

PHYS102  
Inductance  
and  
Circuits

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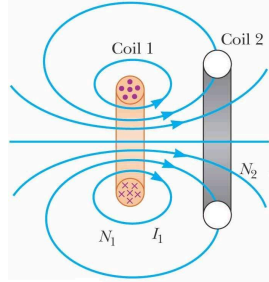
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0.1 Definition

**Mutual Inductance**

- I hope we are all comfortable finding the magnetic flux through various closed loops.
- Let's consider the following situation of two circular coils placed near each other as shown in the figure below.
- Coil 1 has current passing through it (provided by an external battery) and coil 2 is NOT connected to any



battery source.

- $\Phi_2 \propto I_1$

**Flux-Current Connection**

- $\Phi_2 \propto I_1$
- $\Phi_2 = M_{2,1}I_1$  where  $M_{2,1}$  is a constant of proportionality (called Mutual Inductance).
- If the current in circular loop 1 varies, then by Faraday's Law of Induction:

$$\varepsilon_2 = -\frac{d\Phi_2}{dt}$$

$$\varepsilon_2 = -M_{2,1} \frac{dI_1}{dt}$$

is the induced EMF in the circular loop 2.

### **$M$ - The constant.**

- The S.I. Unit for  $M_{2,1}$  is 1 H (read as “one Henry”).
- $M_{2,1}$  depends only on the geometrical arrangement of the closed loops in question.
- $M_{2,1} = M_{1,2}$ .
- We will discuss the use and importance of mutual inductance when working with circuits.

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Inductance – slide 4

### **Mutual Inductance - Example.**

An electric toothbrush has no electrical connection to the power line. But when the toothbrush is in its stand, a coil inside the toothbrush itself rests inside another coil in the stand, and alternating current from the power line flows in the stand coil. The mutual inductance of the two coils results in an induced current in the toothbrush coil, and this current charges the batteries that power the toothbrush. At a given instant the emf in the toothbrush is 4.0V, and current in the stand coil is changing at a rate of 40 A/s. What is the mutual inductance of this arrangement?

$$\left| \frac{dI}{dt} \right| = 40 A/s$$

$$|\varepsilon| = 4V.$$

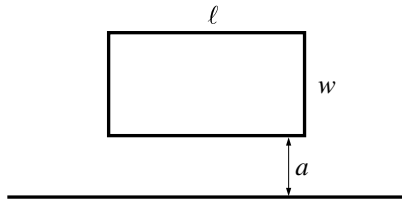
$$M_{1,2} = \frac{|\varepsilon|}{\left| \frac{dI}{dt} \right|} = 4/40 H$$

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Inductance – slide 5

### Mutual Inductance - Example 2.

A rectangular loop of length  $l$  and width  $w$  is located a distance  $a$  from a long, straight wire, as shown in the figure below. What is the mutual inductance of this arrangement?



We calculated the magnetic flux ( $\Phi_B$ ) generated by the straight wire carrying current  $I$  through the rectangular loop.

$$\Phi_B = \frac{\mu_0 I l}{2\pi} \ln\left(1 + \frac{w}{a}\right)$$
$$M_{1,2} = \frac{\Phi_B}{I} = \frac{\mu_0 l}{2\pi} \ln\left(1 + \frac{w}{a}\right)$$

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Inductance – slide 6

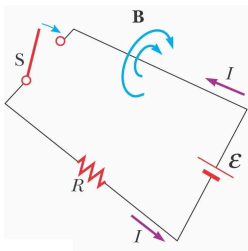
## Self Inductance

slide 7

### 0.2 Definition

#### Self Inductance

- When we were discussing simple circuits (Direct Current) involving a battery, a resistor, and a switch as shown below, we never asked about the magnetic flux through the circuit *generated by the circuit*.



- The current in the circuit after the switch is “flipped” on produces a magnetic flux through the area defined by the circuit.
- There is a changing magnetic flux through the circuit.
- This phenomena is termed “self-inductance”.

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Inductance – slide 7

### Self Inductance - Inductors

- The magnetic flux produced by a circuit is directly proportional to the current in the circuit.
- The constant of proportionality is called the self-inductance constant ( $L$ ).

$$\Phi_B = L I$$

$$L \equiv \frac{\Phi_B}{I} \quad (\text{All terms are produced by a single element.})$$

- The S.I. unit for self-inductance ( $L$ ) is one Henry (H).
- An **inductor** is a device designed specifically to exhibit self-inductance.

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Inductance – slide 8

### Self Inductance and Back EMF

- If the current in the circuit changes (as it would in the previous slide when the switch is closed), then an induced EMF is produced.

$$\varepsilon = -\frac{d\Phi_B}{dt}$$

$$\varepsilon = -L \frac{dI}{dt}$$

- The induced EMF *opposes* the change in current which is why it is often called “**back EMF**”.
- Calculating the self-inductance is typically very hard unless the geometry is simple.

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Inductance – slide 9

## Behavior of Inductors

- An inductor is represented symbolically by the following symbol

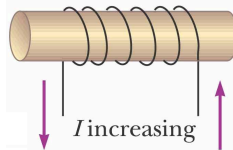


- If  $\frac{dI}{dt} = 0$ , there is no EMF in the inductor, and the inductor acts like a piece of wire.

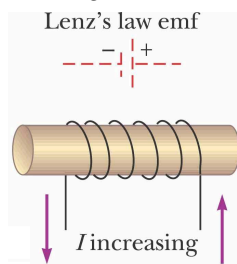
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Inductance – slide 10

## Increasing Currents Through Inductors



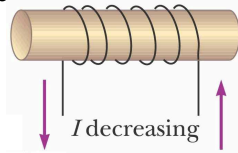
- Current is increasing over time through an inductor indicated above.
- According to Lenz's Law, the induced EMF will try to reduce the increasing current so conceptually the inductor sets up a "voltage" like the following picture.



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Inductance – slide 11

## Decreasing Currents Through Inductors



- Current is decreasing over time through an inductor indicated above.
- According to Lenz's Law, the induced EMF will try to increase the decreasing current so conceptually the inductor sets up a "voltage" like the following picture.

