PHYS 102
Magnetic Fields

Motion of Charged Particles in Magnetic Fields

Circulating Charges (Charge +q in a uniform magnetic field)

$$r = \frac{mv}{|q|B}$$

Since the particle is tracing out a circle of radius of r, we could calculate the time it takes to the particle to undergo one full revolution.

$$T = \frac{2\pi r}{v} = \frac{2\pi}{v}\frac{mv}{|q|B} = \frac{2\pi m}{|q|B}$$

This time is called the ***cyclotron period***.

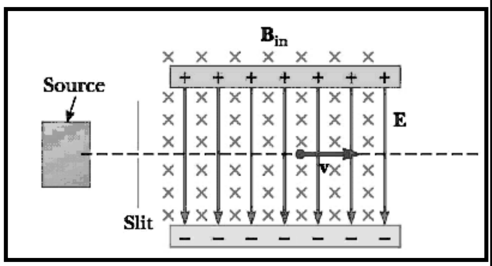
Which implies that the ***frequency*** of the circular motion (***cyclotron frequency***) is

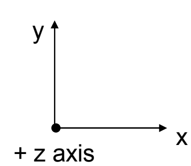
$$f = \frac{1}{T} = \frac{|q|B}{2\pi m} \Rightarrow \omega = 2\pi f = \frac{|q|B}{m}$$

PHYS 102
Magnetic Fields

Question.

- What velocity is needed so that a +q charge moves undeflected in the region defined on the right?
- A uniform electric field is perpendicular to a uniform magnetic field with directions indicated in the figure.



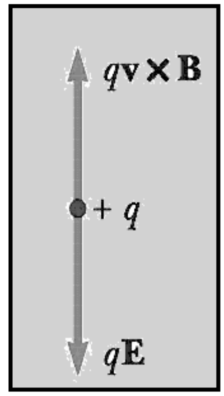


PHYS 102
Magnetic Fields

- Draw a Free Body Diagram (FBD)
- When the two forces have equal magnitudes, the charge +q will NOT deflect.
- This occurs for a speed of
  v = E / B

So

$$\vec{v} = \frac{|\vec{E}|}{|\vec{B}|}\hat{i}$$



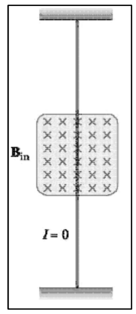
PHYS 102
Magnetic Fields

Magnetic Force on a Current Carrying Conductor

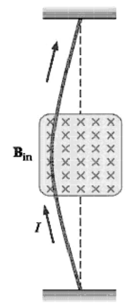
- – The current is a collection of many charged particles in motion
- The direction of the force is given by the right-hand rule

PHYS 102
Magnetic Fields

- In this case, there is no current, so there is no force
- Therefore, the wire remains vertical



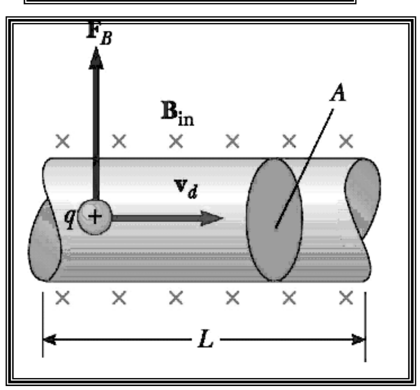
- **B** is into the page
- The current is up the page
- The force is to the left



PHYS 102
Magnetic Fields

Force on a Wire

- The magnetic force is exerted on each moving charge in the wire
  - $\mathbf{F}_1 = q\,\mathbf{v}_d \times \mathbf{B}$
- The total force is the product of the force on one charge with the number of charges
  - $\mathbf{F} = (q\,\mathbf{v}_d \times \mathbf{B})nAL$



PHYS 102
Magnetic Fields

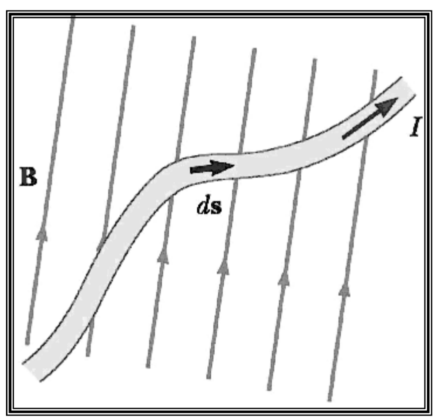
- In terms of the current, this become
- $\mathbf{F} = I\,\mathbf{L} \times \mathbf{B}$
  - **L** is a vector that points in the direction of the current
    - Its magnitude is the length *L* of the segment
  - I is the current
  - **B** is the magnetic field

Force on a Wire

PHYS 102
Magnetic Fields

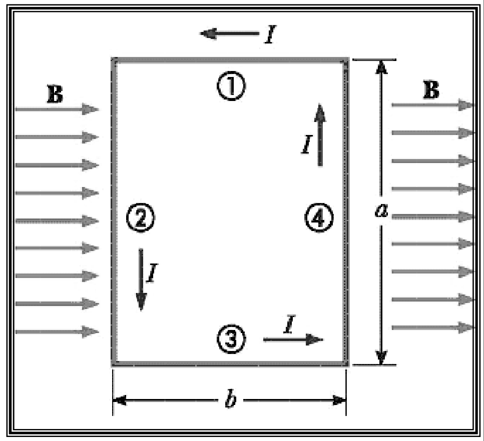
- What if the wire is not straight? (go back to 17th century mathematics)
- Consider a small segment of the wire, $d\mathbf{s}$
- The force exerted on this segment is $d\mathbf{F} = I\, d\mathbf{s} \times \mathbf{B}$
- The total force is

$$\mathbf{F} = I \int_{\mathbf{a}}^{b} d\mathbf{s} \times \mathbf{B}$$



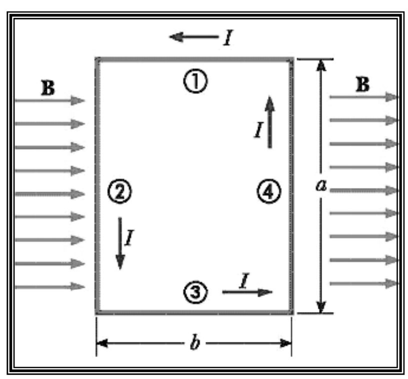
PHYS 102
Magnetic Fields

- The rectangular loop of wire on the right carries a current I in a uniform magnetic field.
- No magnetic force acts on sides 1 & 3
  - The wires are parallel to the field and **L** x **B** = 0



PHYS 102
Magnetic Fields

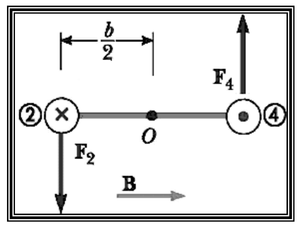
- There is a force on sides 2 & 4 -> perpendicular to the field
- The magnitude of the magnetic force on these sides will be:
  - $F_2 = F_4 = IaB$
- The direction of $\mathbf{F}_2$ is out of the page
- The direction of $\mathbf{F}_4$ is into the page



PHYS 102 Magnetic Fields

- The forces are equal and in opposite directions, but not along the same line of action
- The forces produce a torque around point *O*

Torque on a Current Loop



PHYS 102 Magnetic Fields

- The maximum torque is found by:

$$\tau_{max} = F_2\frac{b}{2} + F_4\frac{b}{2} = (I\,aB)\frac{b}{2} + (I\,aB)\frac{b}{2}$$
$$= I\,abB$$

- The area enclosed by the loop is *ab*, so $\tau_{max} = IAB$
  - This maximum value occurs only when the field is parallel to the plane of the loop

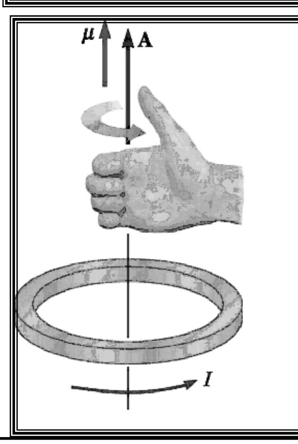
PHYS 102 Magnetic Fields

- The torque has a maximum value when the field is perpendicular to the normal to the plane of the loop
- The torque is zero when the field is parallel to the normal to the plane of the loop
- $\tau = I\mathbf{A} \times \mathbf{B}$ where A is perpendicular to the plane of the loop and has a magnitude equal to the area of the loop

PHYS 102 Magnetic Fields

- The right-hand rule can be used to determine the direction of **A** for a closed loop.
- Curl your fingers in the direction of the current in the loop
- Your thumb points in the direction of **A**

Direction of **A**



PHYS 102
Magnetic Fields

Magnetic Dipole Moment

- The product $I\mathbf{A}$ is defined as the **magnetic dipole moment**, μ, of the loop
  - Often called the magnetic moment
- SI units: A · m^2
- Torque in terms of magnetic moment:

$$\tau = \mu \times \mathbf{B}$$

  - Analogous to $\tau = \mathbf{p} \times \mathbf{E}$ for electric dipole

PHYS 102
Magnetic Fields

- **B**-field does work in rotating a current carrying loop through an angle dθ given by

Negative because torque tends to decrease θ

$$dW = -\tau\, d\theta \longrightarrow dW = -\mu B \sin\theta\, d\theta$$

$$dU = -dW = \mu B \sin\theta\, d\theta$$

$$U = -\mu B \cos\theta + U_0$$

Choosing $U = 0$ when θ = π/2 yields the following expression:

PHYS 102
Magnetic Fields

$$U = -\vec{\mu} \cdot \vec{B}$$

This equation gives the potential energy of a magnetic dipole in a magnetic field.