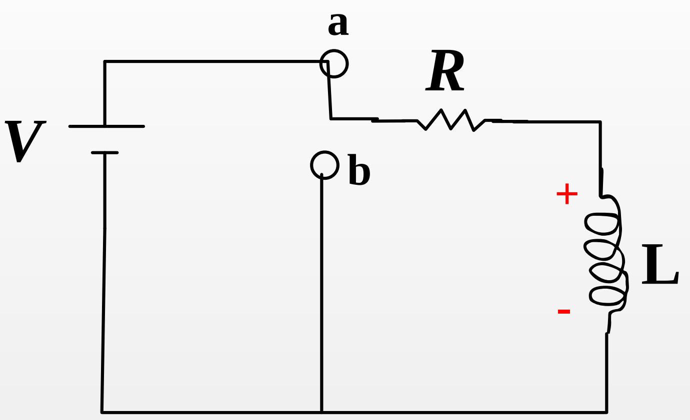
PHYS102
DC-Circuits
with
Inductors

Dr. Suess

April 13, 2007

LR Circuit - Equation for Current



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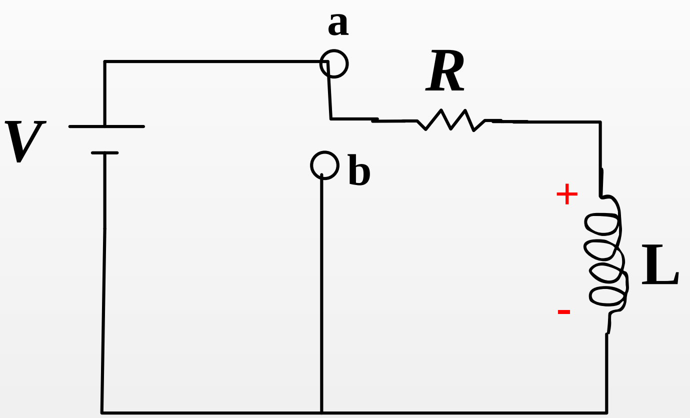

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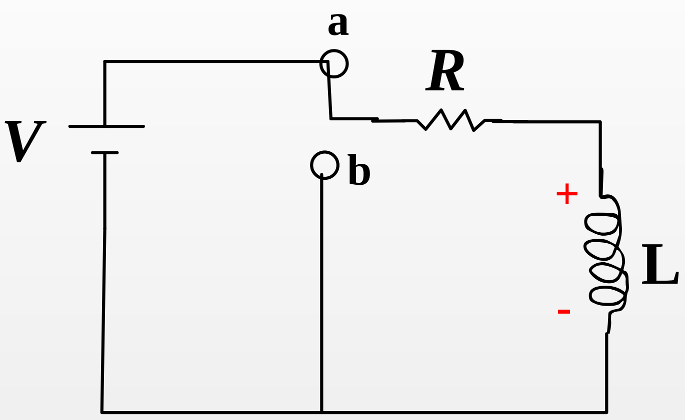

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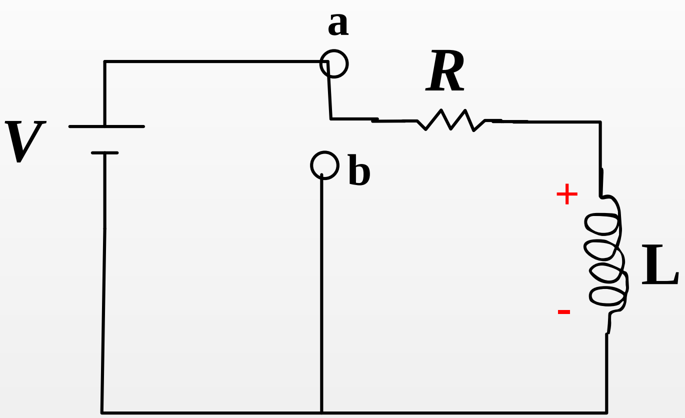

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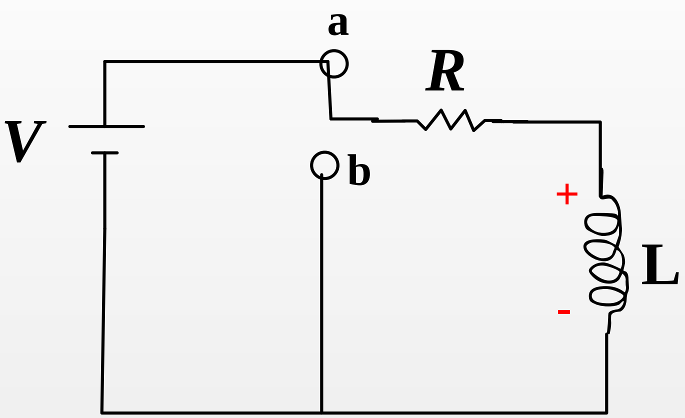

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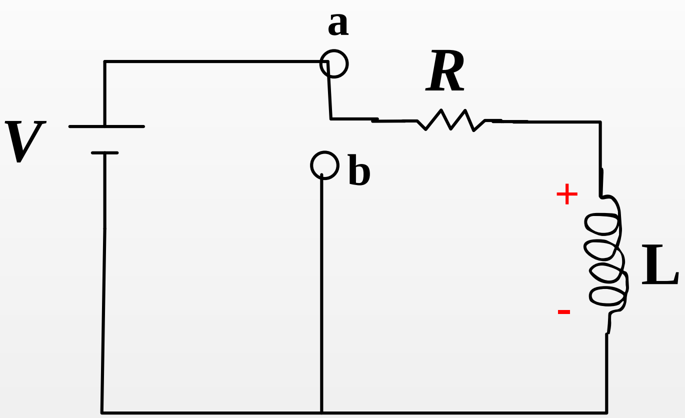

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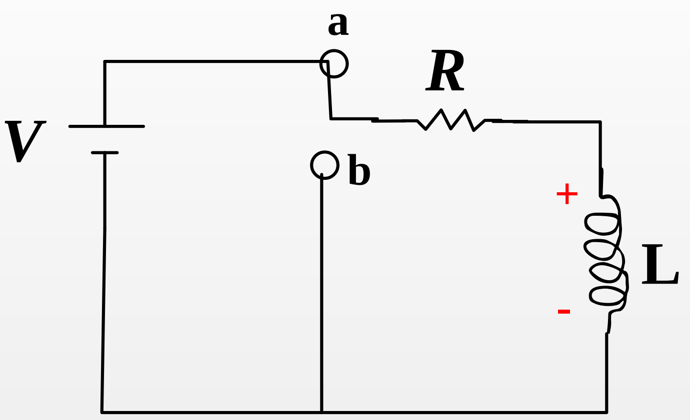

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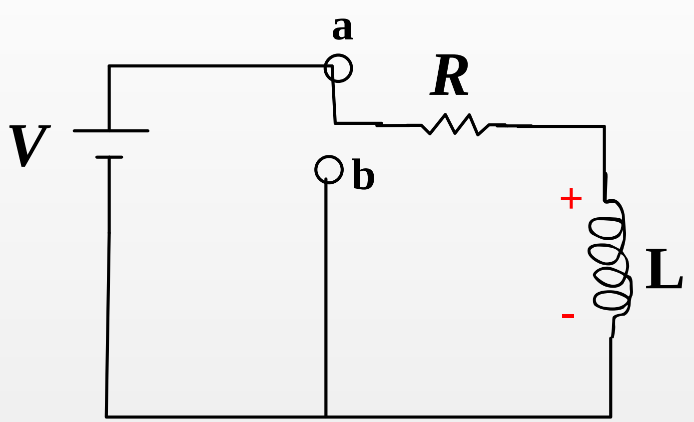

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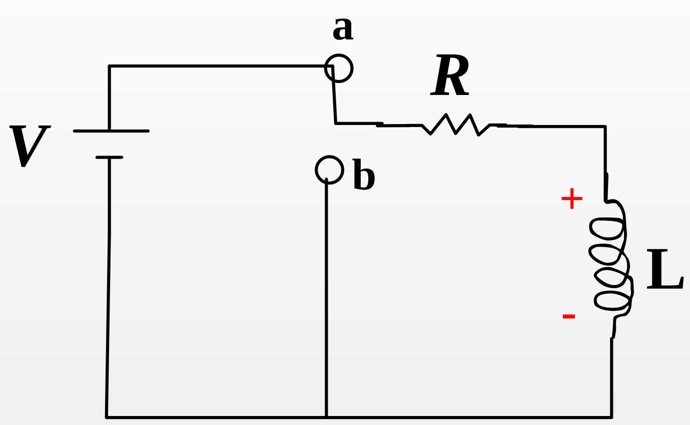

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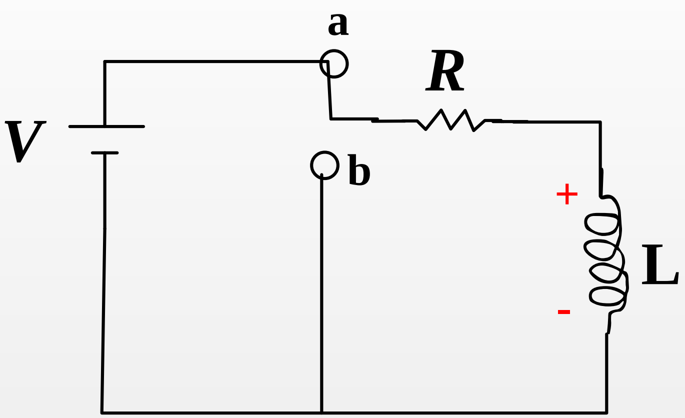

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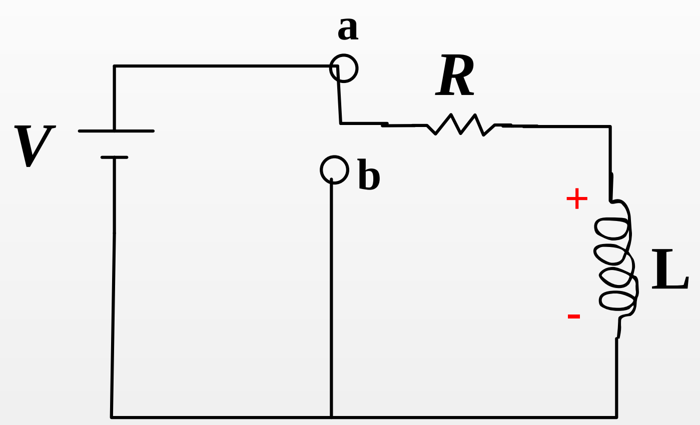

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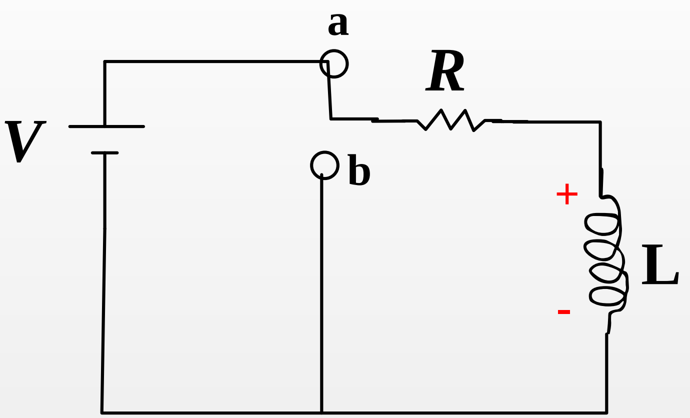

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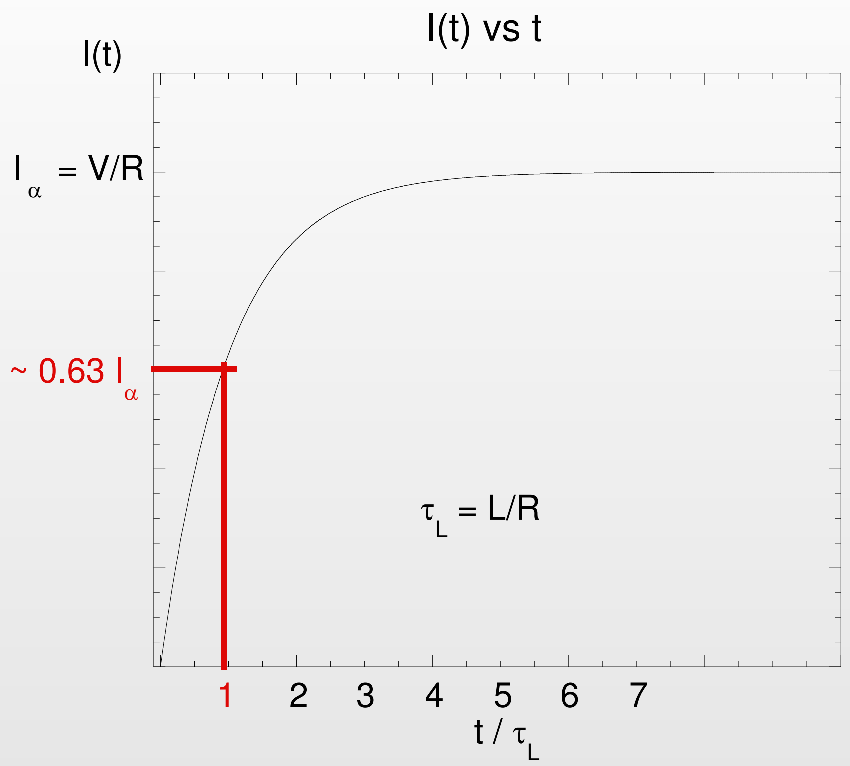

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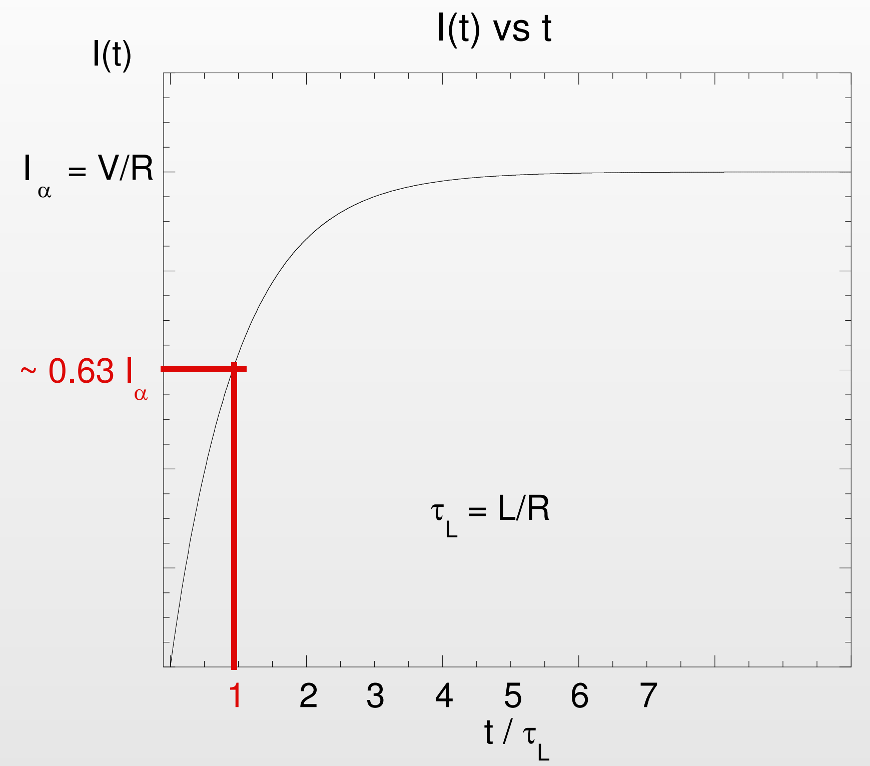

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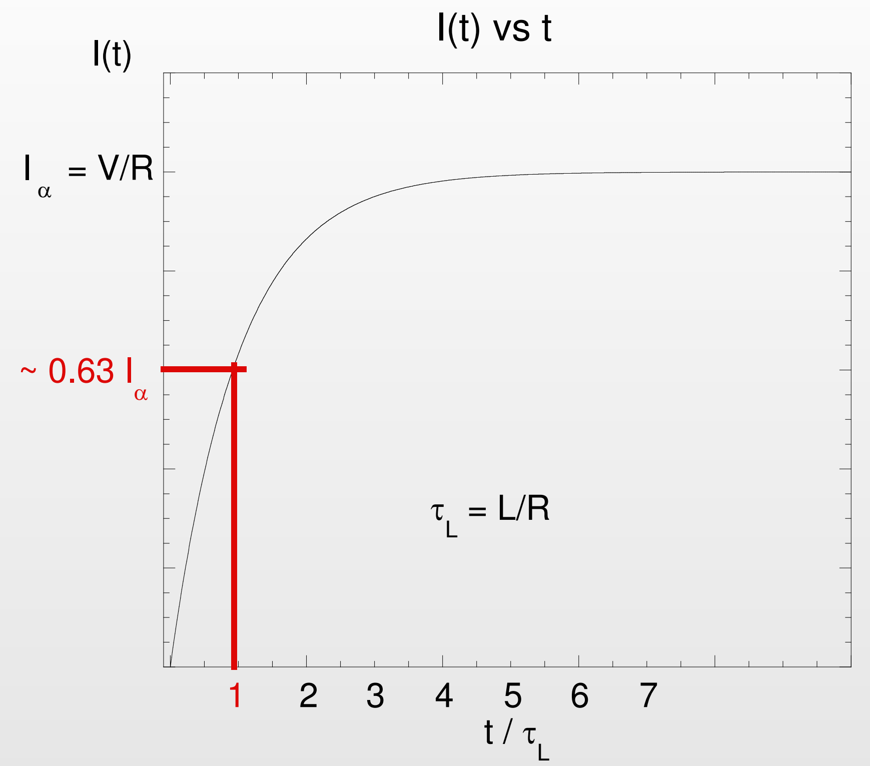

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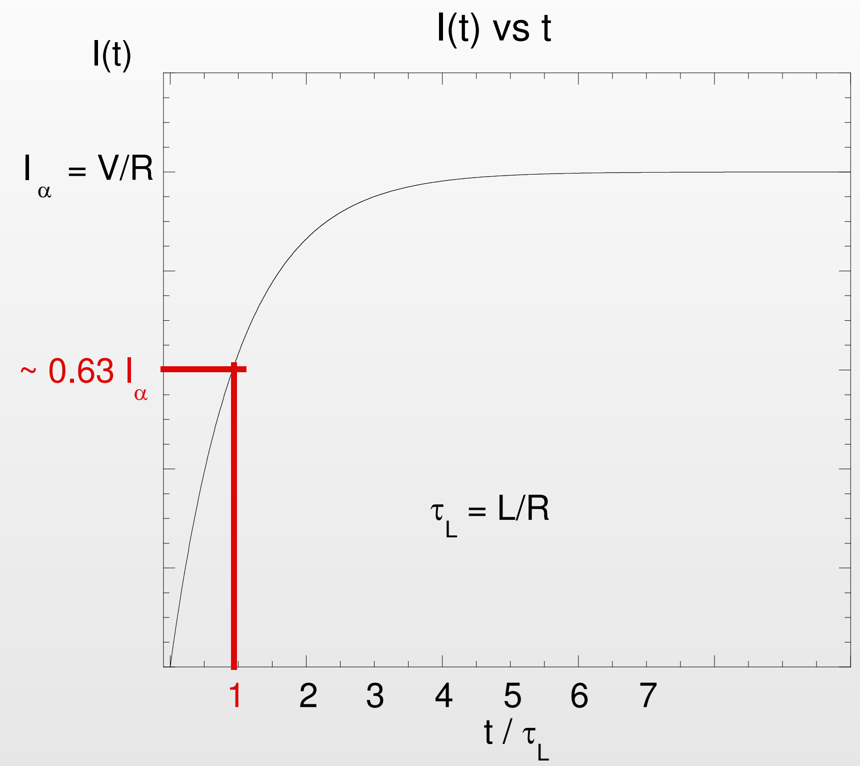

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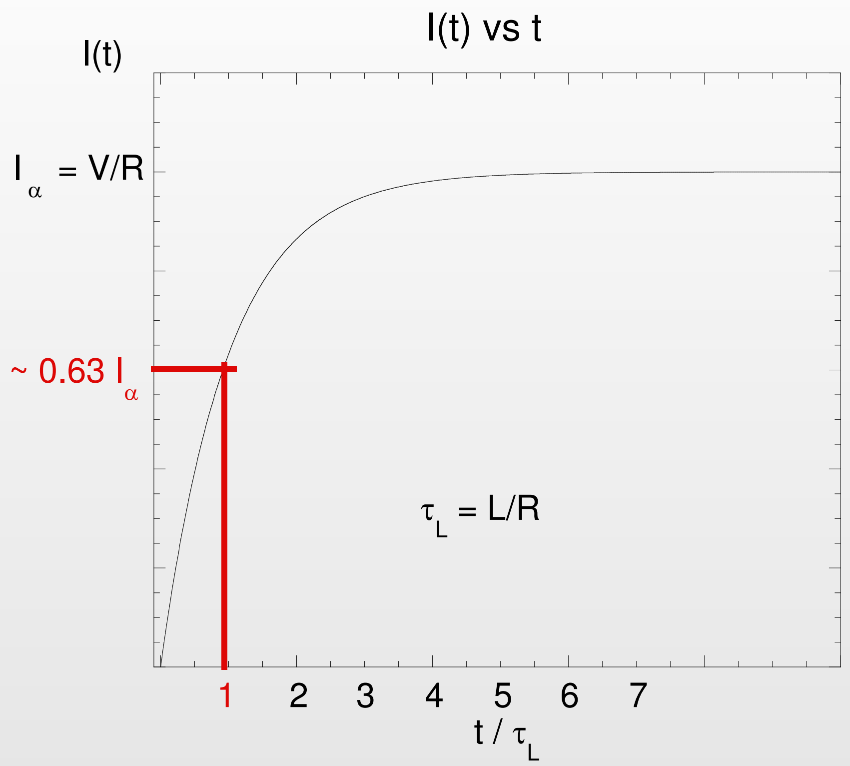

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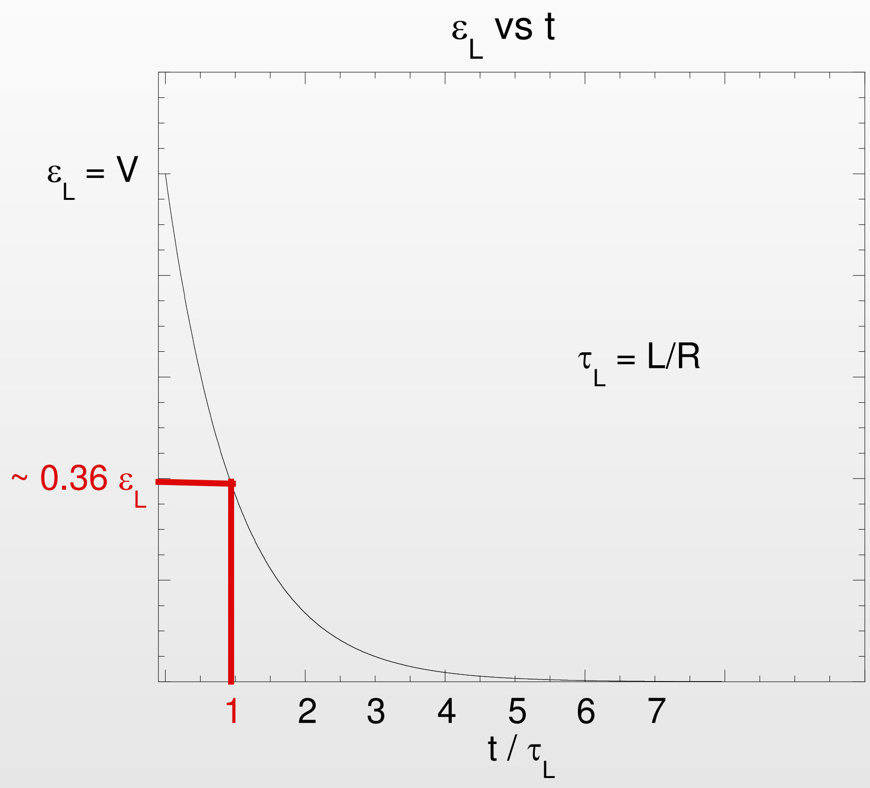
- What happens to the EMF in the inductor?

Graphing - EMF of Inductor



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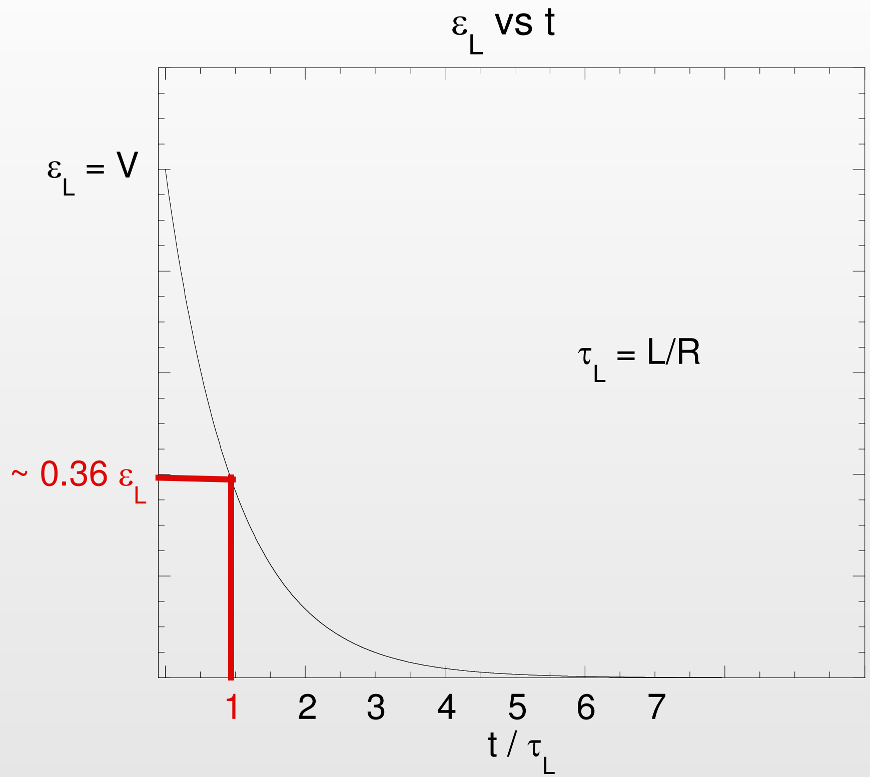

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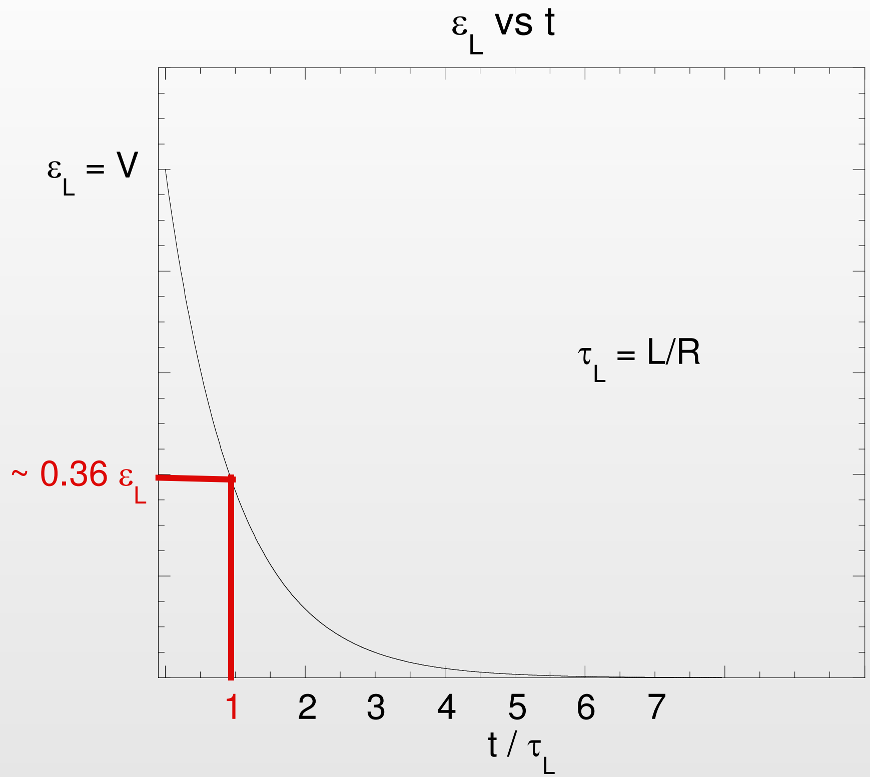

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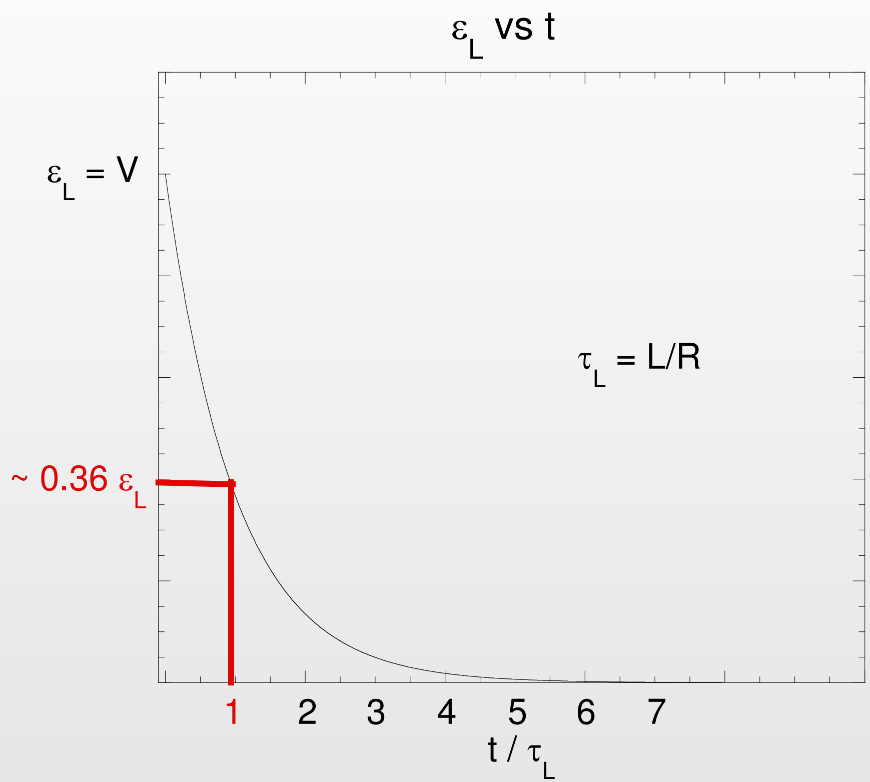

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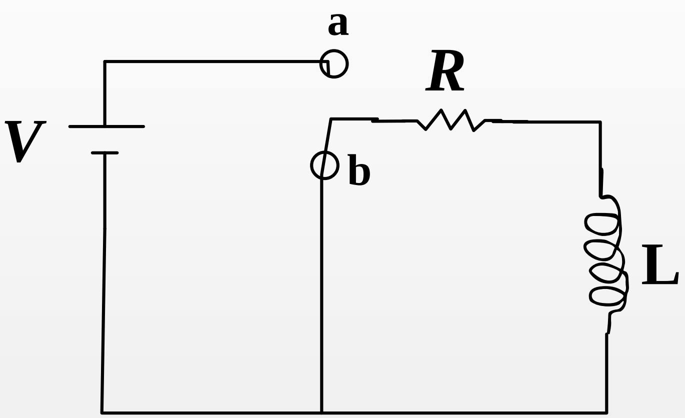
- The EMF in the inductor approaches zero as the current in the circuit reaches equilibrium (i.e., current does not fluctuate).

Disconnecting the Battery



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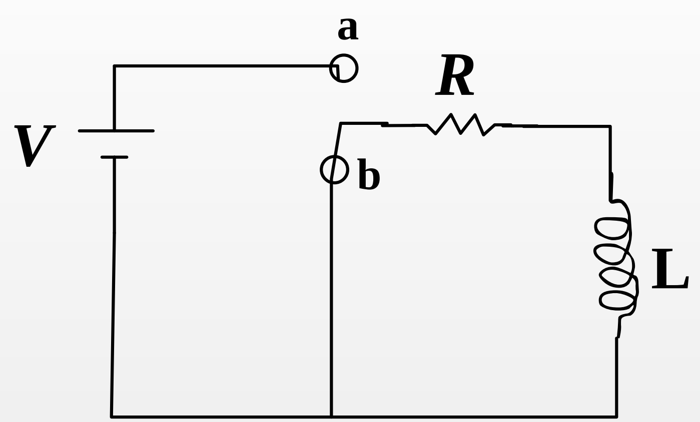

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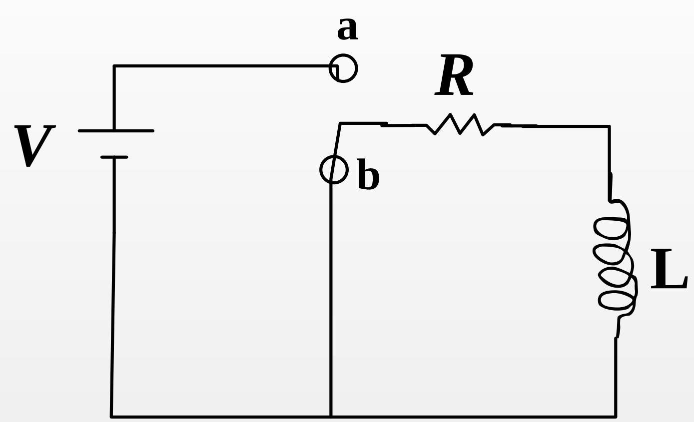

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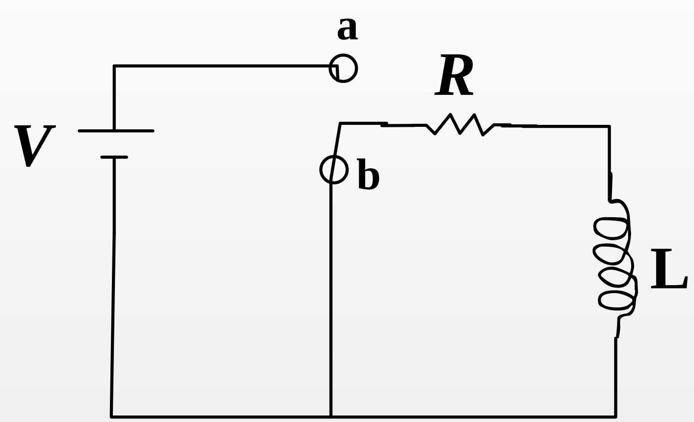

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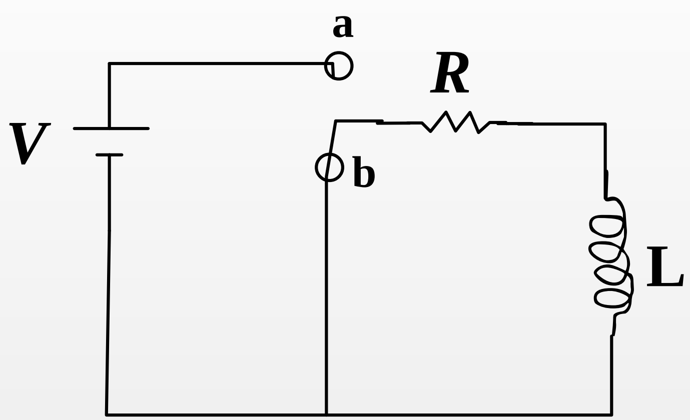

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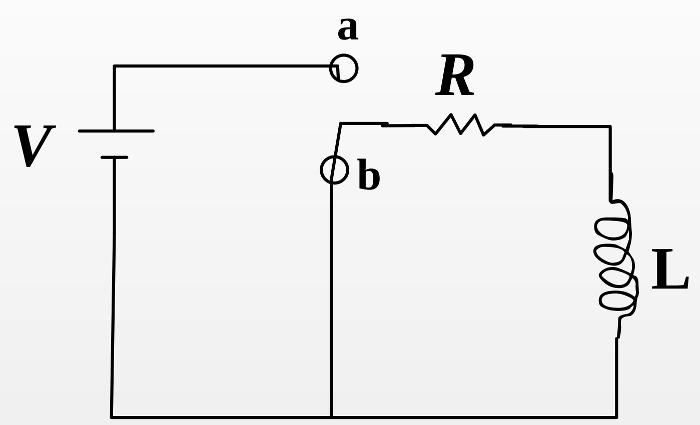

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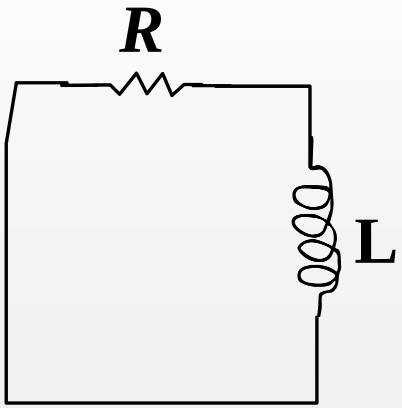
Current Decay through LR Circuits



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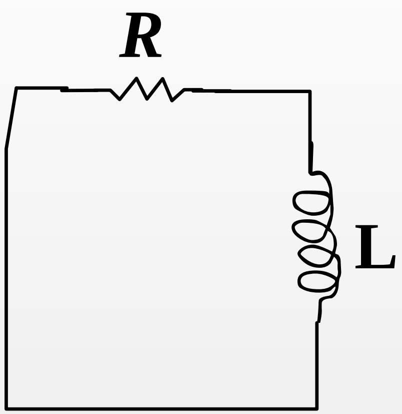

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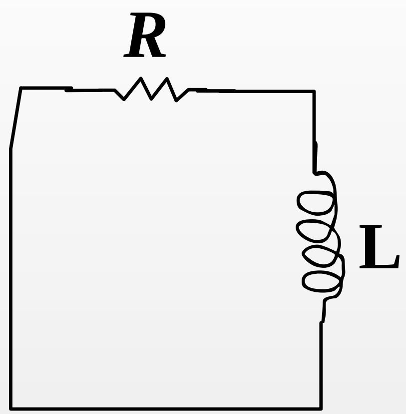

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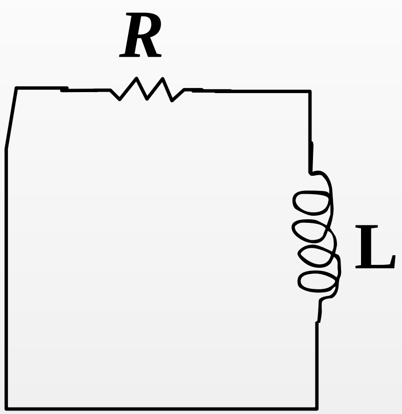

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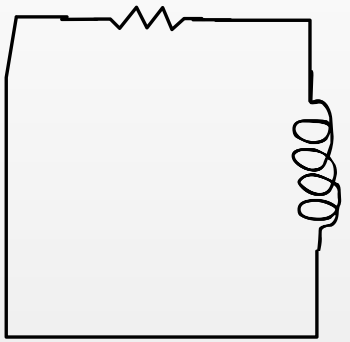

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- I_0 represents the current through the inductor right before the switch was thrown into position (b).

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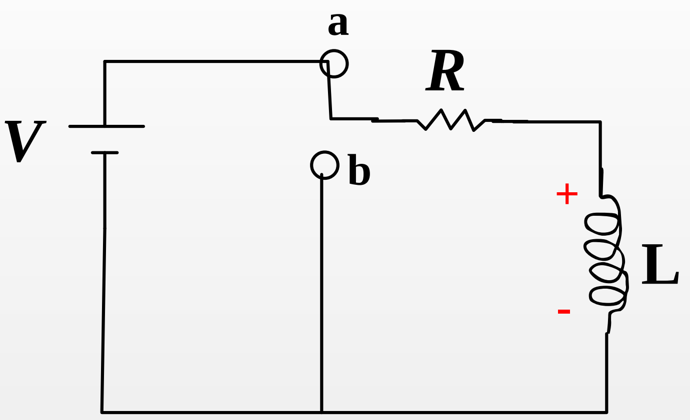
- Start with the circuit first discussed in this lecture!

Energy Stored in Inductors



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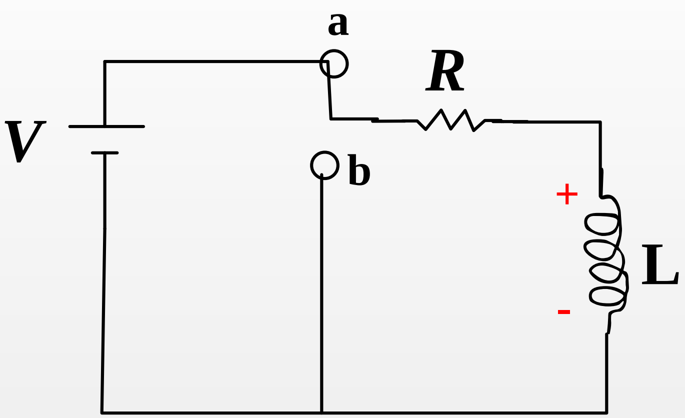

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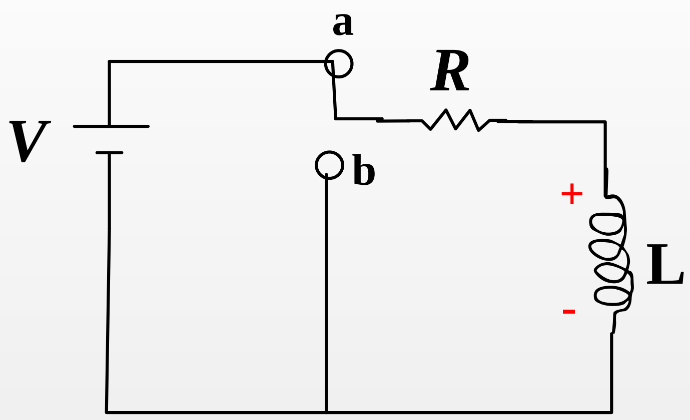

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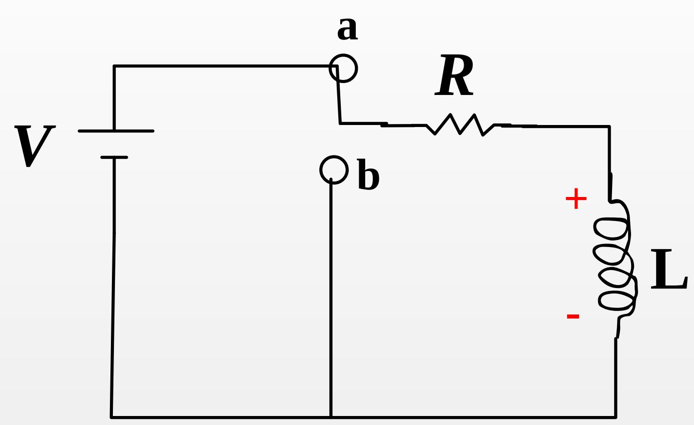

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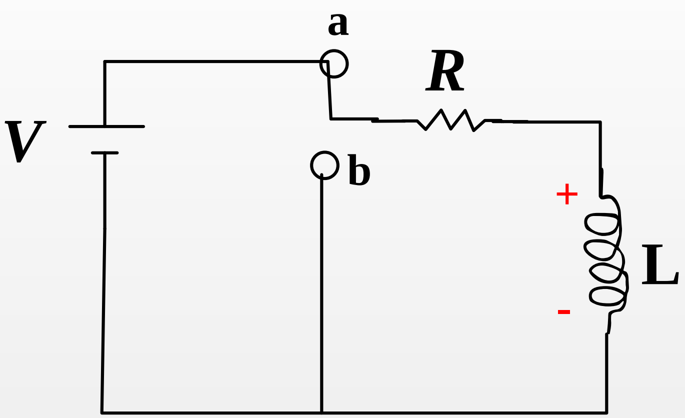

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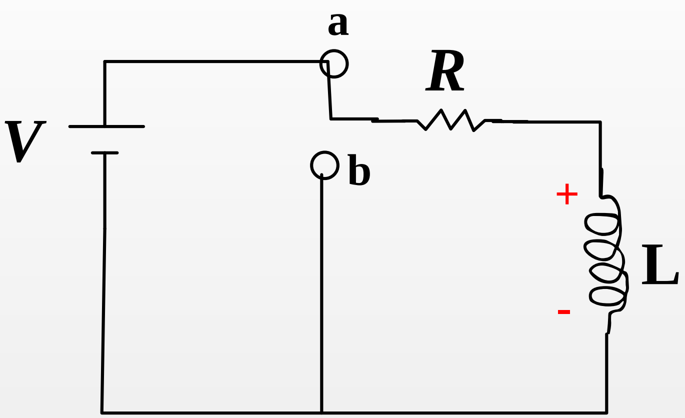

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- The VI term is the power delivered by the battery.The $I^2\,R$ term is the power dissipated by the resistor.

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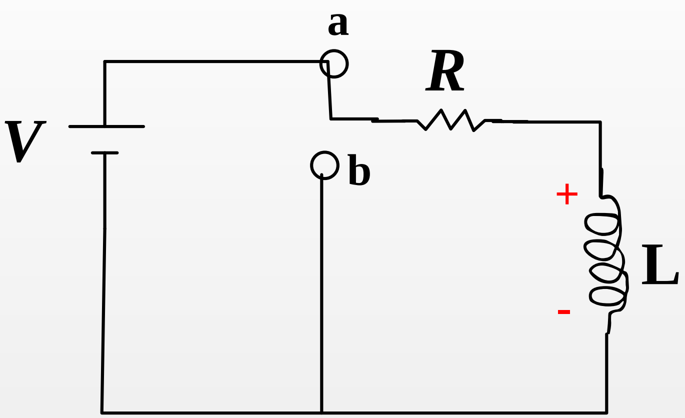

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- The $V\,I$ term is the power delivered by the battery.The $I^2\,R$ term is the power dissipated by the resistor.This means the $L\,I\,\frac{\mathrm{d}I}{\mathrm{d}t}$ term is the power stored in the inductor.

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- The energy stored in the inductor varies as I^2 (much like Q^2 for a capacitor).
- Is there a general expression that does not depend on the self-inductance? In other words, is there a relationship between the amount of energy and the magnetic field?

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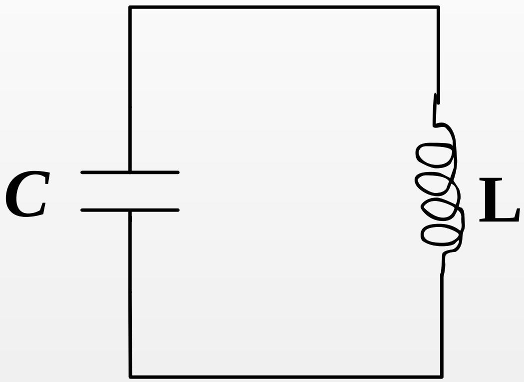
- Since energy stored in a capacitor is proportional to Q^2, it suffices to prove that the charge on the capacitor "oscillate".

LC - Circuit



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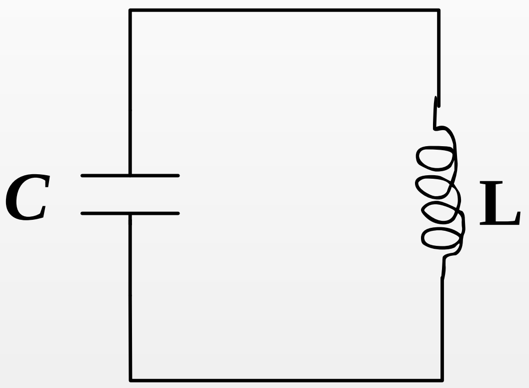

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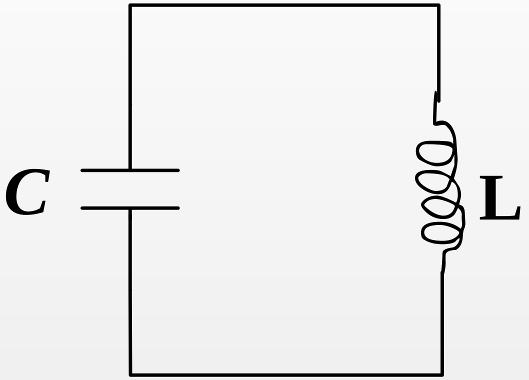

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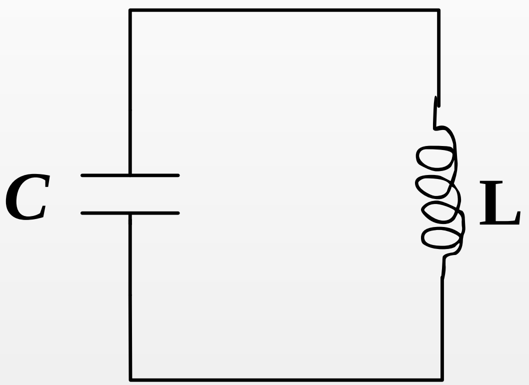

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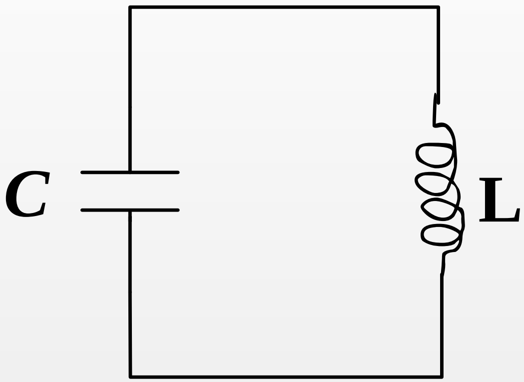

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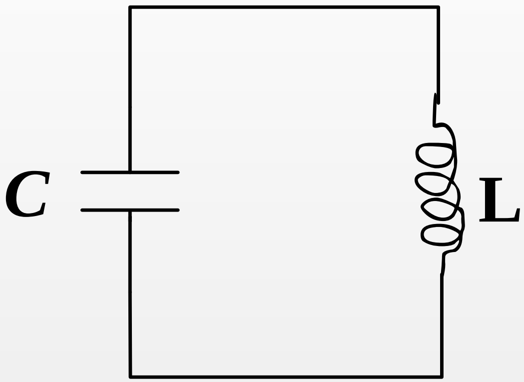

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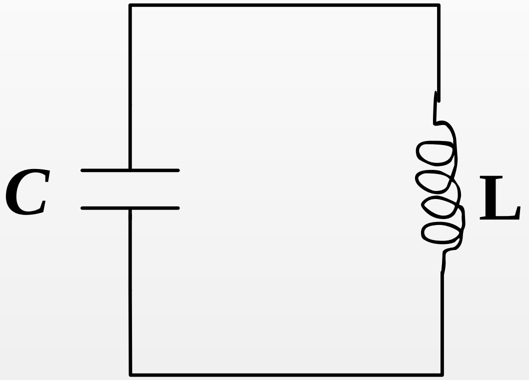

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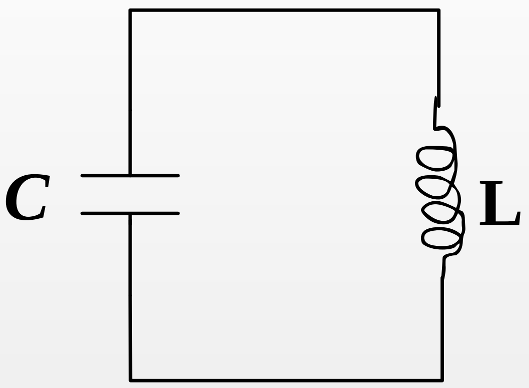

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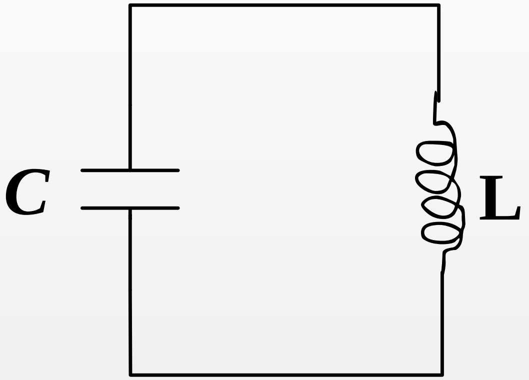
$$0 = \frac{Q}{C} + L\frac{d^2Q}{dt^2}$$

Charge Oscillations



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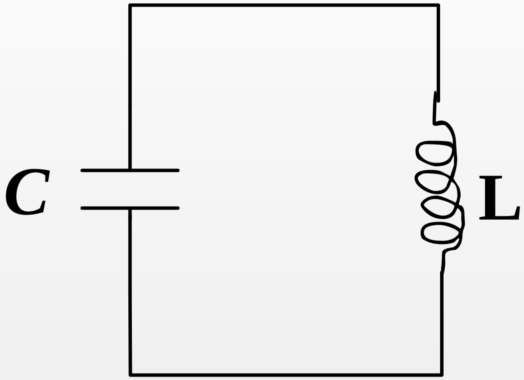

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- This equation defines simple harmonic motion with an angular frequency $\omega = \frac{1}{\sqrt{LC}}$.

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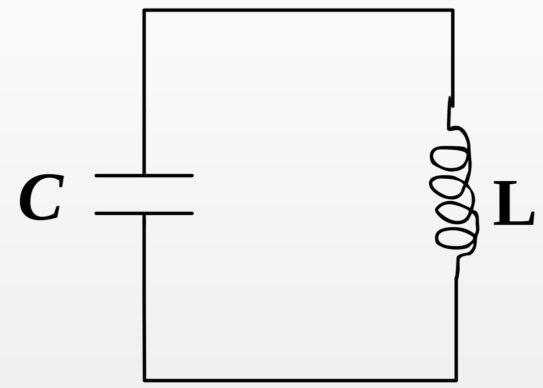

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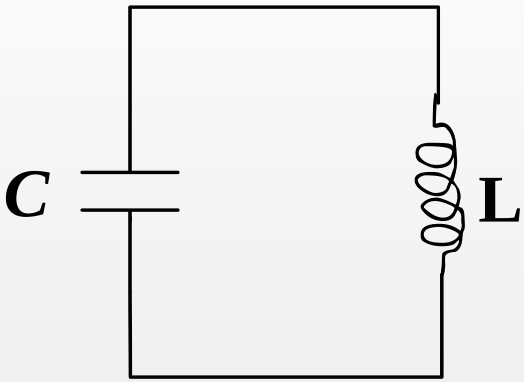

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- From mechanics, we know a solution for $Q(t)$:

$$Q(t) = Q_0 \sin(\omega t + \phi)$$