PHYS102
Inductance
and
Circuits

Dr. Suess

April 9, 2007

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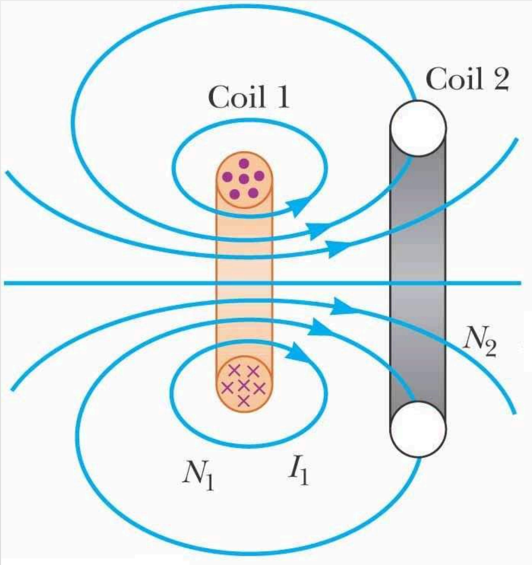

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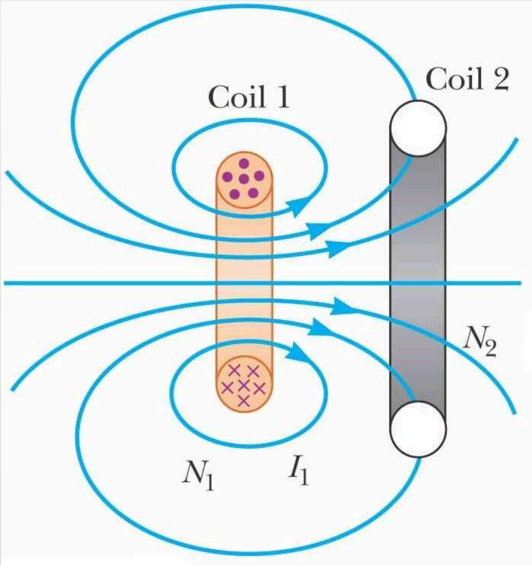

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- Flux-Current Connection
- M - The constant.
- Mutual Inductance - Example.
- Mutual Inductance - Example 2.

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Mutual Inductance - Example.

An electric toothbrush has no electrical connection to the power line. But when the toothbrush is in its stand, a coil inside the toothbrush itself rests inside another coil in the stand, and alternating current from the power line flows in the stand coil. The mutual inductance of the two coils results in an induced current in the toothbrush coil, and this current charges the batteries that power the toothbrush. At a given instant the emf in the toothbrush is 4.0V, and current in the stand coil is changing at a rate of 40 A/s. What is the mutual inductance of this arrangement?

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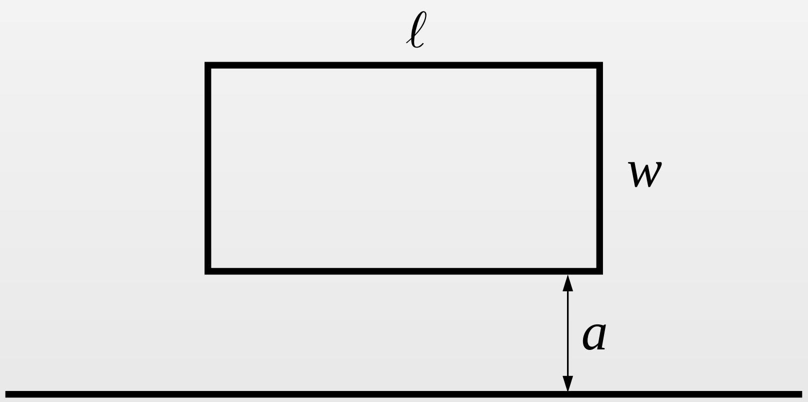

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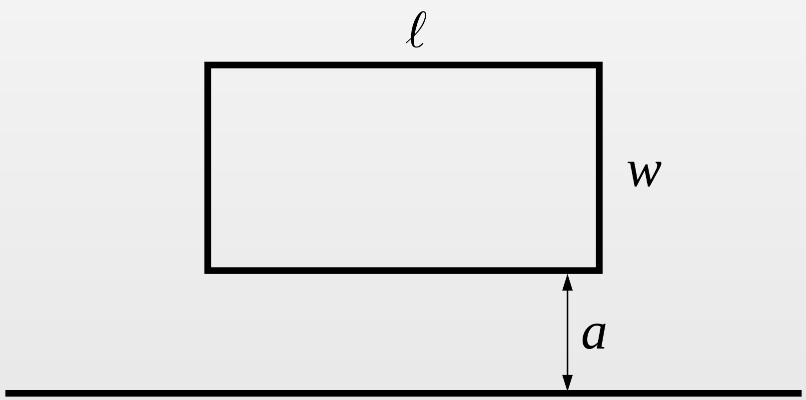

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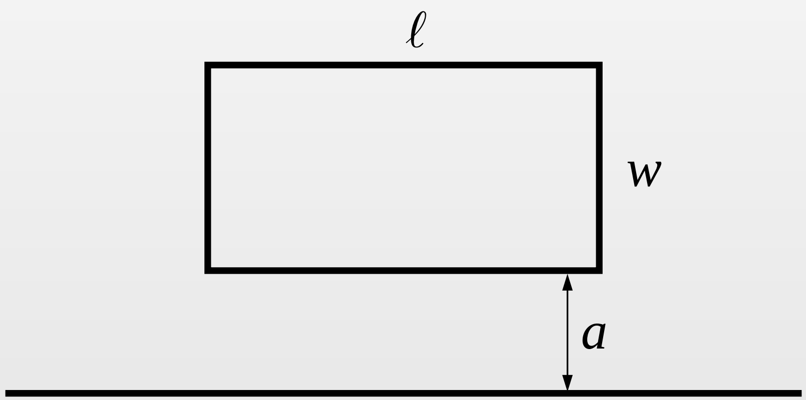

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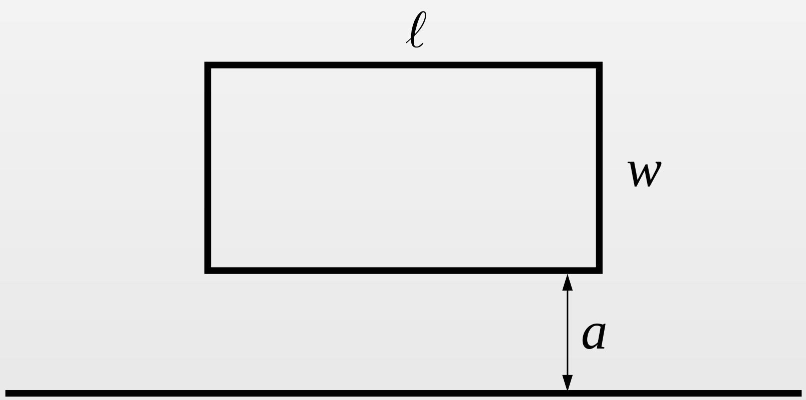

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- When we were discussing simple circuits (DirectCurrent)involving a battery, a resistor, and a switch as shown below, we never asked about the magnetic flux through the circuit *generated by the circuit*.

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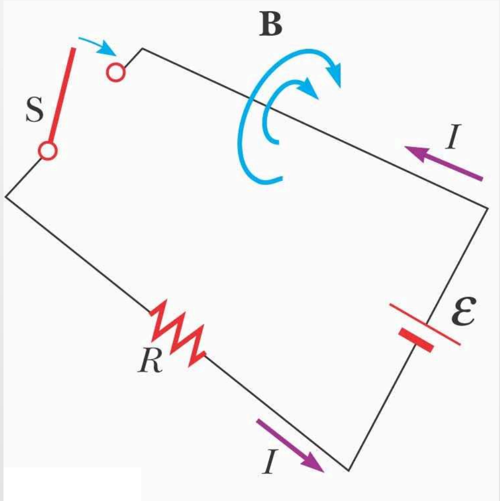

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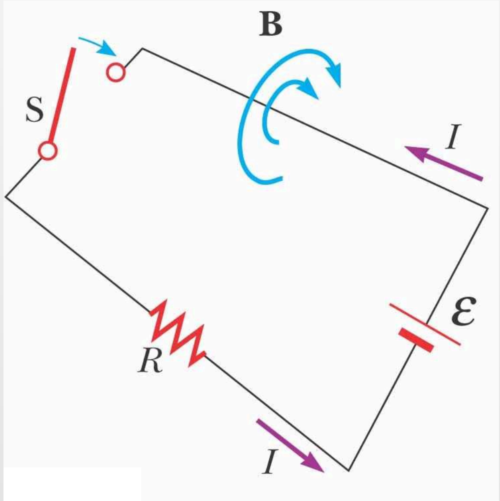

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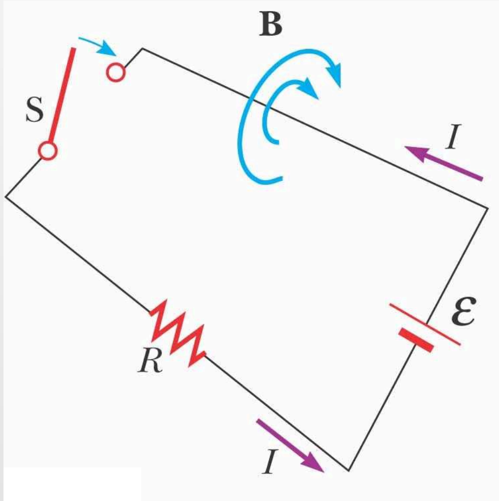

- The current in the circuit after the switch is “flipped” on produces a magnetic flux through the area defined by the circuit.
- There is a changing magnetic flux through the circuit.
- This phenomena is termed “self-inductance”.

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- An **inductor** is a device designed specifically to exhibit self-inductance.

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- Calculating the self-inductance is typically very hard unless the geometry is simple.

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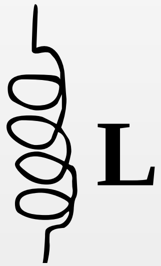

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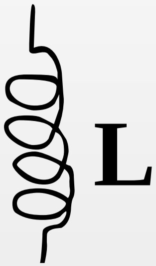
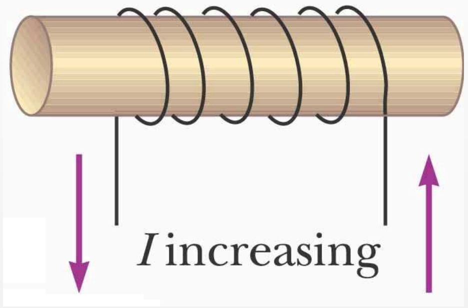


- If $\frac{dI}{dt} = 0$, there is no EMF in the inductor, and the inductor acts like a piece of wire.

Increasing Currents Through Inductors



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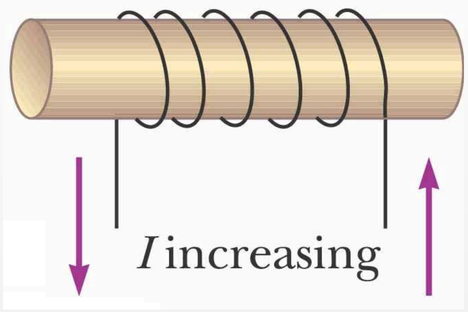

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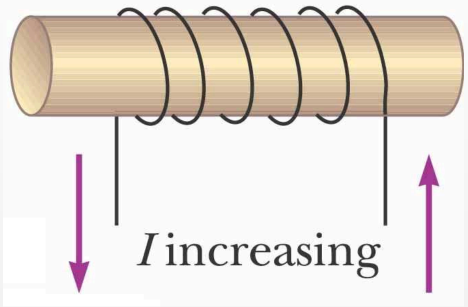

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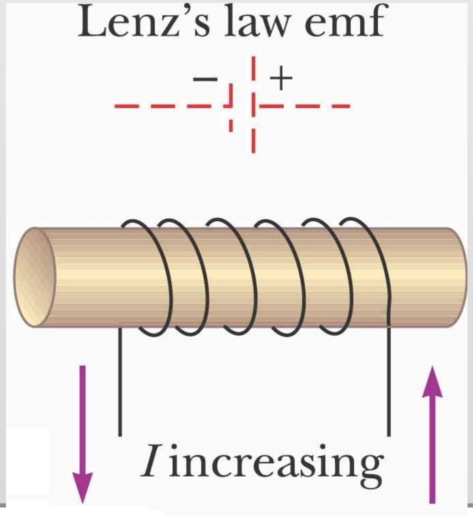

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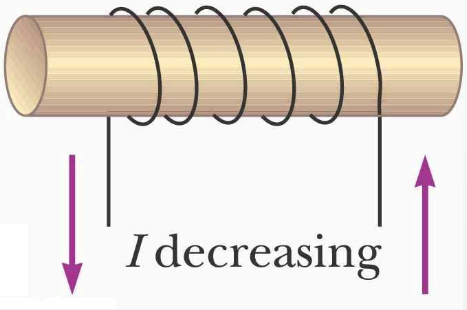

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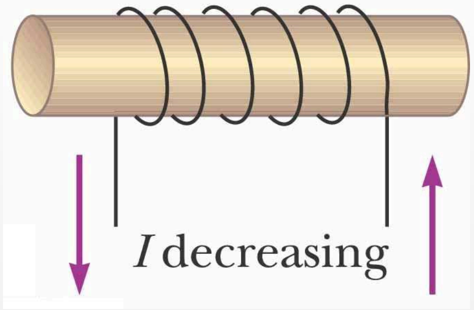

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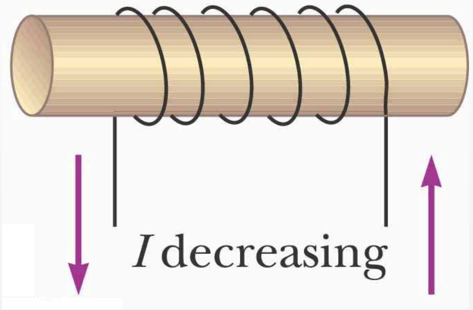

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