PHYS102
DC-Circuits
with
Inductors

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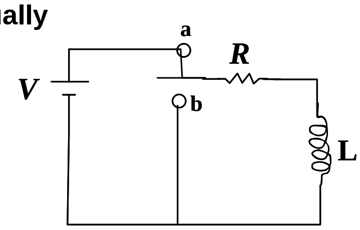
0.1 LR - Circuits - General

LR - Circuits

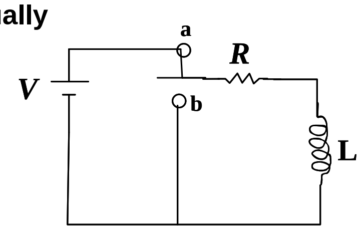
- We already analyzed the behavior of a solenoid with increasing and decreasing currents.
- Let's now look at circuit containing a resistor and an inductor.
- The circuit still has a direct-current (DC) source i.e., a battery.
- A switch will be used to increase or decrease current through the circuit.

LR - Circuit Conceptually



- The magnetic flux through the circuit is initially zero (No current flows in the circuit).
- As soon as the switch is thrown into position (a), a current begins to flow so a magnetic flux through the circuit now exists.
- Initially, the inductor behaves like an infinite resistor "opposes the change in current".

LR - Circuit Conceptually



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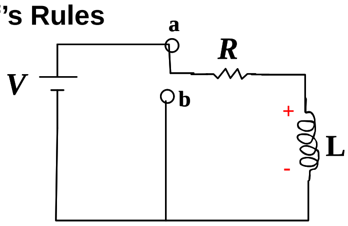
- After some time, the current in the circuit reaches an equilibrium value.
  - The inductor behaves like a piece of wire ($\varepsilon_L = 0$ since $\frac{\mathrm{d}I}{\mathrm{d}t} = 0$.)

Analysis

0.2 Ramping Current

LR Circuits - Kirchhoff's Rules



- Begin by applying Kirchhoff's rules to the closed circuit.

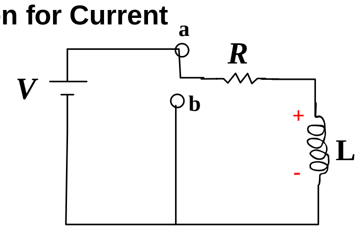
$$-I\,R - |\varepsilon_L| + V = 0.$$

$$|\varepsilon_L| = L\frac{\mathrm{d}I}{\mathrm{d}t}$$

$$\Rightarrow V = I\,R + L\frac{\mathrm{d}I}{\mathrm{d}t}$$

PHYS102 Circuits with Inductors – slide 5

Mathematical Equation for Current



- Continuation from last slide.

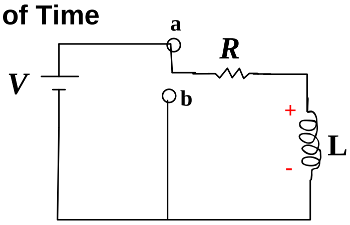
$$\Rightarrow V/R = I + L/R\,\frac{\mathrm{d}I}{\mathrm{d}t}$$

$$\Rightarrow \frac{\mathrm{d}I}{(I - V/R)} = -\frac{R\,\mathrm{d}t}{L}$$

$$\Rightarrow \int \frac{\mathrm{d}I}{(I - V/R)} = -\int \frac{R\,\mathrm{d}t}{L}$$

PHYS102 Circuits with Inductors – slide 6

Current as a Function of Time



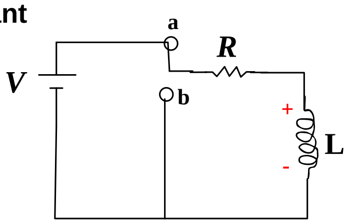
- Continuation from last slide.

$$\Rightarrow \int \frac{\mathrm{d}I}{(I - V/R)} = -\int \frac{R\,\mathrm{d}t}{L}$$

$$I = \frac{V}{R}\,(1 - e^{-Rt/L})$$

NOTE: L/R has units of seconds.

Inductive Time Constant



- Continuation from last slide.

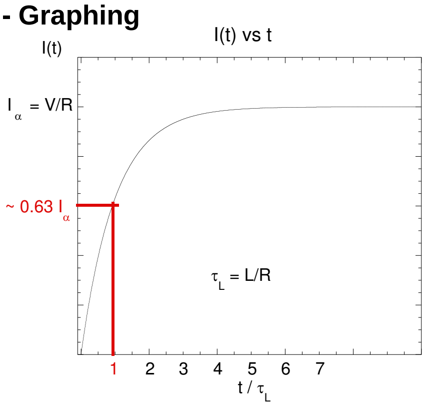
$$I = \frac{V}{R}\,(1 - e^{-Rt/L})$$

$$I = \frac{V}{R}\,(1 - e^{-t/\tau_L}) \qquad (\text{where } \tau_L \equiv L/R)$$

τ_L is called the “inductive” time constant for the circuit.

0.3 Graphing Current and EMF

EMF Through Inductor - Graphing



$$I(t) = \frac{V}{R}\left(1 - e^{-Rt/L}\right)$$

- The current through the inductor builds up over time (just like we stated conceptually).

- What happens to the EMF in the inductor?