PHYS102 - Charge and Force

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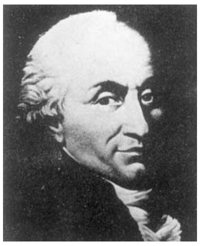
Electrostatics

Throughout the beginning of this course, we will deal with electrostatics.

This means we concern ourselves with charges that are at rest with respect to each other.

How do we quantify the forces between stationary charged particles (like in the demo)?

Thank you Mr. Charles Augustin Coulomb:



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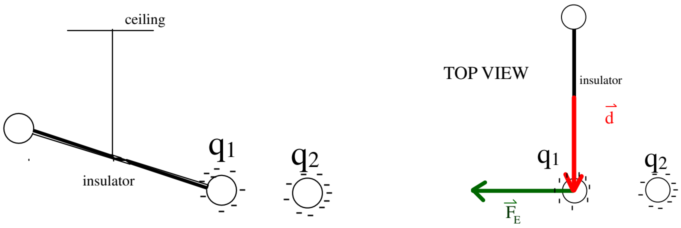
0.1 Forces

Electrostatic Forces

Coulomb used a torsional balance with two insulated conducting spheres in a dumbbell configuration as shown below.



- Charge q_2 and the distance between the spheres carrying charge q_1 and q_2 were controlled.
- The repulsive force between the charged spheres produces a torque about the pivot point:

$$\tau = d\,F_E = \kappa\,\theta$$

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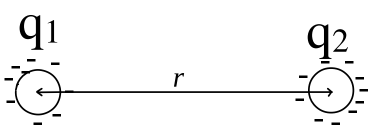
Electrostatic Forces II

$$F_E \propto \theta$$

- "twist" is proportional to the magnitude of electrostatic force, F_E.
- allows for a measure of relative values of F_E.
- Coulomb showed that $F_E \propto \frac{1}{r^2}$
  - r is the separation of the charges.
- NOTE: for uniformly charged spheres, r is the distance between their centers



Coulomb's Law

Coulomb also showed:

$$F_E \propto q_1\, q_2$$

$$\Rightarrow F_E \propto \frac{q_1\, q_2}{r^2}.$$

$$F_E = \frac{k\, q_1\, q_2}{r^2}. \quad (1)$$

This is a mathematical statement of Coulomb's law.

NOTE: This resembles Newton's Universal law of Gravitation:

$$F_G = \frac{G\, m_1\, m_2}{r^2}$$

Coulomb's Law II

What are the units of k? What is the value of k?

- Units: $[q]$ = C; $[r]$ = m; $[F_E]$ = N.
- Using $F_E = \frac{k\, q_1\, q_2}{r^2} \Rightarrow [k] = \frac{N\, m^2}{C^2}$

Experimentally:

$$k = 8.987 \times 10^9\ N\, m^2\, C^{-2} \approx 9.0 \times 10^9\ N\, m^2\, C^{-2}$$

We will also use a convenient "rationalized" system in which:

$$F_E = \frac{1}{4\,\pi\,\epsilon_0}\frac{q_1\, q_2}{r^2}$$

where $\epsilon_0 = 8.85 \times 10^{-12}\, C^2\, N^{-1}\, m^{-2}$ is called the "permittivity of free space".

Forces slide 8

0.2 Magnitude and Direction

Vector Nature

Since we are science-minded people, we know that force is a vector so we need to specify a direction.

We may proceed in one of two ways:

- Method (1) - directional vector method.
  - $\vec{F} = \frac{k\, q_1\, q_2}{r^2}\, \hat{r}$
    - $\hat{r}$ is a directional unit vector - it has a length of 1 unit.
  - NOTE: The signs of q_1 and q_2 are important as the product will give the sign of the force.

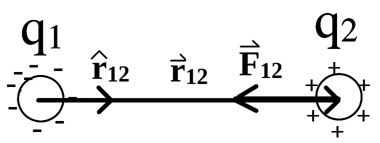
Vectors

- Method (2) - using knowledge of attraction and repulsion between charged particles.
  - $|\vec{F}| = \frac{k\, |q_1|\, |q_2|}{r^2}$
  - NOTE: The signs of q_1 and q_2 are **NOT** important as this deals with the magnitude of the force.
  - The direction is later implied by the attractive or repulsive behavior between the charged particles.

0.3 Analytical - Vector

Directional Vector

From PHYS101 - the directional vector $\hat{r}$ is what we need. Consider an example of two charged spheres. Let's calculate the force $\vec{F}_{12}$ (read as "the force exerted by charge q_1 on charge q_2"). First find $\vec{r}_{12}$ as this will help give the directional vector. Then find $\hat{r}_{12} = \frac{\vec{r}_{12}}{r_{12}}$.



Since the Force is given by: $\vec{F}_{12} = \frac{k\, q_1\, q_2}{r^2}\, \hat{r}_{12} = -\frac{k\, |q_1|\, |q_2|}{r^2}\, \hat{r}_{12}$

NOTE: $\vec{F}_{21}$ and $\vec{F}_{12}$ form an action reaction-pair as explicit in Coulomb's law.