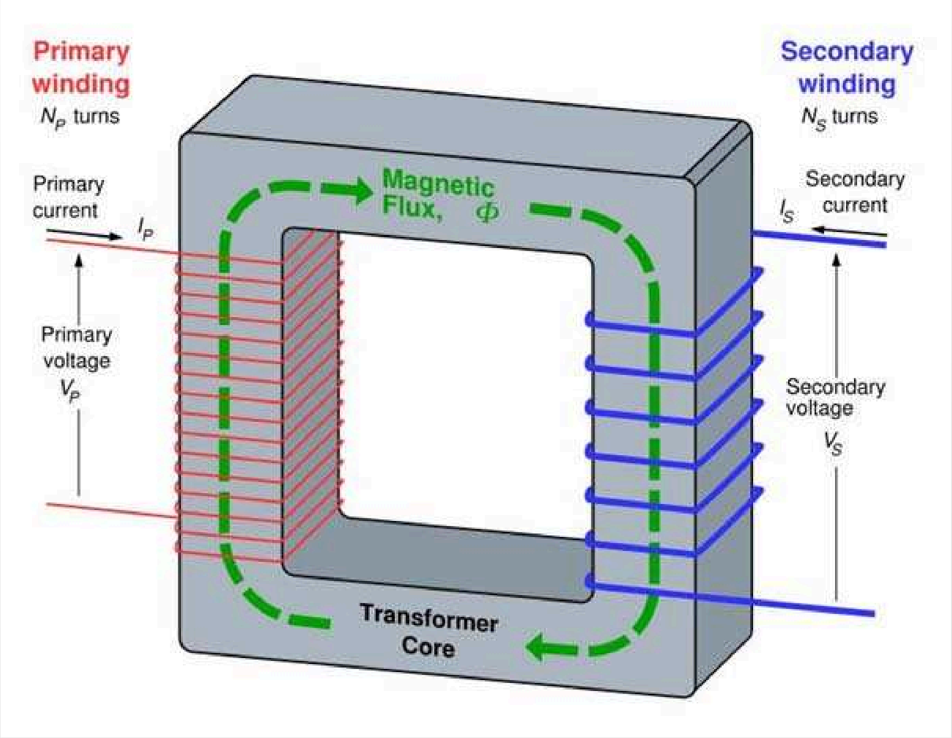
Transformers - Picture

- The artist rendition below is that of a typical transformer.



- Iron core used to concentrate magnetic flux which ensures
  - the magnetic flux through primary and secondary coils is the same.

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$$I_P V_P = I_S V_S \quad \text{(Statement of Conservation of Energy)}$$