PHYS102
DC-Circuits
with
Inductors

Dr. Suess

April 11, 2007

LR - Circuits

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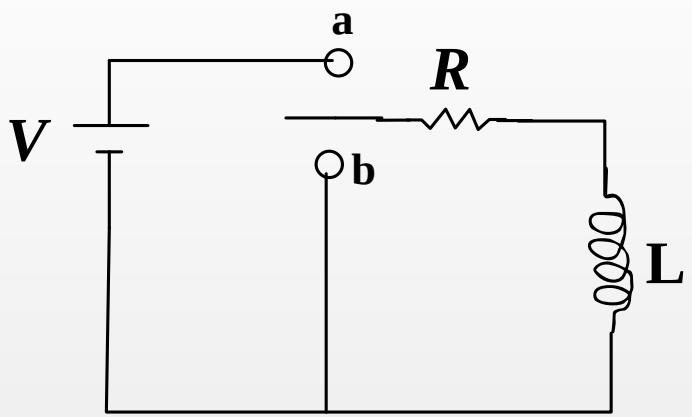
- Let's now look at circuit containing a resistor and an inductor.
- The circuit still has a direct-current (DC) source i.e., a battery.
- A switch will be used to increase or decrease current through the circuit.

LR - Circuit Conceptually



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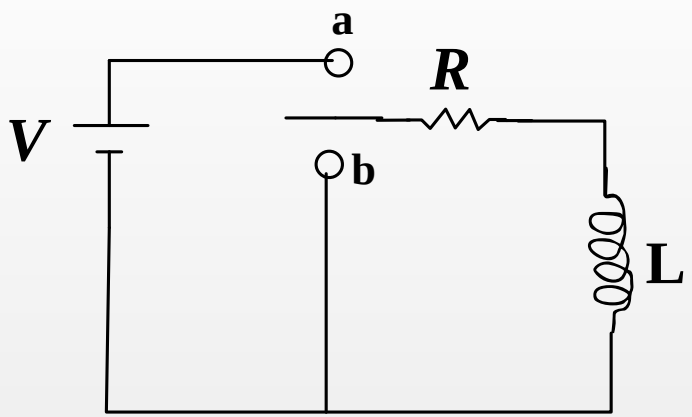

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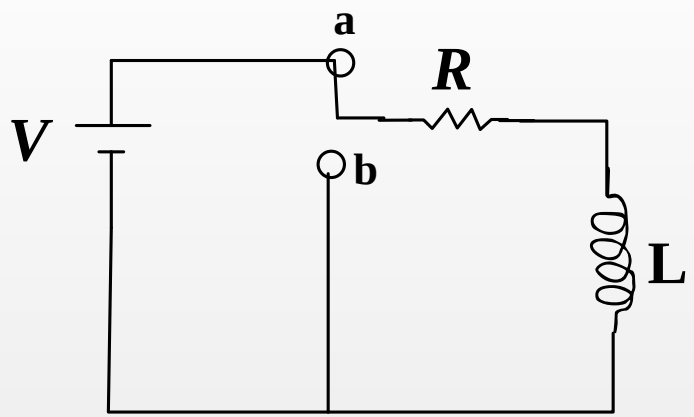

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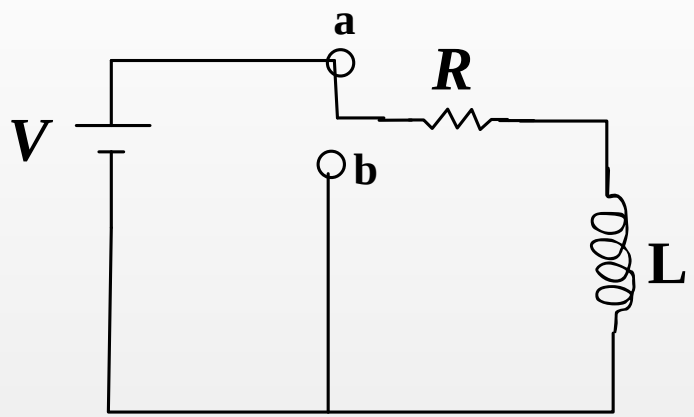

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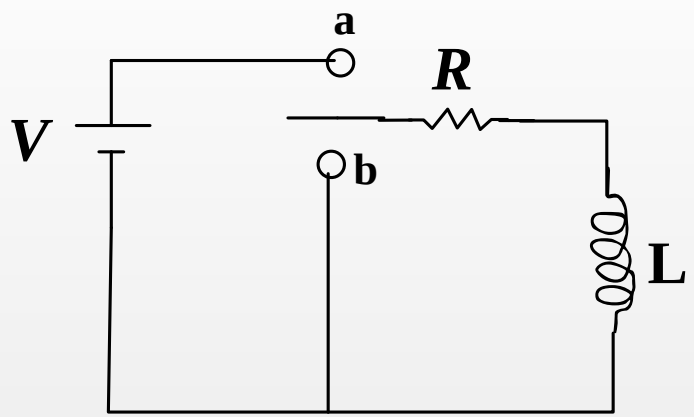

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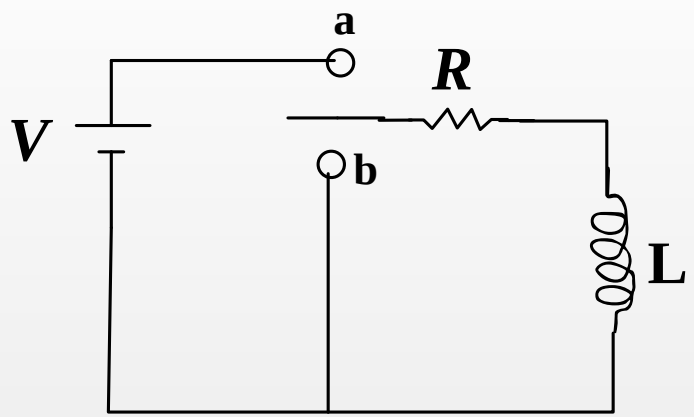

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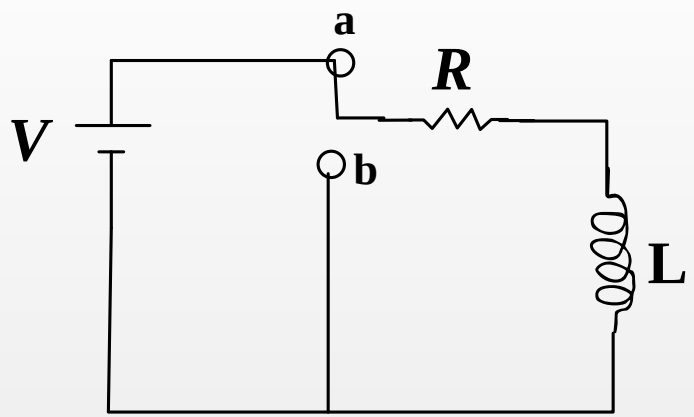

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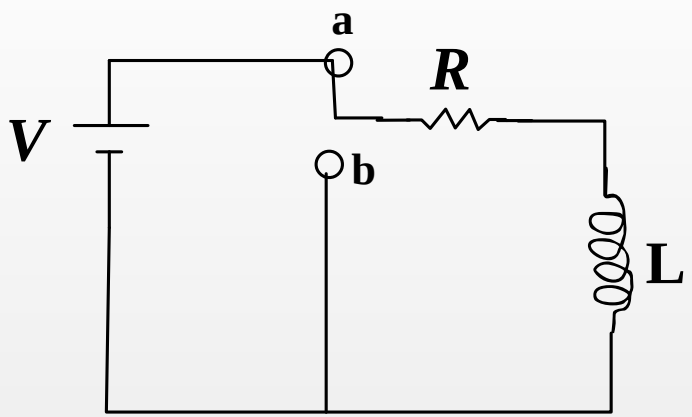
- After some time, the current in the circuit reaches an equilibrium value.
  - The inductor behaves like a piece of wire ($\varepsilon_L = 0$ since $\frac{\mathrm{d}I}{\mathrm{d}t} = 0$.)

LR Circuits - Kirchhoff's Rules



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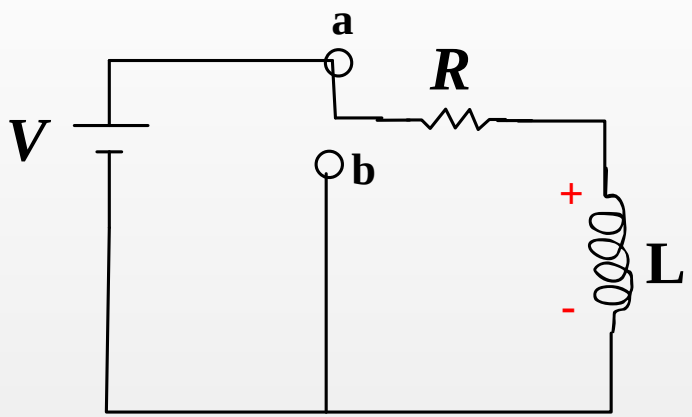

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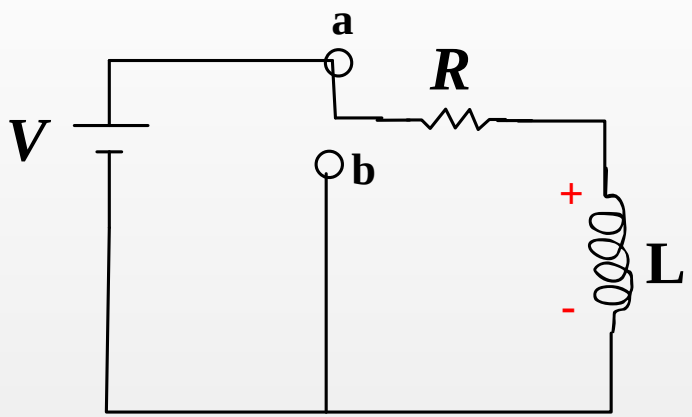

- Begin by applying Kirchhoff's rules to the closed circuit.

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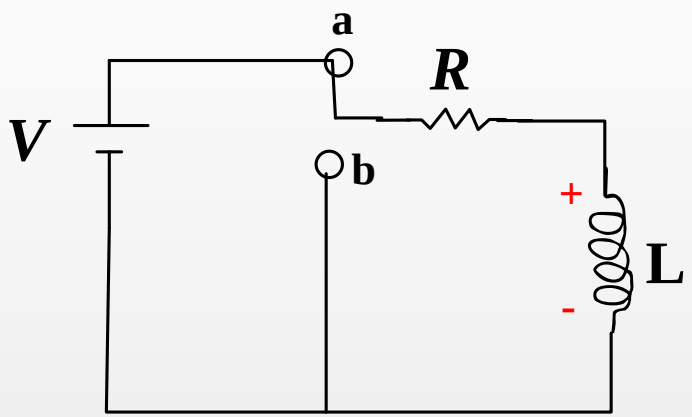

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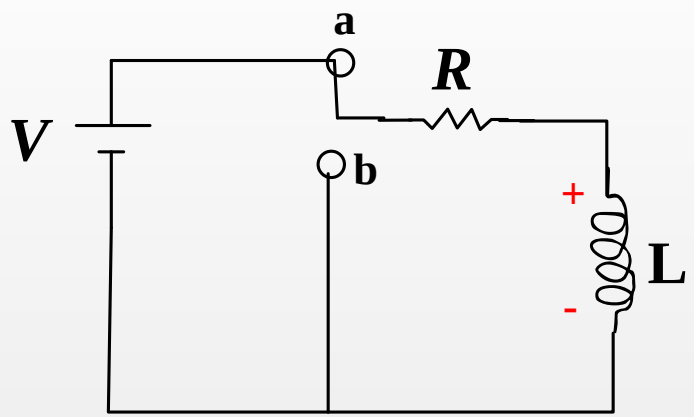

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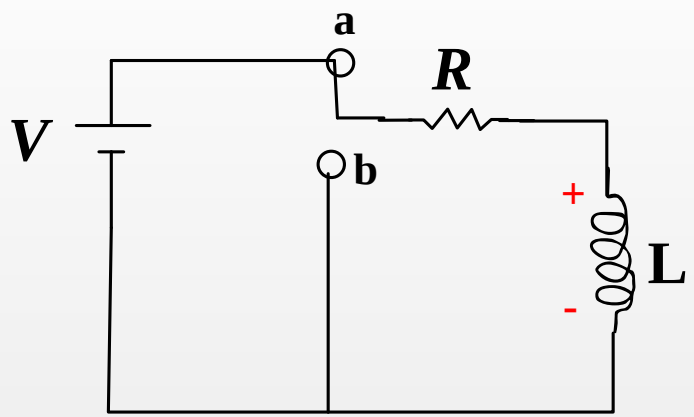

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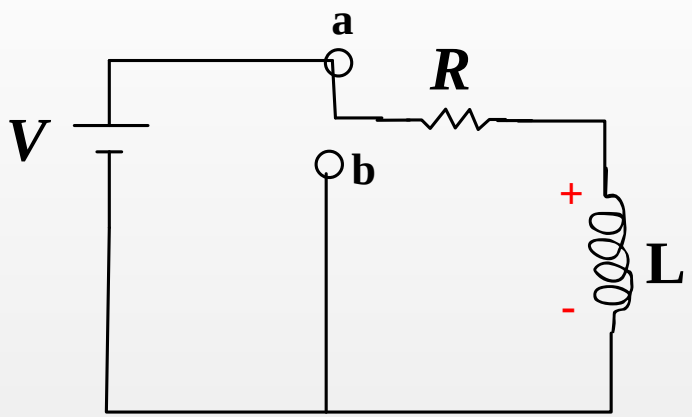
$$-I\,R - |\varepsilon_L| + V = 0.$$

$$|\varepsilon_L| = L\frac{\mathrm{d}I}{\mathrm{d}t}$$

$$\Rightarrow V = I\,R + L\frac{\mathrm{d}I}{\mathrm{d}t}$$

Mathematical Equation for Current



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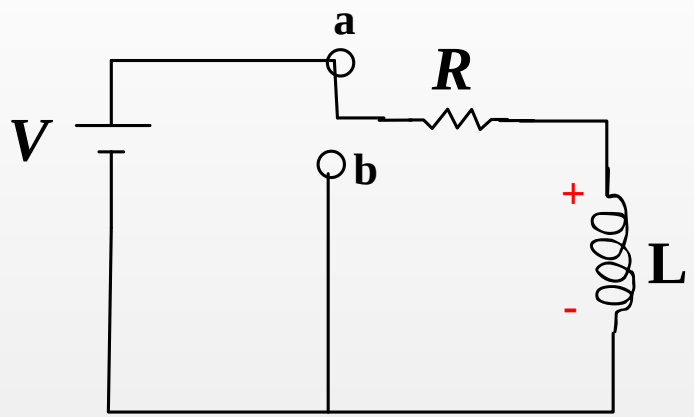

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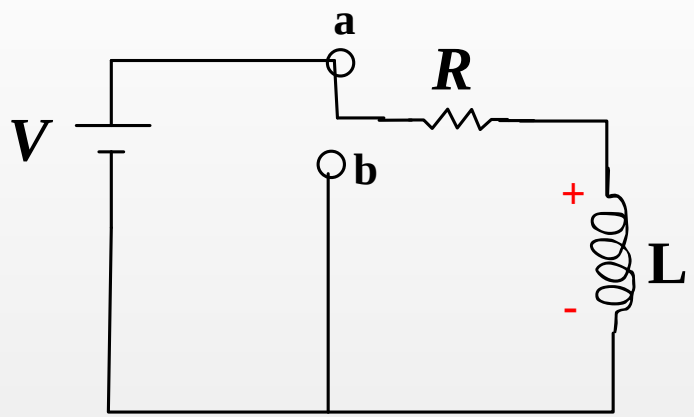

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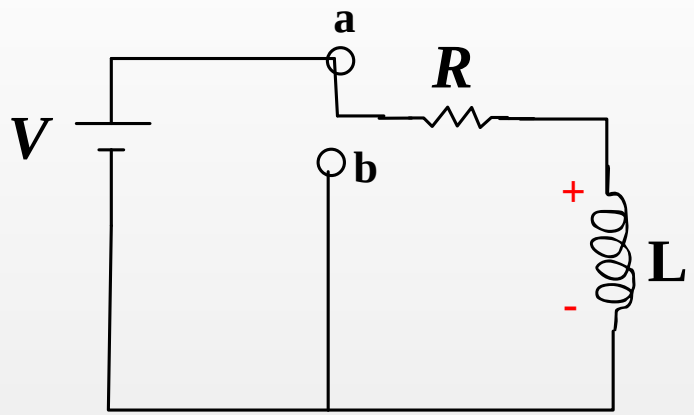

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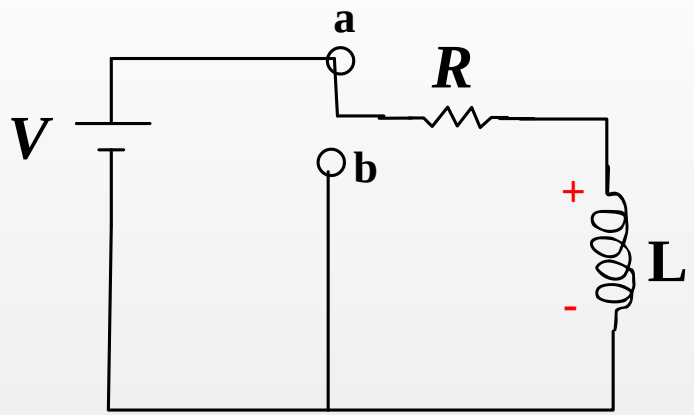

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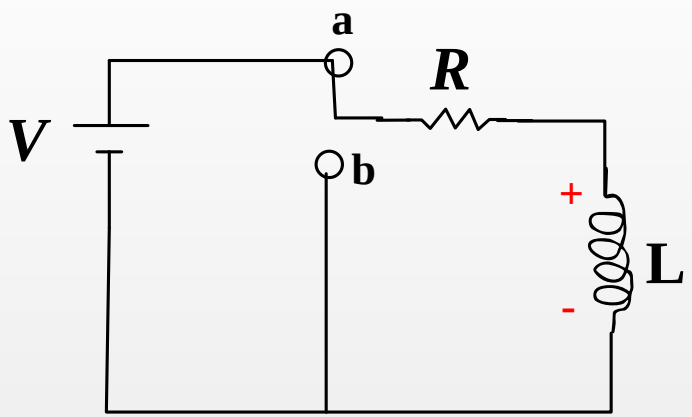

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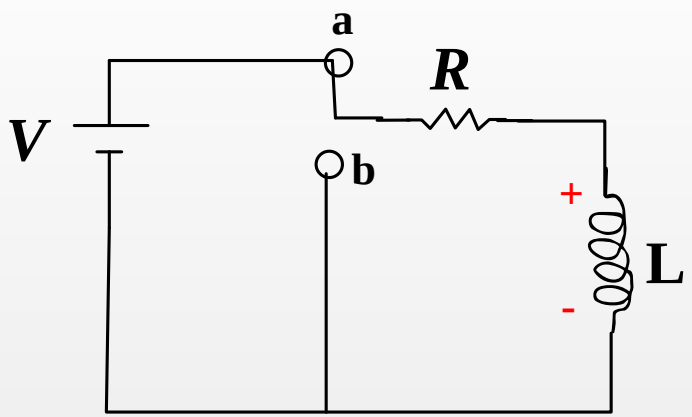

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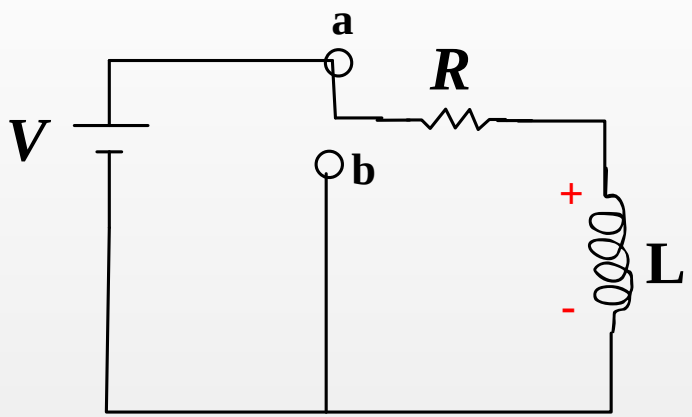

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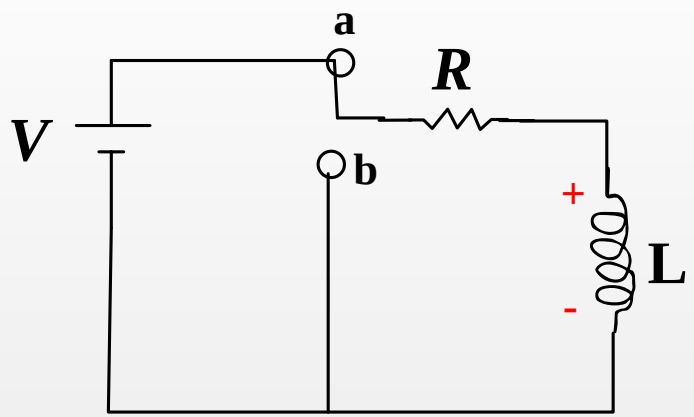

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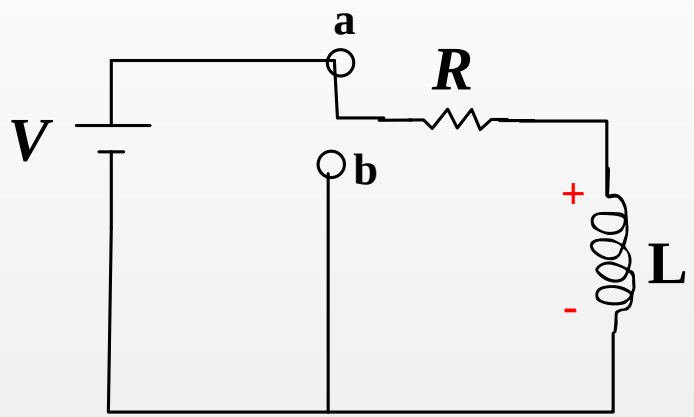

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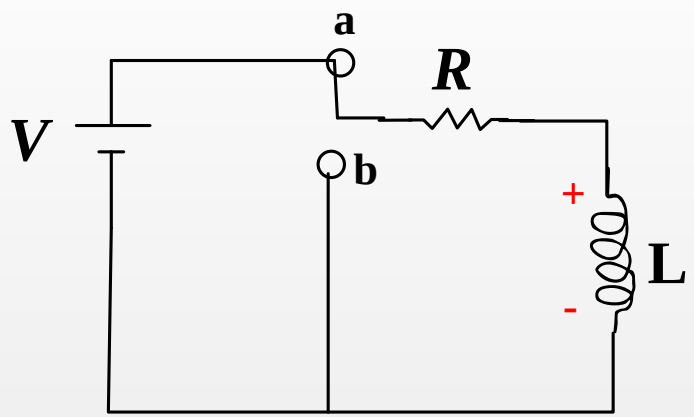

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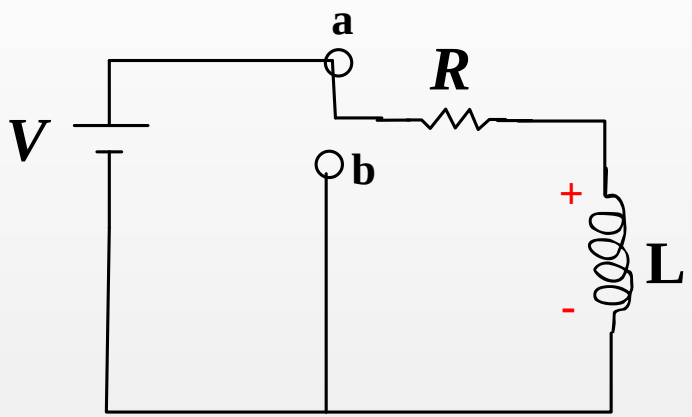

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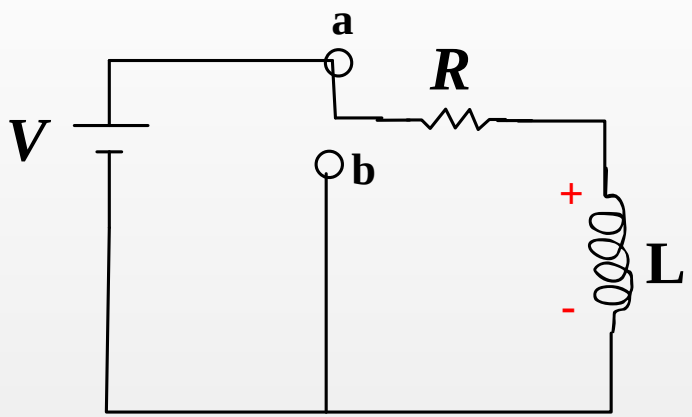

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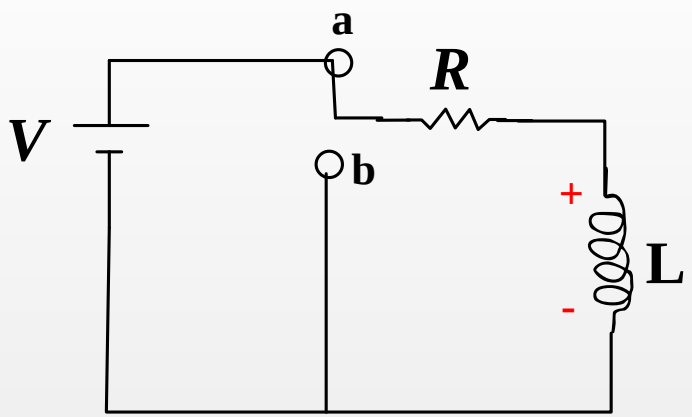

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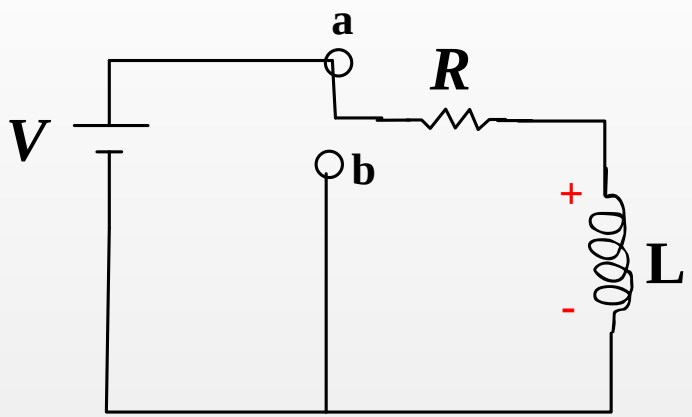

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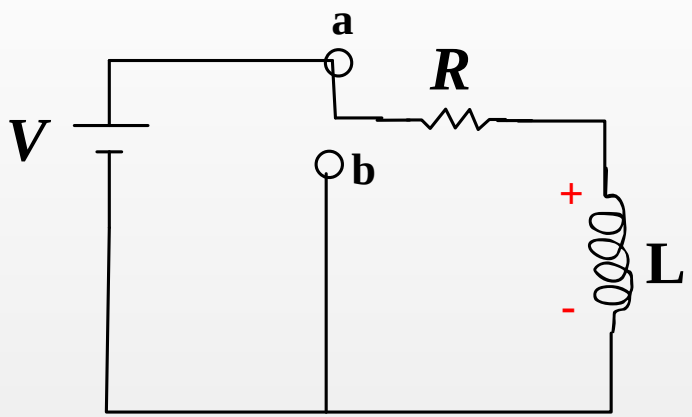

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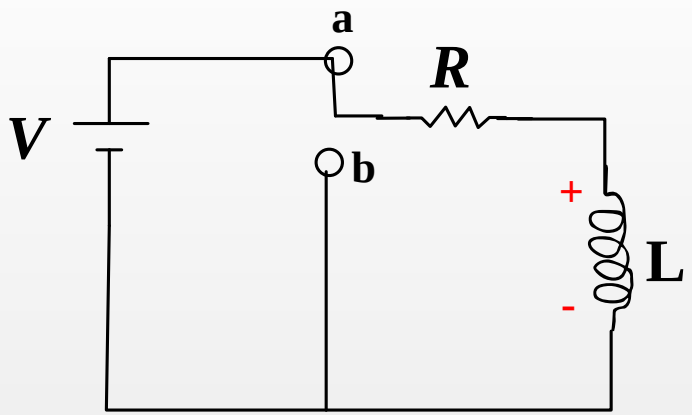

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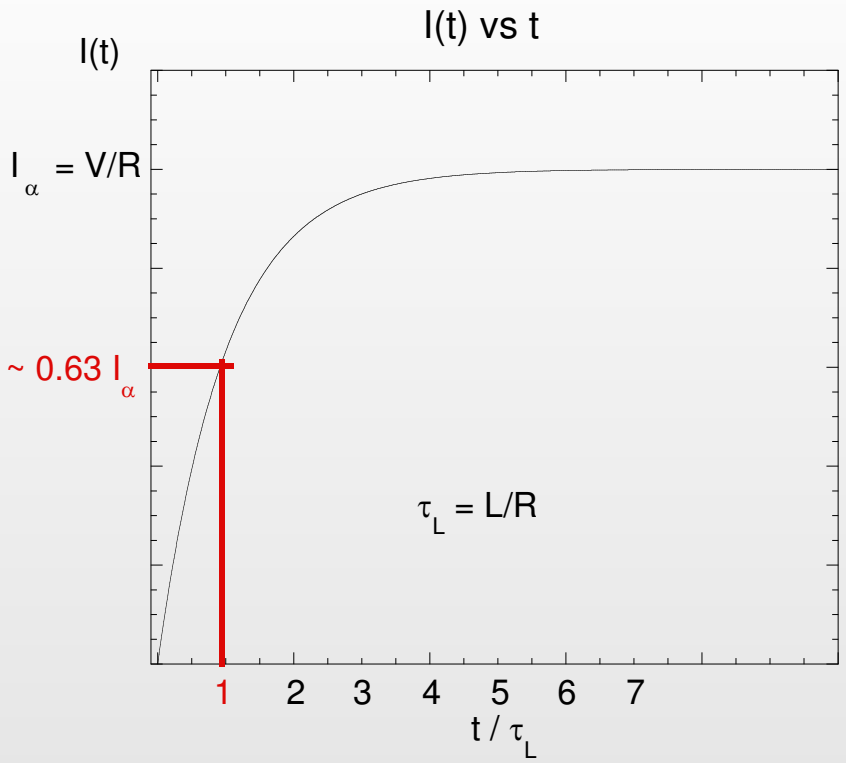

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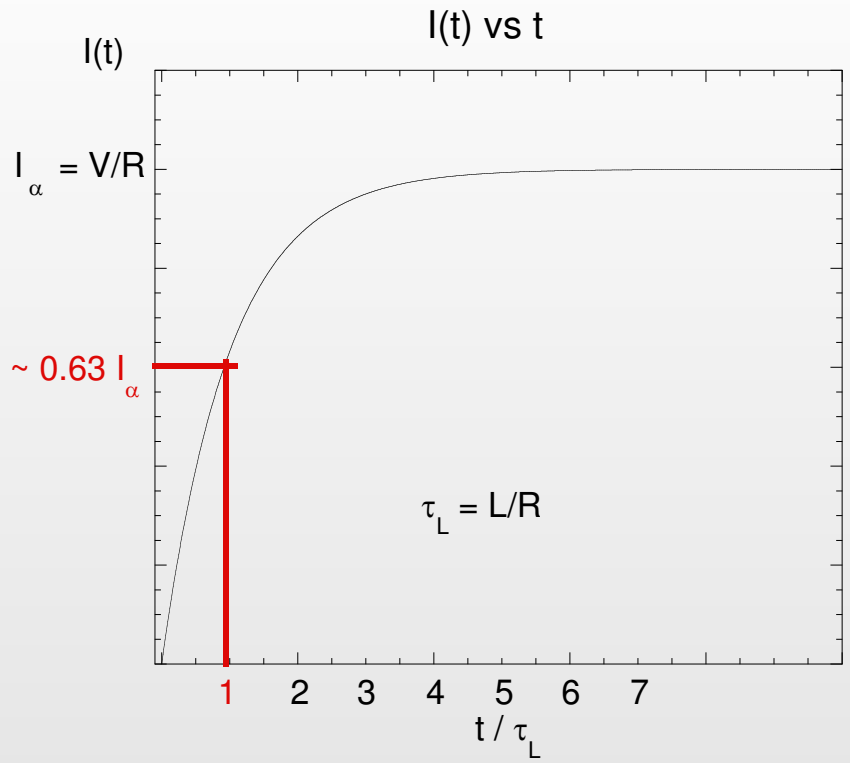

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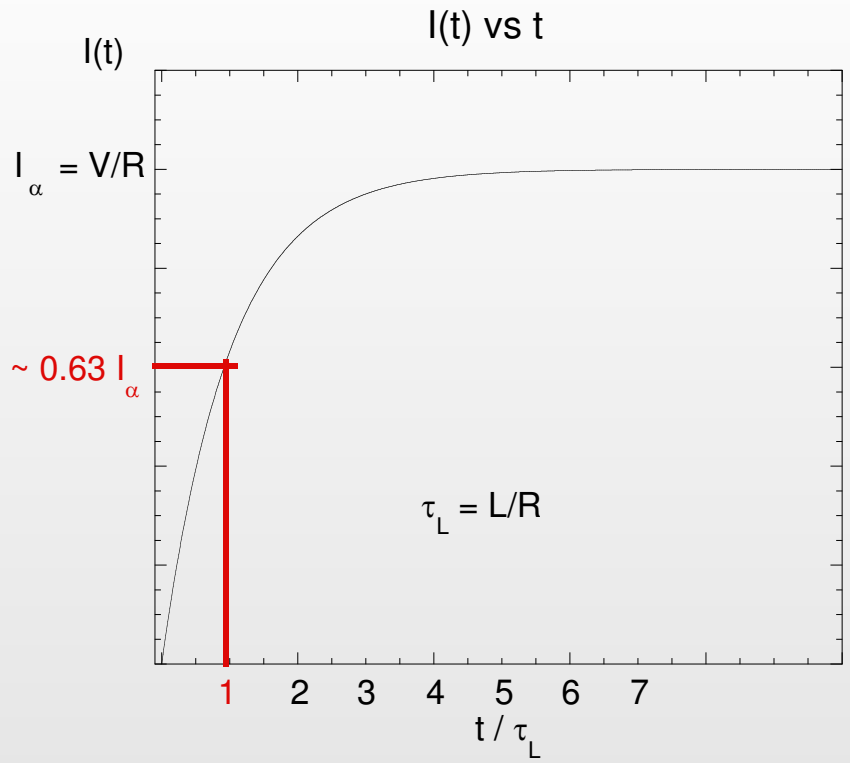

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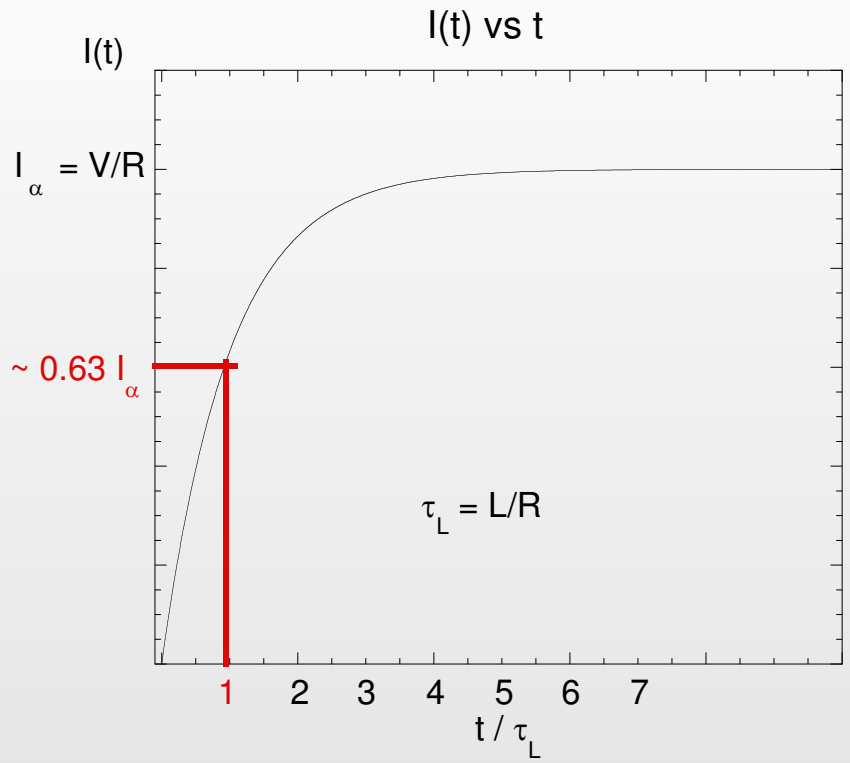

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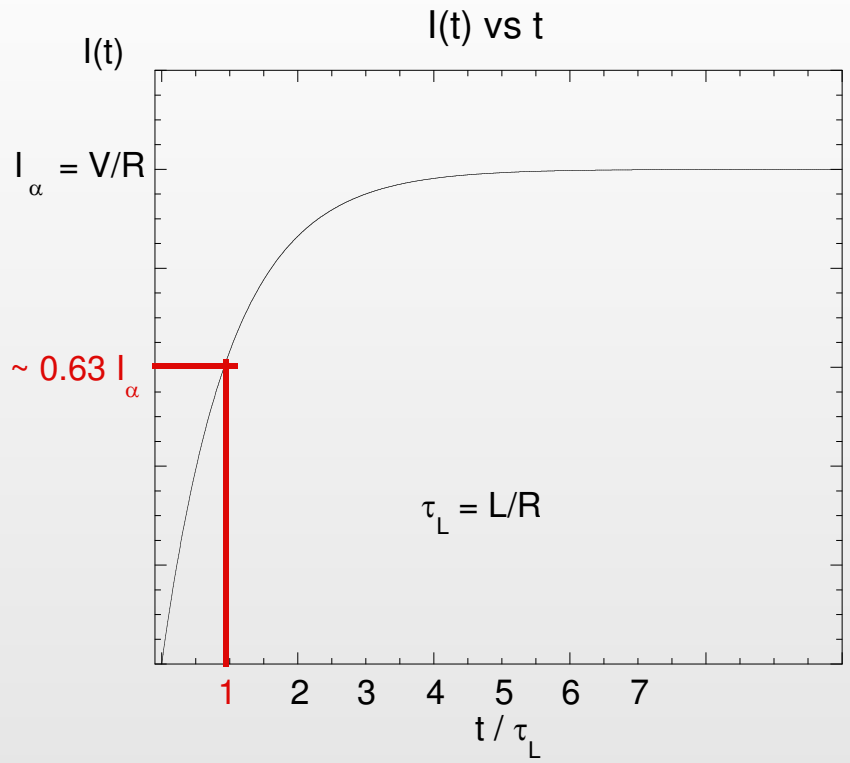

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- What happens to the EMF in the inductor?