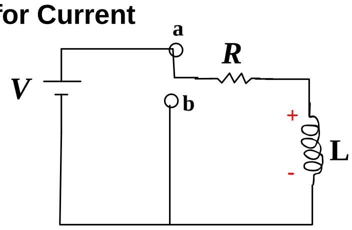
PHYS102
DC-Circuits
with
Inductors

Dr. Suess

April 13, 2007

LR Circuit - Equation for Current

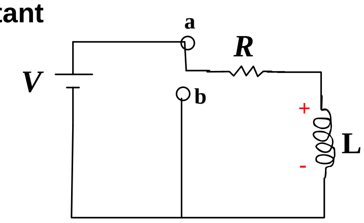


$$\Rightarrow \int \frac{\mathrm{d}I}{(I - V/R)} = -\int \frac{R\,\mathrm{d}t}{L}$$

$$I = \frac{V}{R}(1 - e^{-Rt/L})$$

NOTE: $L/RL/R$ has units of secondsseconds.

Inductive Time Constant



- Continuation from last slide.

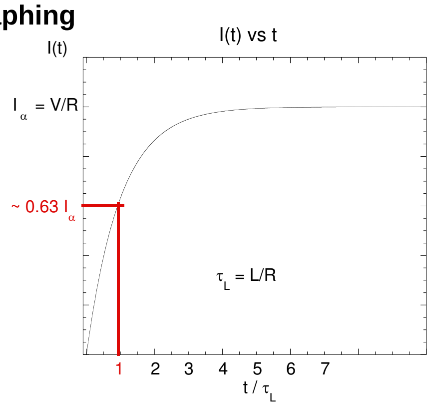
$$I = \frac{V}{R}(1 - e^{-Rt/L})$$

$$I = \frac{V}{R}(1 - e^{-t/\tau_L \tau_L}) \qquad (\text{where } \tau_L \equiv L/R)(\text{where } \tau_L \equiv L/R)$$

$\tau_L \tau_L$ is called the "inductive" time constant"inductive" time constant for the circuit.

0.1 Graphing Current and EMF

Current Behavior - Graphing



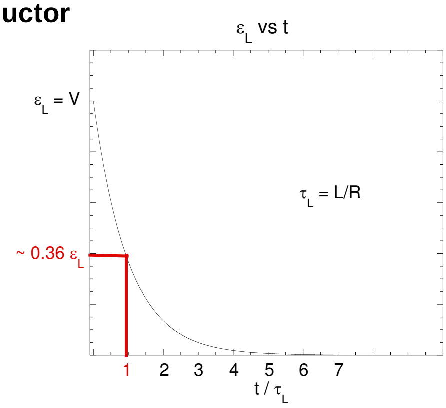
$$I(t) = \frac{V}{R}\left(1 - e^{-Rt/L}\right)$$

- The current through the inductor builds up over time (just like we stated conceptually).

PHYS102 Circuits with Inductors – slide 4

- What happens to the EMF in the inductor?

Graphing - EMF of Inductor



$$I(t) = \frac{V}{R}\left(1 - e^{-Rt/L}\right)$$

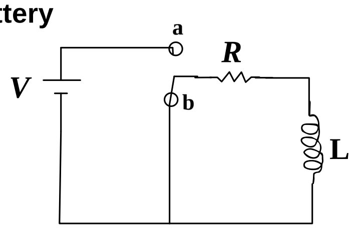
$$\varepsilon_L = -L\,\frac{\mathrm{d}I}{\mathrm{d}t} = V\,e^{-Rt/L}$$

PHYS102 Circuits with Inductors – slide 5

- The EMF in the inductor approaches zero as the current in the circuit reaches equilibrium (i.e., current does not fluctuate).

0.2 Removing the Battery

Disconnecting the Battery

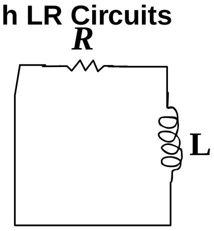


- After a very long time, the switch is thrown into position (b).
  - The battery is disconnected from the rest of the circuit.
- Writing Kirchhoff's Rules for this loop:

$$-I\,R - |\varepsilon_L| = 0.$$
$$\Rightarrow L\frac{\mathrm{d}I}{\mathrm{d}t} = -I\,R$$

PHYS102 Circuits with Inductors – slide 6

Current Decay through LR Circuits



- Continuation from the last slide.

$$L\frac{\mathrm{d}I}{\mathrm{d}t} = -I\,R$$
$$\Rightarrow \frac{\mathrm{d}I}{I} = -\frac{R}{L}\mathrm{d}t$$
$$\Rightarrow I(t) = I_0\,e^{-t/\tau_L}$$

- I_0 represents the current through the inductor right before the switch was thrown into position (b).

PHYS102 Circuits with Inductors – slide 7

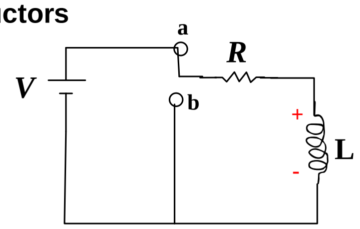
LR - RC Circuit Comparison

- The equations used to describe the current through the inductor (RL circuit) and the charge on a capacitor (RC circuit) look identical in form.
- We calculated the energy stored in a capacitor by considering the buildup of charge.
  - What is the energy of an inductor with current flowing through it?
- Start with the circuit first discussed in this lecture!

PHYS102 Circuits with Inductors – slide 8

0.3 Power

Energy Stored in Inductors



- From Kirchhoff's Rules:

$$\Rightarrow V = IR + L\,\frac{\mathrm{d}I}{\mathrm{d}t}$$

- Multiply both sides of the equation by I.

$$\Rightarrow VI = I^2 R + L\,I\,\frac{\mathrm{d}I}{\mathrm{d}t}$$

- The VI term is the power delivered by the battery.The $I^2\,R$ term is the power dissipated by the resistor.This means the $L\,I\,\frac{\mathrm{d}I}{\mathrm{d}t}$ term is the power stored in the inductor.

0.4 Power in Inductors

Current and Energy in Inductors

- Power is the rate of change of Energy per time.

$$P_L = L\,I\,\frac{\mathrm{d}I}{\mathrm{d}t} = \frac{dU}{dt}$$
$$\Rightarrow dU_L = L\,I\,dI$$
$$\Rightarrow U_L = \int L\,I\,dI = \frac{1}{2}\,L\,I^2$$

- The energy stored in the inductor varies as I^2 (much like Q^2 for a capacitor).
- Is there a general expression that does not depend on the self-inductance? In other words, is there a relationship between the amount of energy and the magnetic field?

0.5 Energy and B-fields

Magnetic Energy Density

- Consider a solenoid with radius a, length l, and n number of turns per unit length.

$$B = \mu_0 \, n \, I$$
$$L = n^2 \, \mu_0 \, l \, (\pi \, a^2) = n^2 \, \mu_0 \, (A \, l)$$
$$\Rightarrow U_B = \frac{1}{2} \, L \, I^2 = \frac{1}{2} \, n^2 \, \mu_0 \, (A \, l) \, \left(\frac{B}{\mu_0 \, n} \right)^2$$
$$\Rightarrow U_B = \frac{1}{2 \, \mu_0} \, B^2 \, (A \, l)$$

- $(A \, l)$ is the volume where the magnetic field exists. Think energy per volume.

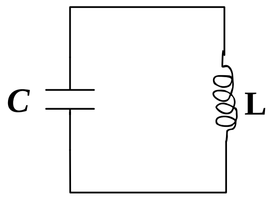
$$u_B = U_B / V = \frac{B^2}{2 \, \mu_0}$$

LC - Oscillator slide 12

Energy Oscillations

- Since capacitors store electrical energy and inductors store magnetic energy, we could place a fully charged capacitor in series with an inductor.
- The electrical energy should be transferred to magnetic energy ,and then the magnetic energy should get transferred back into electrical energy. This cycle should repeat itself. Let's prove it.
- Since energy stored in a capacitor is proportional to Q^2, it suffices to prove that the charge on the capacitor "oscillate".

LC - Circuit

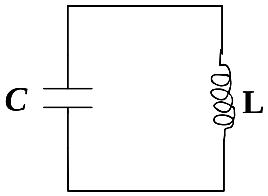


- Start with a *fully charged capacitor* and place it in series with an inductor as shown above.
- Write down the total energy of the system at some time t after the capacitor is connected to the inductor.

$$U_T = \frac{1}{2} \frac{Q^2}{C} + \frac{1}{2} \, L \, I^2$$

$$\frac{dU_T}{dt} = 0 = \frac{Q}{C} \frac{dQ}{dt} + L \, I \, \frac{dI}{dt}$$
$$0 = \frac{Q}{C} + L \, \frac{d^2Q}{dt^2}$$

Charge Oscillations



$$\frac{d^2 Q}{dt^2} = -Q/LC = -\omega^2 Q$$

- This equation defines simple harmonic motion with an angular frequency $\omega = \frac{1}{\sqrt{LC}}$. The charge on the capacitor, Q, is undergoing simple harmonic motion.
- From mechanics, we know a solution for $Q(t)$:

$$Q(t) = Q_0 \sin(\omega t + \phi)$$