PHYS102
AC-Circuits

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0.1 Qualitative Behavior

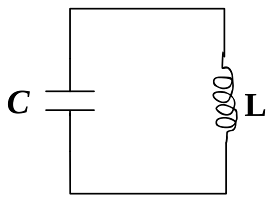
Energy Oscillations

- Since capacitors store electrical energy and inductors store magnetic energy, we could place a fully charged capacitor in series with an inductor.
- The electrical energy should be transferred to magnetic energy ,and then the magnetic energy should get transferred back into electrical energy. This cycle should repeat itself. Let's prove it.
- Since energy stored in a capacitor is proportional to Q^2, it suffices to prove that the charge on the capacitor "oscillate".

PHYS102 AC Circuits – slide 2

0.2 LC - Circuits - Analysis

LC - Circuit



- Start with a *fully charged capacitor* and place it in series with an inductor as shown above.
- Write down the total energy of the system at some time t after the capacitor is connected to the inductor.

$$U_T = \frac{1}{2}\frac{Q^2}{C} + \frac{1}{2}L\,I^2$$
$$\frac{dU_T}{dt} = 0 = \frac{Q}{C}\frac{dQ}{dt} + L\,I\,\frac{dI}{dt}$$
$$0 = \frac{Q}{C} + L\,\frac{d^2Q}{dt^2}$$

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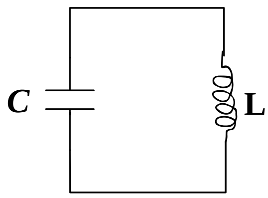
Charge Oscillations



$$\frac{d^2 Q}{dt^2} = -Q/LC = -\omega^2 Q$$

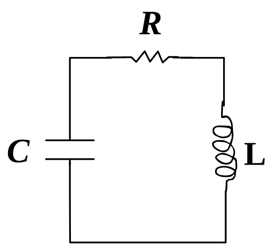
- This equation defines simple harmonic motion with an angular frequency $\omega = \frac{1}{\sqrt{LC}}$. The charge on the capacitor, Q, is undergoing simple harmonic motion.
- From mechanics, we know a solution for $Q(t)$:

$$Q(t) = Q_0 \sin(\omega t + \phi)$$

PHYS102 AC Circuits – slide 4

0.3 RLC - Circuits - No External Driving Voltage

Effects of Resistance on LC Circuits



- The charge on the capacitor, Q, will undergo simple harmonic motion when the capacitor is directly connected across an inductor.
- What if we were to consider the finite resistance R of the wires connecting the inductor and the capacitor?
  - What would happen to the period of oscillation?
  - What would happen to the current in the circuit after a very long time?

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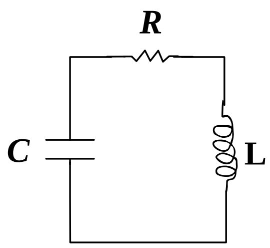
RLC - Damped Oscillations

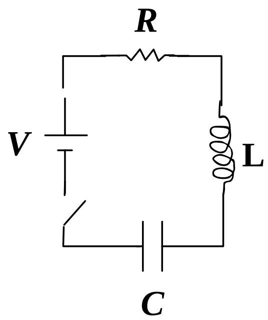


- The overall behavior of the charge oscillation will be affected because the resistance of the wire will cause energy loss (due to heating). The larger the resistance, the faster energy gets dissipated (and the longer the period of oscillation becomes).

$$P_R = I^2\,R$$

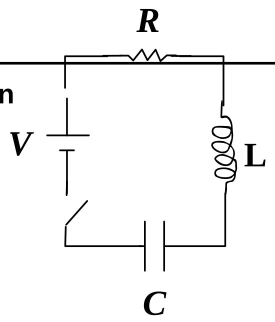
- Since the charge (or current) oscillations decrease over time, this type of oscillation behavior is termed a *"damped oscillation"*.

RLC - DC - Situation



- What would happen if we connected a battery, a resistor, an inductor, and a capacitor in series with a switch?
- There would be NO current at the moment the switch is closed (WHY?) and the current would be zero after a very long time after the switch is closed (WHY?).
- Not very interesting is it.

RLC - AC - Introduction



- What if we connected the inductor, capacitor, and resistor in series with a time varying voltage source (like the wall socket in this room)?
- I would say that we need to understand the behavior of each component individually under a time varying voltage.

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Alternating Current Circuits slide 9

0.4 Introduction

Introduction

- So far all of the circuits mentioned in this course have been **D**irect **C**urrent (DC) circuits.
- **A**lternating **C**urrent (AC) circuits are circuits that have time varying currents.
- Time varying currents are produced by time varying voltage sources.
- We use a new symbol to indicate time varying voltages.

PHYS102 AC Circuits – slide 9

Time Varying Voltages

- We will restrict discussion to sinusoidally varying voltages of the form

$$V(t) = V_P \sin(\omega t + \phi)$$

where V_P is the "peak" voltage (amplitude), ω is the angular frequency, and ϕ is the phase.

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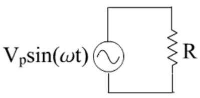
Time Varying Voltages and Resistors

- Consider a resistor (R) connected in series with an alternating voltage ($V_P \sin\omega t$) as shown below.

  - Applying Kirchhoff's rules:



$$V(t) = I(t)\,R$$

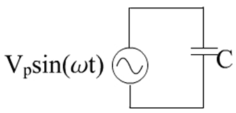
$$I(t) = \frac{V(t)}{R} = \frac{V_P \sin\omega t}{R}$$

- The current through a resistor is "in-phase" with the driving voltage source.

Time Varying Voltages and Capacitors

- Consider a capacitor (C) connected in series with an alternating voltage ($V_P \sin\omega t$) as shown below.

  - Let's calculate the current through the capacitor:



$$q(t) = C\,V(t)$$

$$I(t) = \frac{dq}{dt} = C\,\frac{dV}{dt}$$

$$I(t) = C\,\omega\,V_P \cos\omega t$$

- The current through a capacitor is "out-of-phase" with the driving voltage source.

"Out-of-Phase?"

- The time dependent voltage was given by

$$V(t) = V_P \sin\omega t$$

- The current through the capacitor is given by

$$I(t) = C\,\omega\,V_P \cos\omega t \qquad I(t) = C\,\omega\,V_P \cos\omega t \rightarrow C\,\omega\,V_P \sin\left(\omega t + \frac{\pi}{2}\right)$$

- From trigonometry:

$$\cos\omega t = \sin\left(\omega t + \frac{\pi}{2}\right)$$

Capacitive Reactance

- The current through the capacitor is

$$I(t) = C\,\omega\,V_P\,\sin\left(\omega t + \frac{\pi}{2}\right)$$

- The current through the capacitor is $\frac{\pi}{2}$ out of phase with the driving voltage.
  - Current leads the voltage by 90°.
- The peak current through the capacitor is $I_P = C\,\omega\,V_P$
  - This resembles Ohm's Law with $I_P = \frac{V_P}{\chi_c}$
  - The term $\chi_c = 1/(C\,\omega)$ has a unit of Ohm and is called capacitive reactance (χ_c)

Please note this correction.

Properties of Capacitive Reactance

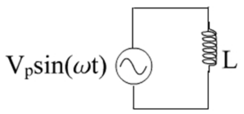
- The reactance for a capacitor describes the behavior of a capacitor placed in a circuit with a time-varying voltage source.

$$\chi_c = \frac{1}{C\,\omega}$$

- When ω is large, χ_c is small so the capacitor offers little "resistance" to current flow.
- When ω is small, χ_c is large so the capacitor offers greater "resistance" to current flow.
- χ_c is NOT the same as resistance because NO POWER IS DISSIPATED THROUGH A CAPACITOR.

Time Varying Voltages and Inductors

- Consider an inductor (L) connected in series with an alternating voltage ($V_P\,\sin\omega t$) as shown below.



  - Let's calculate the current through the inductor:

$$V(t) = L\,\frac{dI}{dt}$$

$$L\,I(t) = \int V_P\,\sin\omega t\,dt$$

$$I(t) = -\frac{V_P}{\omega L}\,\cos\omega t$$

- The current through a inductor is "out-of-phase" with the driving voltage source.

“Out-of-Phase?” for Inductor

- The time dependent voltage was given by

$$V(t) = V_P \sin \omega t$$

- The current through the inductor is given by

$$I(t) = -\frac{V_P}{\omega L} \cos \omega t \qquad I(t) = \frac{V_P}{\omega L} (-\cos \omega t) \rightarrow \frac{V_P}{\omega L} \sin\left(\omega t - \frac{\pi}{2}\right)$$

- From trigonometry:

$$-\cos \omega t = \sin\left(\omega t - \frac{\pi}{2}\right)$$

Inductive Reactance

- The current through the inductor is

$$I(t) = \frac{V_P}{\omega L} \sin\left(\omega t - \frac{\pi}{2}\right)$$

- The current through the inductor is $\frac{\pi}{2}$ out of phase with the driving voltage.
  - Current lags behind the driving voltage by 90°.
- The peak current through the capacitor is $I_P = \frac{V_P}{\omega L}$
  - This resembles Ohm’s Law with $I_P = \frac{V_P}{\chi_L}$
  - The term $\chi_L = \omega L$ has a unit of Ohm and is called inductive reactance (χ_L)

Properties of Inductive Reactance

- The reactance for an inductor describes the behavior of a inductor placed in a circuit with a time-varying voltage source.

$$\chi_L = \omega L$$

- When ω is large, χ_L is large so the inductor offers greater “resistance” to current flow.
- When ω is small, χ_L is small so the inductor offers less “resistance” to current flow.
- χ_L is NOT the same as resistance because NO POWER IS DISSIPATED THROUGH A INDUCTOR.