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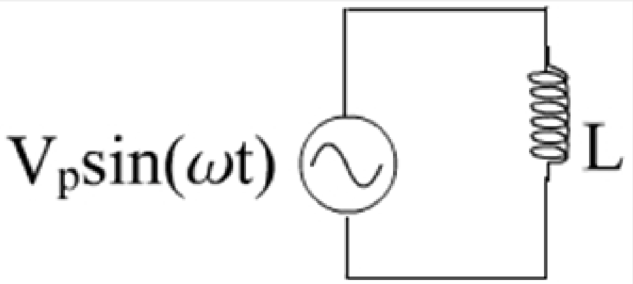

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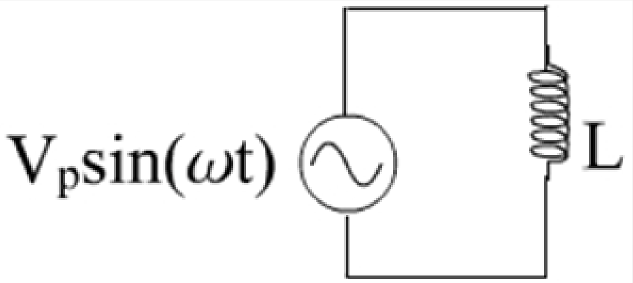

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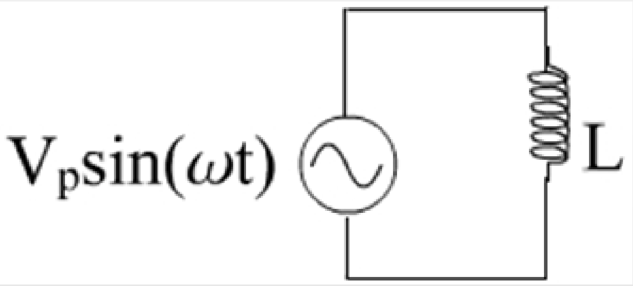

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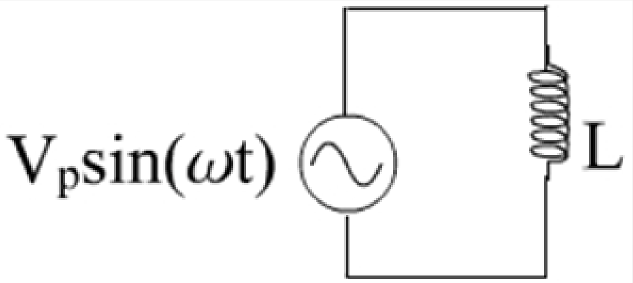

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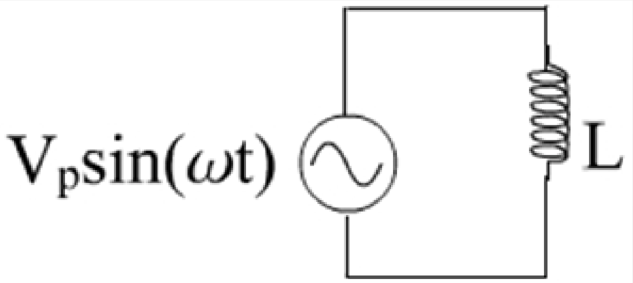

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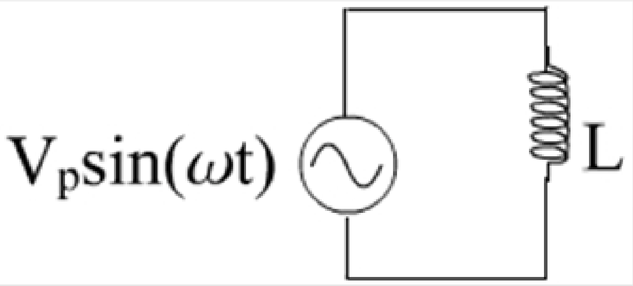

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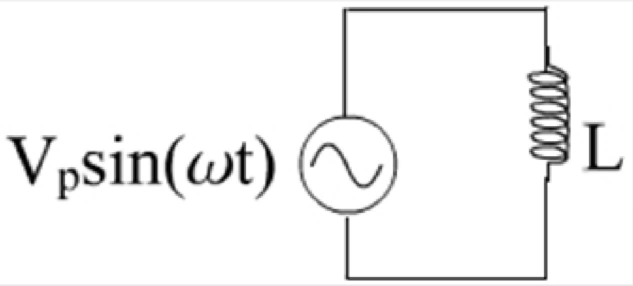

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- The current through a inductor is "out-of-phase" with the driving voltage source.

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  - The term $\chi_L = \omega\, L$ has a unit of Ohm and is called inductive reactance (χ_L)

Properties of Inductive Reactance

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- When ω is small, χ_L is small so the inductor offers less "resistance" to current flow.
- χ_L is NOT the same as resistance because NO POWER IS DISSIPATED THROUGH A INDUCTOR.

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  - This method for adding up potentials (or currents) is called "Phasor analysis"

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  - Click here for phasor animation.