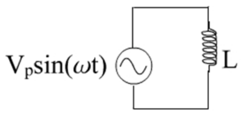
0.1 Inductors in an AC circuit

Time Varying Voltages and Inductors

- Consider an inductor (L) connected in series with an alternating voltage ($V_P\ \sin\omega t$) as shown below.



- ◦ Let's calculate the current through the inductor:

$$V(t) = L\,\frac{dI}{dt}$$

$$L\,I(t) = \int V_P\ \sin\omega t\,dt$$

$$I(t) = -\frac{V_P}{\omega\,L}\ \cos\omega t$$

- The current through a inductor is "out-of-phase" with the driving voltage source.

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"Out-of-Phase?" for Inductor

- The time dependent voltage was given by

$$V(t) = V_P\ \sin\omega t$$

- The current through the inductor is given by

$$I(t) = -\frac{V_P}{\omega\,L}\cos\omega t \qquad\qquad I(t) = \frac{V_P}{\omega\,L}\left(-\cos\omega t\right) \rightarrow \frac{V_P}{\omega\,L}\sin\left(\omega t - \frac{\pi}{2}\right)$$

- From trigonometry:

$$-\cos\omega t = \sin\left(\omega t - \frac{\pi}{2}\right)$$

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Inductive Reactance

- The current through the inductor is

$$I(t) = \frac{V_P}{\omega L} \sin\left(\omega t - \frac{\pi}{2}\right)$$

- The current through the inductor is $\frac{\pi}{2}$ out of phase with the driving voltage.
  - Current lags behind the driving voltage by 90°.
- The peak current through the capacitor is $I_P = \frac{V_P}{\omega L}$
  - This resembles Ohm's Law with $I_P = \frac{V_P}{\chi_L}$
  - The term $\chi_L = \omega L$ has a unit of Ohm and is called inductive reactance (χ_L)

Properties of Inductive Reactance

- The reactance for an inductor describes the behavior of a inductor placed in a circuit with a time-varying voltage source.

$$\chi_L = \omega L$$

- When ω is large, χ_L is large so the inductor offers greater "resistance" to current flow.
- When ω is small, χ_L is small so the inductor offers less "resistance" to current flow.
- χ_L is NOT the same as resistance because NO POWER IS DISSIPATED THROUGH A INDUCTOR.

Adding up voltages (currents) in AC circuits

- When several components are connected in *series*, their potential differences add.
- When several components are connected in *parallel*, their currents add.
- Adding sines and cosines of differing amplitude *and phases* is algebraically awkward.
  - We could graphically add up the potentials (or currents) (THINK VECTORS).
  - This method for adding up potentials (or currents) is called "Phasor analysis"

Phasors

- Any quantity that has a harmonic time dependence can be associated with a *rotating* vector known as a **phasor**.
- For the function

  $$V(t) = V_0 \sin(\omega t)$$

  - The phasor lies in the xy-plane with it tail fixed at the origin.
  - The amplitude of the vector is V_0.
  - Time dependence is described by a *counterclockwise* rotation with angular speed ω.
  - The function $V(t)$ is the instantaneous projection of the phasor on the y-axis.
  - Click here for phasor animation.