PHYS102
AC-Circuits

Dr. Suess

April 16, 2007

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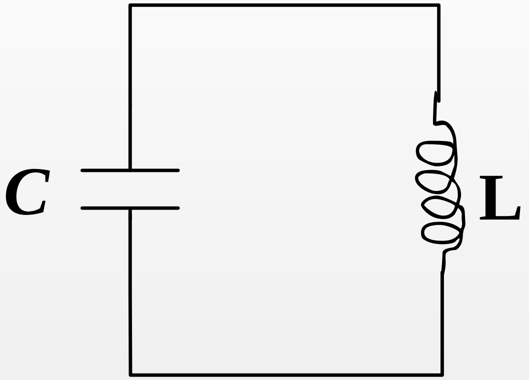
- The electrical energy should be transferred to magnetic energy ,and then the magnetic energy should get transferred back into electrical energy. This cycle should repeat itself. Let's prove it.
- Since energy stored in a capacitor is proportional to Q^2, it suffices to prove that the charge on the capacitor "oscillate".

LC - Circuit



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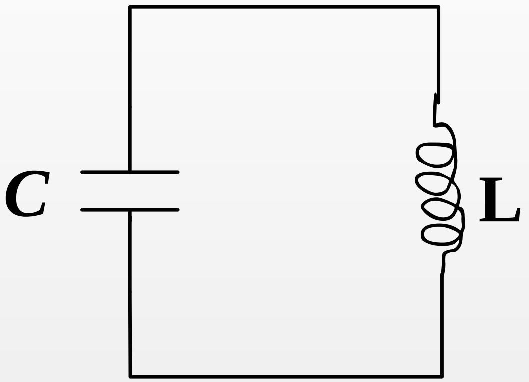

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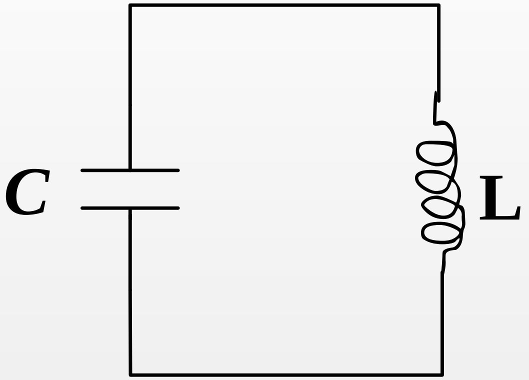

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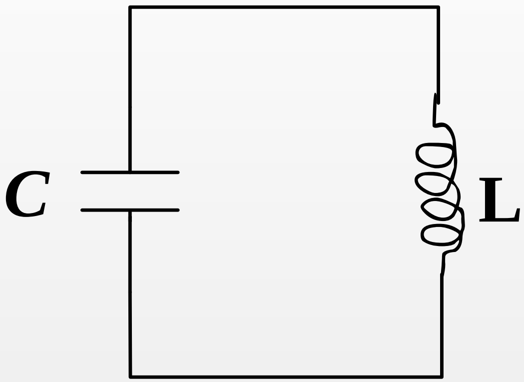

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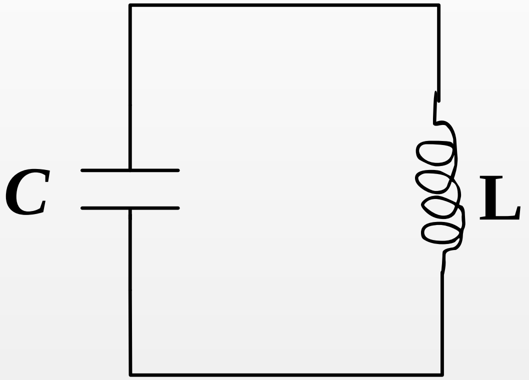

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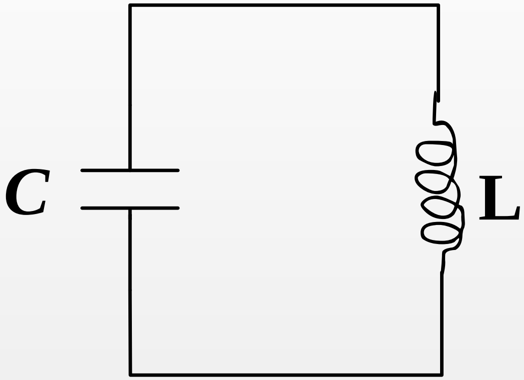

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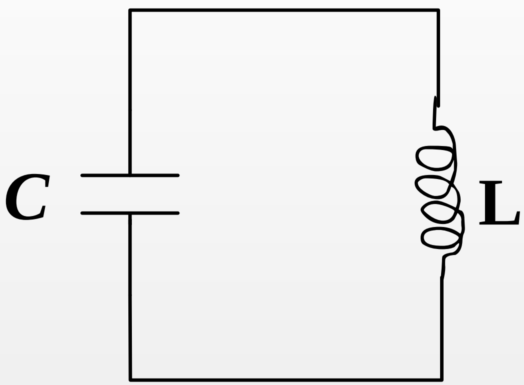

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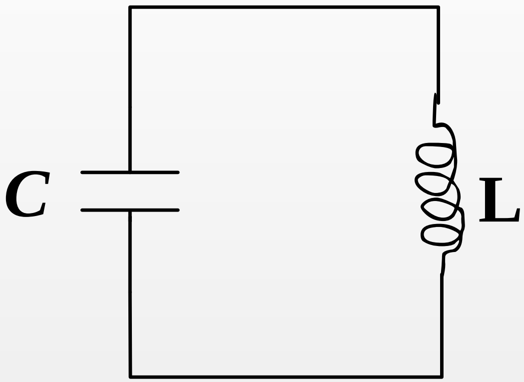

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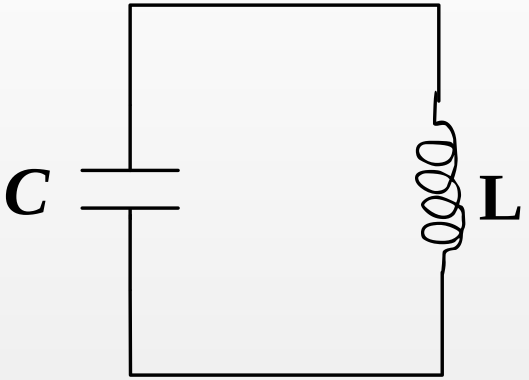
$$0 = \frac{Q}{C} + L\,\frac{d^2Q}{dt^2}$$

Charge Oscillations



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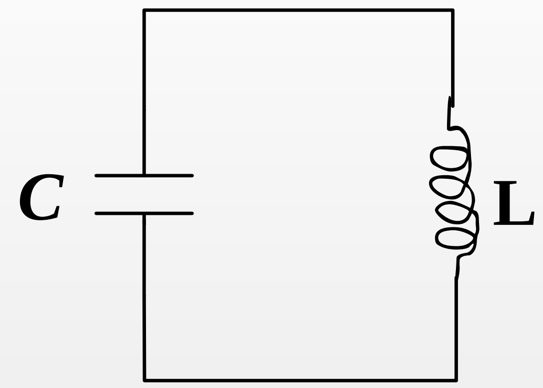

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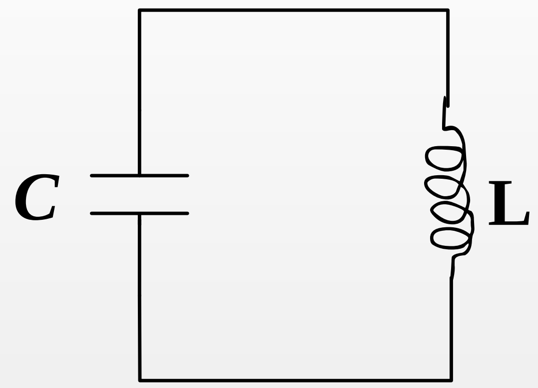

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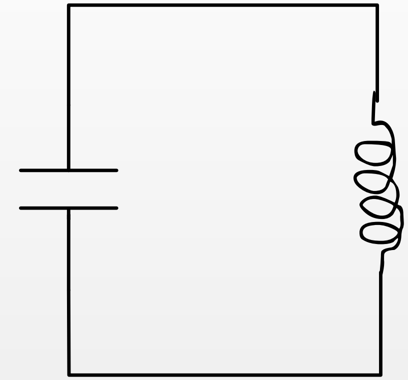

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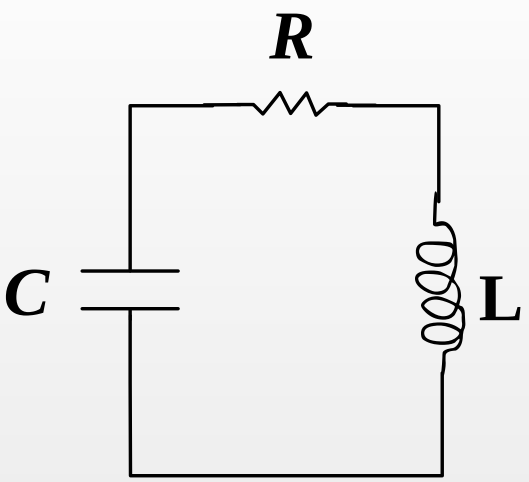
- This equation defines simple harmonic motion with an angular frequency $\omega = \frac{1}{\sqrt{LC}}$. The charge on the capacitor, Q, is undergoing simple harmonic motion.
- From mechanics, we know a solution for $Q(t)$:

$$Q(t) = Q_0 \sin(\omega t + \phi)$$

Effects of Resistance on LC Circuits



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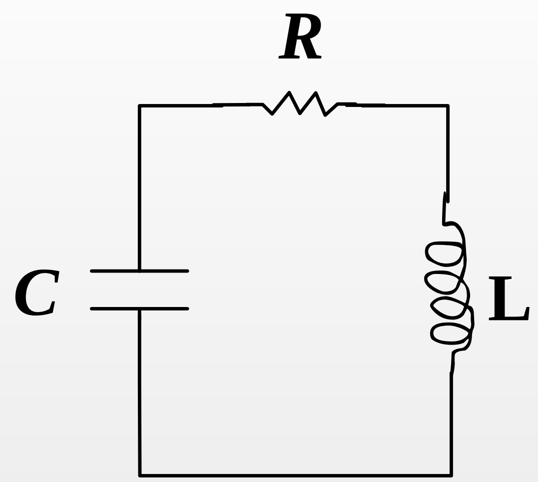

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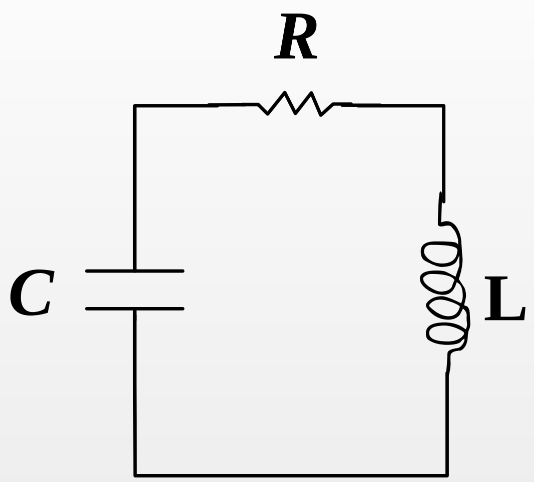

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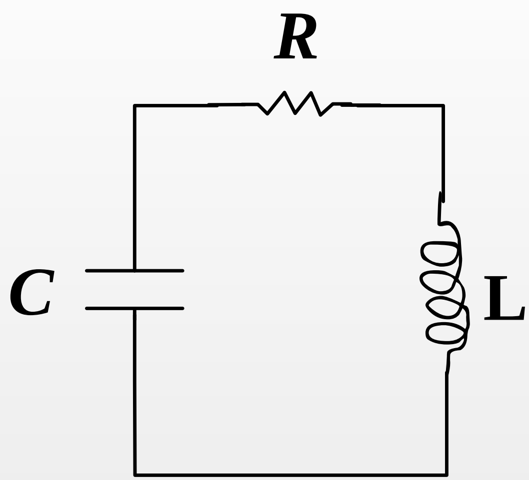

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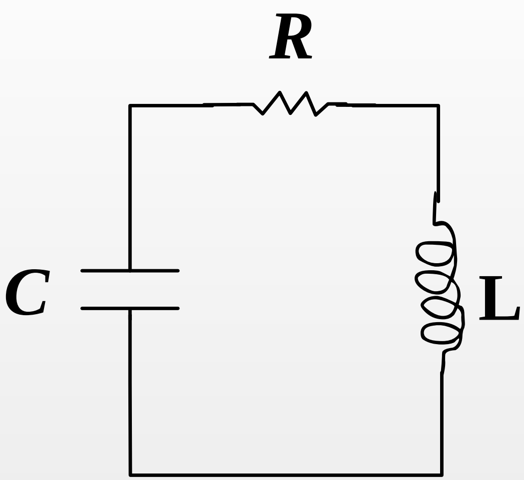

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- What if we were to consider the finite resistance R of the wires connecting the inductor and the capacitor?
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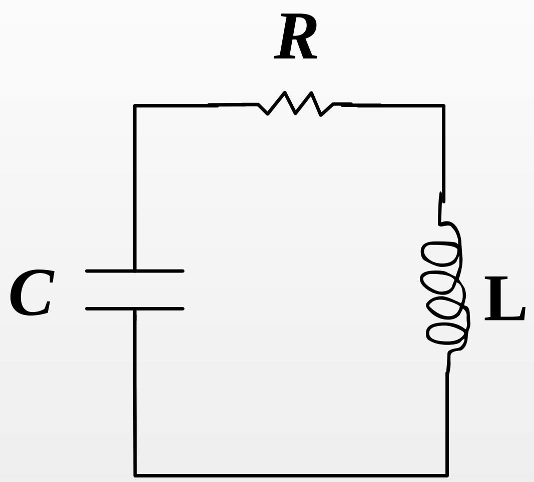
  - What would happen to the current in the circuit after a very long time?

RLC - Damped Oscillations



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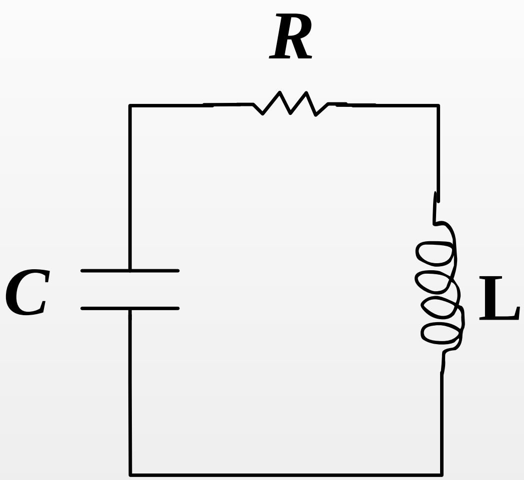

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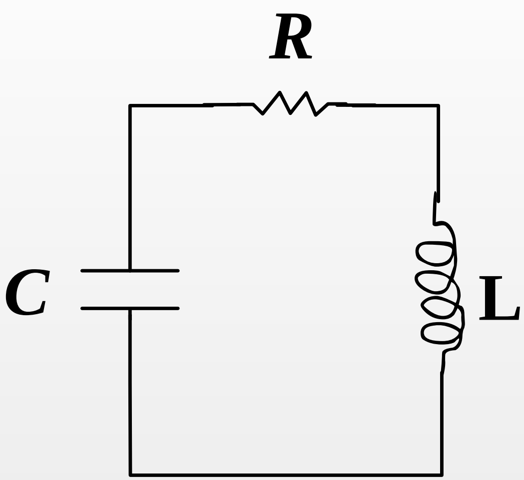

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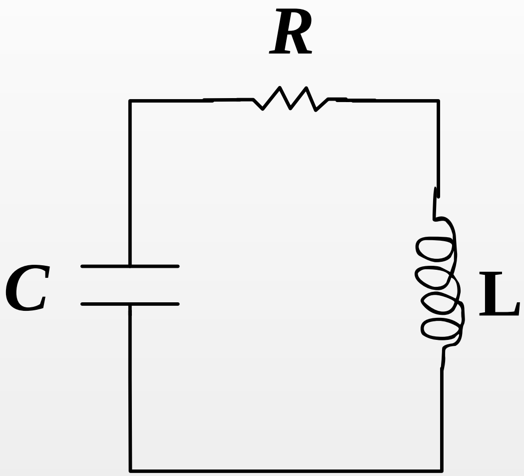

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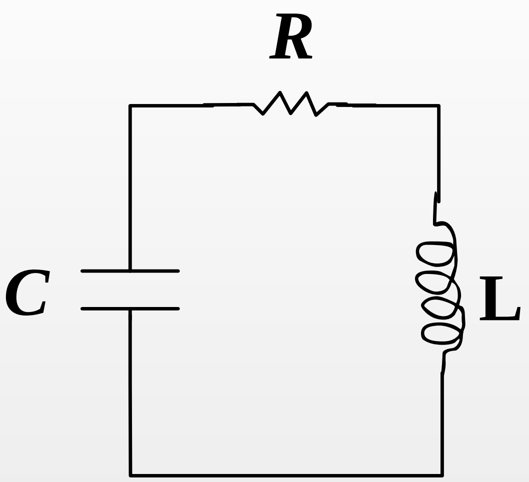

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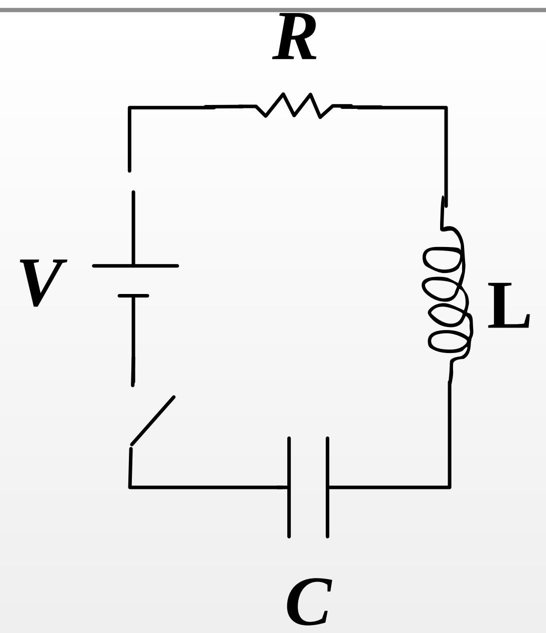
- Since the charge (or current) oscillations decrease over time, this type of oscillation behavior is termed a *“damped oscillation”*.

RLC - DC - Situation



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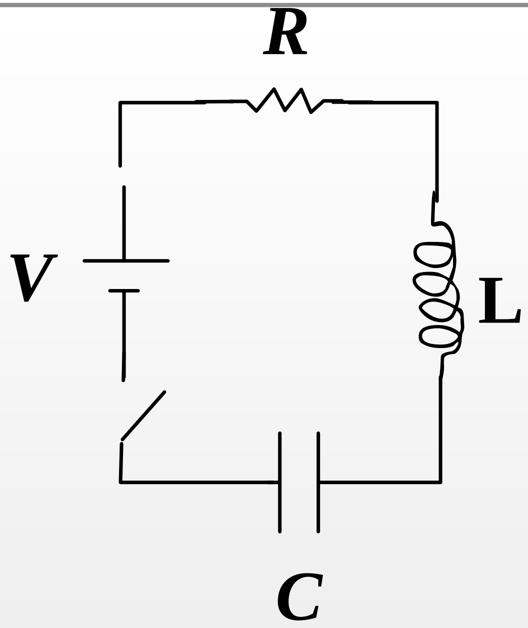

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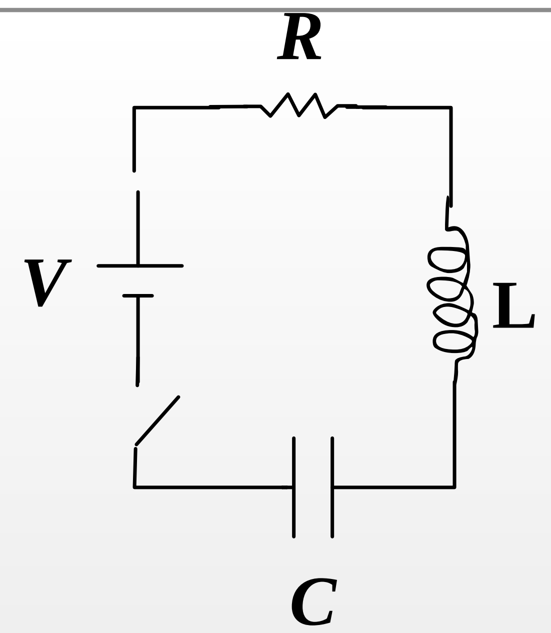

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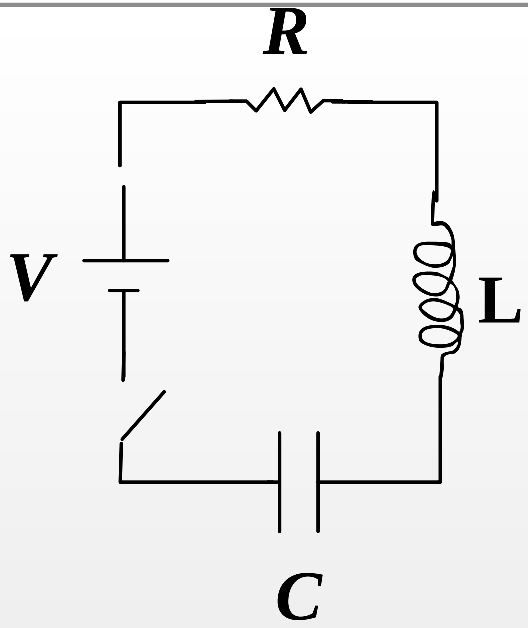

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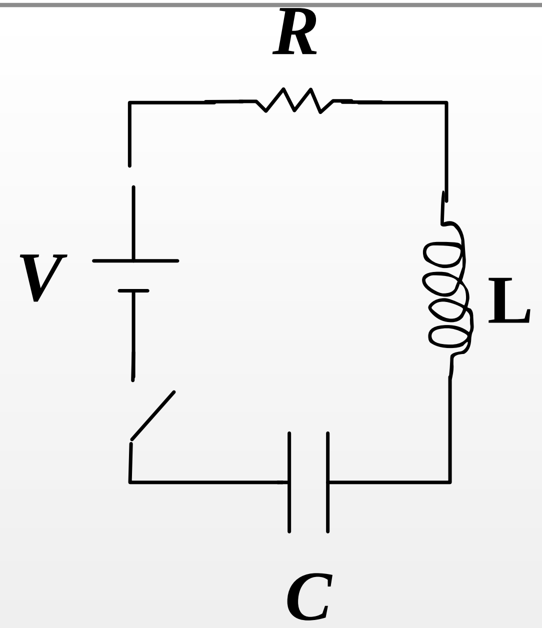

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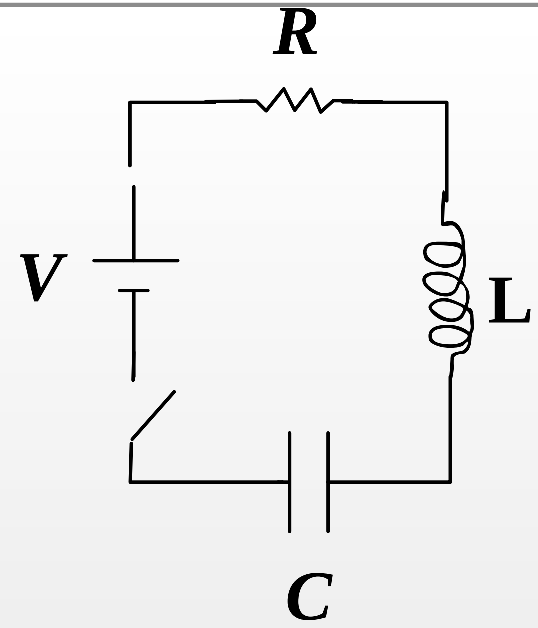
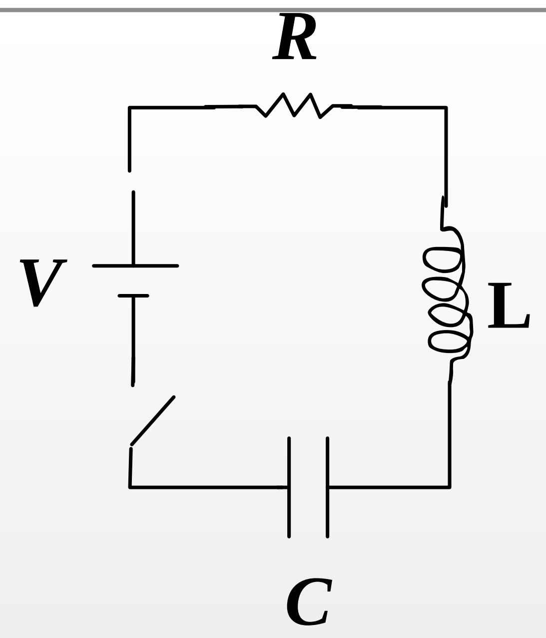


- What if we connected the inductor, capacitor, and resistor in series with a time varying voltage source (like the wall socket in this room)?

RLC - AC - Introduction



- What if we connected the inductor, capacitor, and resistor in series with a time varying voltage source (like the wall socket in this room)?
- I would say that we need to understand the behavior of each component individually under a time varying voltage.

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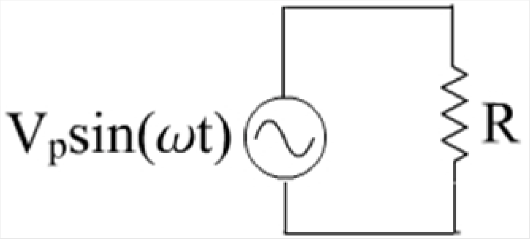
where V_P is the "peak" voltage (amplitude), ω is the angular frequency, and ϕ is the phase.

Time Varying Voltages and Resistors

- Consider a resistor (R) connected in series with an alternating voltage ($V_P \sin \omega t$) as shown below.



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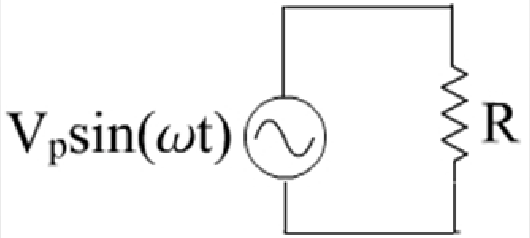

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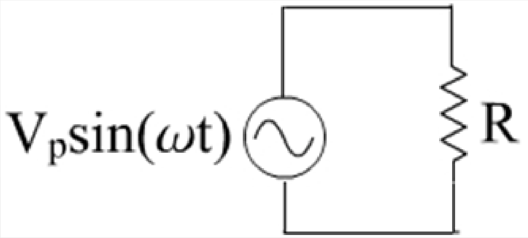

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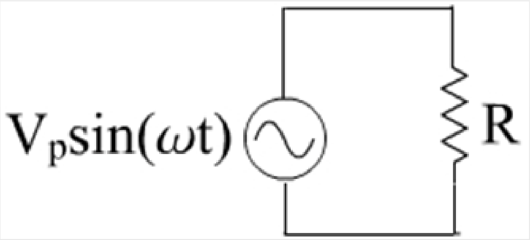

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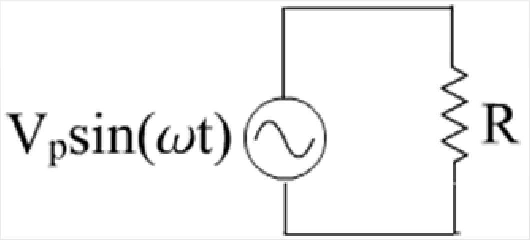

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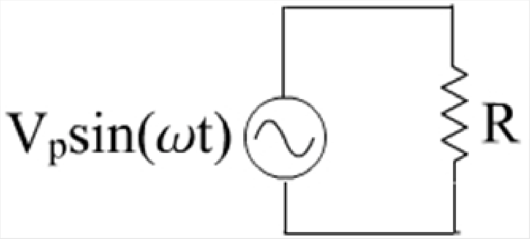

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- The current through a resistor is "in-phase" with the driving voltage source.

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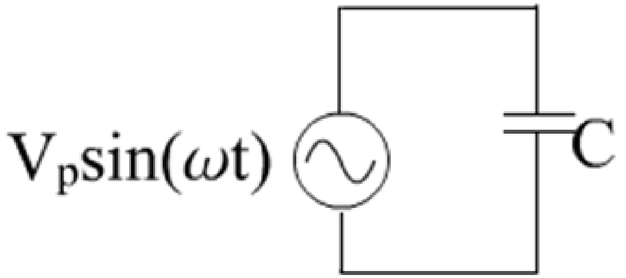

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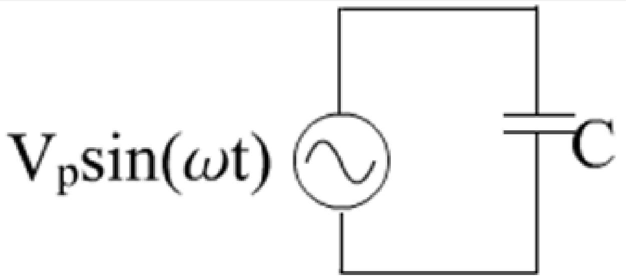

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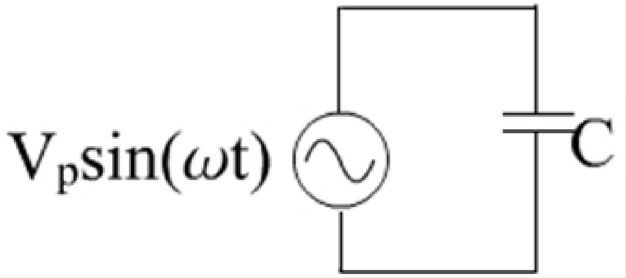

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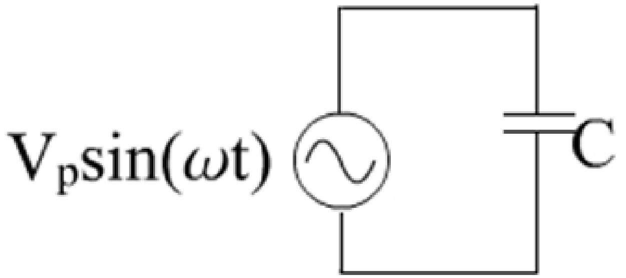

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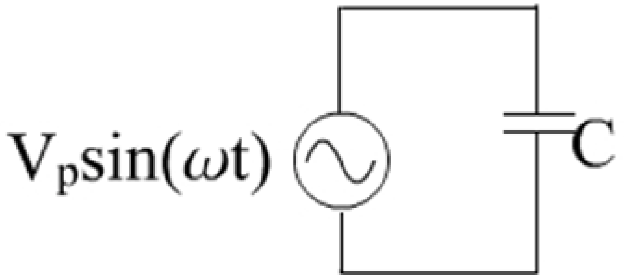

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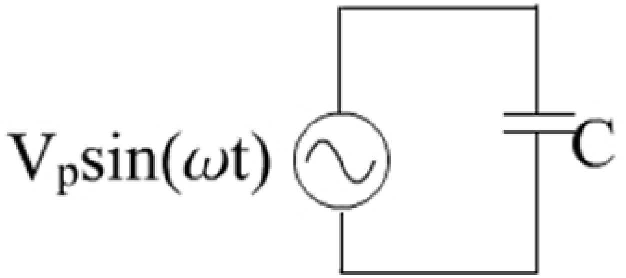

$$q(t) = C\,V(t)$$

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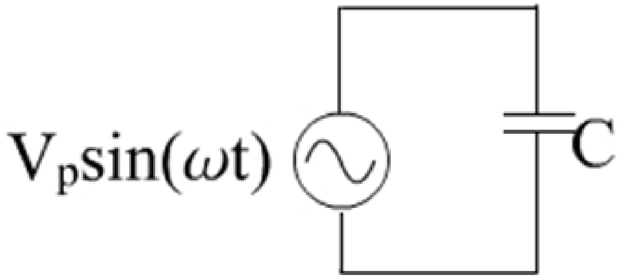
$$I(t) = C\,\omega\,V_P\,\cos \omega t$$

Time Varying Voltages and Capacitors

- Consider a capacitor (C) connected in series with an alternating voltage ($V_P \sin \omega t$) as shown below.
    - Let’s calculate the current through the capacitor:



$$q(t) = C\,V(t)$$

$$I(t) = \frac{dq}{dt} = C\,\frac{dV}{dt}$$

$$I(t) = C\,\omega\,V_P\,\cos \omega t$$

- The current through a capacitor is “out-of-phase” with the driving voltage source.

“Out-of-Phase?”

- The time dependent voltage was given by

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$$V(t) = V_P \sin \omega t$$

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- The time dependent voltage was given by

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“Out-of-Phase?”

- The time dependent voltage was given by

$$V(t) = V_P \sin \omega t$$

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$$I(t) = C\,\omega\,V_P \cos \omega t$$

- From trigonometry:

$$\cos \omega t = \sin \left(\omega t + \frac{\pi}{2} \right)$$

“Out-of-Phase?”

- The time dependent voltage was given by

$$V(t) = V_P \sin \omega t$$

- The current through the capacitor is given by

$$I(t) = C\,\omega\,V_P \cos \omega t \rightarrow C\,\omega\,V_P \sin\left(\omega t + \frac{\pi}{2}\right)$$

- From trigonometry:

$$\cos \omega t = \sin\left(\omega t + \frac{\pi}{2}\right)$$

Capacitive Reactance

- The current through the capacitor is

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$$I(t) = C\,\omega\,V_P\,\sin\left(\omega t + \frac{\pi}{2}\right)$$

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  - Current leads the voltage by 90°.

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Please note this correction.

Properties of Capacitive Reactance

- The reactance for a capacitor describes the behavior of a capacitor placed in a circuit with a time-varying voltage source.

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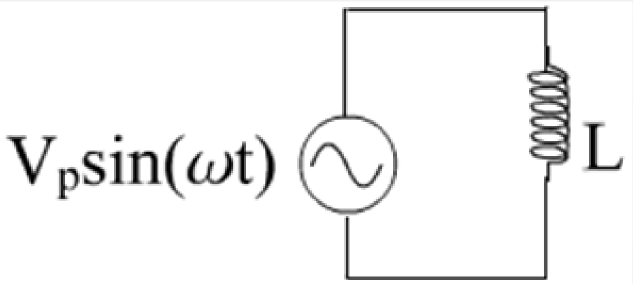
- χ_c is NOT the same as resistance because NO POWER IS DISSIPATED THROUGH A CAPACITOR.

Time Varying Voltages and Inductors

- Consider an inductor (L) connected in series with an alternating voltage ($V_P \sin \omega t$) as shown below.



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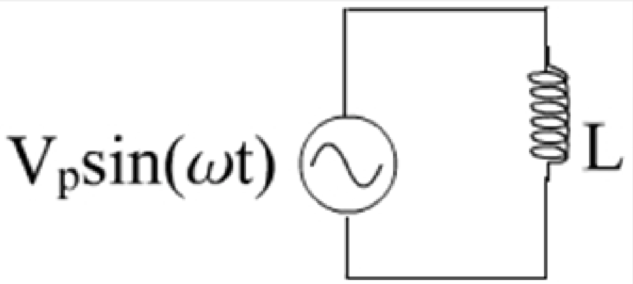

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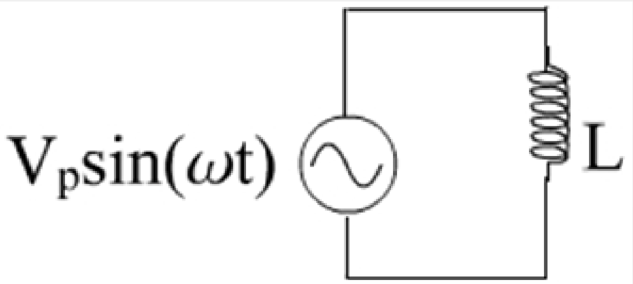

$$V(t) = L\frac{dI}{dt}$$

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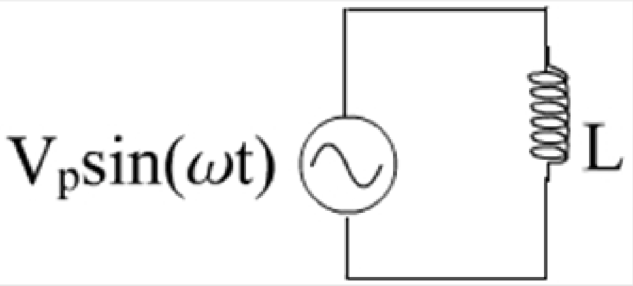

$$V(t) = L\,\frac{dI}{dt}$$

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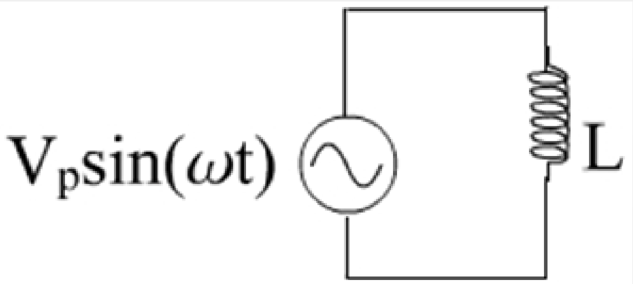

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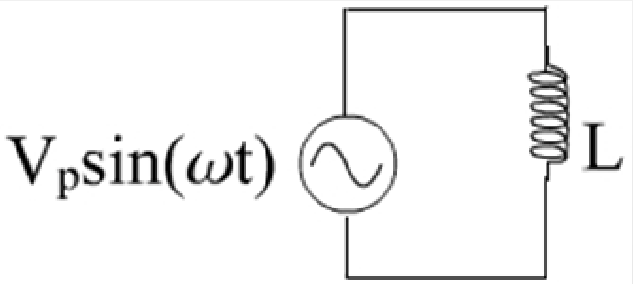

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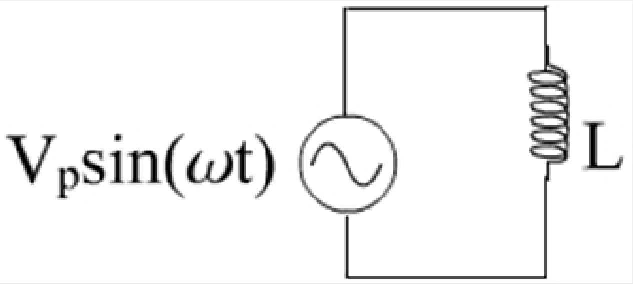

$$V(t) = L\,\frac{dI}{dt}$$

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$$I(t) = -\frac{V_P}{\omega\, L}\,\cos \omega t$$

- The current through a inductor is "out-of-phase" with the driving voltage source.

“Out-of-Phase?” for Inductor

- The time dependent voltage was given by

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- From trigonometry:

$$-\cos \omega t = \sin\left(\omega t - \frac{\pi}{2}\right)$$

“Out-of-Phase?” for Inductor

- The time dependent voltage was given by

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- The current through the inductor is given by

$$I(t) = \frac{V_P}{\omega L} (-\cos \omega t) \rightarrow \frac{V_P}{\omega L} \sin\left(\omega t - \frac{\pi}{2}\right)$$

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Inductive Reactance

- The current through the inductor is

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Inductive Reactance

- The current through the inductor is

$$I(t) = \frac{V_P}{\omega L} \sin\left(\omega t - \frac{\pi}{2}\right)$$

- The current through the inductor is $\frac{\pi}{2}$ out of phase with the driving voltage.
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  - The term $\chi_L = \omega L$ has a unit of Ohm and is called inductive reactance (χ_L)

Properties of Inductive Reactance

- The reactance for an inductor describes the behavior of a inductor placed in a circuit with a time-varying voltage source.

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