PHYS102 - Capacitors

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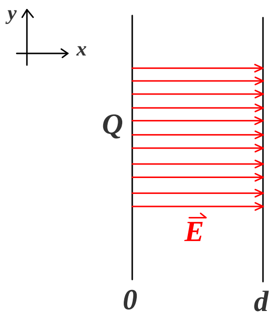
0.1 Parallel plate example

Capacitors

- When using a pair of conductors to store energy, we term the pair of conductors a capacitor. capacitor.
- Capacitors typically used to store short-termshort-term electrical energy.
- Consider our previous example:

$$|\Delta V| = \frac{Q\,d}{\varepsilon_0\,A} \rightarrow |\Delta V| \propto Q$$

$$|\Delta V| = Q\left(\frac{d}{\varepsilon_0\,A}\right)$$



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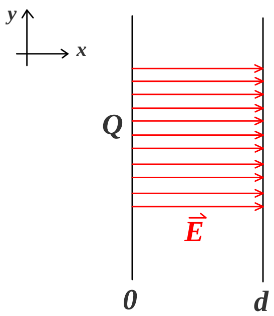
Capacitors II

$$|\Delta V| = Q\left(\frac{d}{\varepsilon_0\,A}\right)$$

- Rewriting to find the amount of charge Q.

$$Q = |\Delta V|\left(\frac{\varepsilon_0\,A}{d}\right) \rightarrow \frac{Q}{|\Delta V|} = \frac{\varepsilon_0\,A}{d}$$

- Notice that the ratio Q/V depends only the geometry of the specific problem.
- The ratio $C \equiv Q/V$ is termed the capacitance.



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0.2 Capacitance

Capacitors II

- C is a measure of the capacity to store charge for a given potential difference across two conductors.
- The unit of capacitance is one "Farad".

$$[C] = \frac{[Q]}{[V]} = 1\text{C} \;/\; 1\ \text{V} \equiv 1\text{Farad}$$

- 1 Farad is a very large value, and typical values of capacitance are:pF, nF, and μF.Note: 1 miliFarad is also large.

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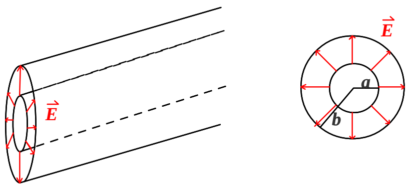
Calculating Capacitance

- Calculate the capacitance for a long coaxial cable of length L. Represent the cable as two concentric cylindrical conductors with radii a and b ($b > a$) as shown on the right.
  - Let the inner conductor carry a charge $+Q$ uniformly distributed over its length.

$$\Delta V_{ba} = -\int_b^a \vec{E} \cdot d\vec{l}$$

$$\vec{E}(a < r < b) = \frac{\lambda}{2\,\pi\,\varepsilon_0\,r}\hat{r}$$

$$\Rightarrow \Delta V_{ba} = \frac{Q}{2\,\pi\,\varepsilon_0\,L}\,\ln(b/a) \rightarrow \Delta V_{ba} > 0.\checkmark$$



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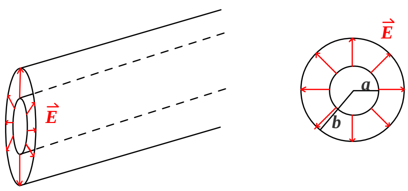
Calculating Capacitance II

$$\Delta V_{ba} = \frac{Q}{2\,\pi\,\varepsilon_0\,L}\,\ln(b/a)$$

$$\Rightarrow C = Q/V = \frac{2\,\pi\,\varepsilon_0\,L}{\ln(b/a)}$$

- The capacitance of a coaxial cable varies as the length of the cable varies. This is a very important result for many experiments.
- It is convenient to talk about the capacitance per unit length for a coaxial cable.

$$C/L = \frac{2\,\pi\,\varepsilon_0}{\ln(b/a)}$$



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Energy slide 7

0.3 Capacitance

Capacitance and Energy

- We can rewrite the energy required to charge a conductor to final charge Q in terms of capacitance and potential difference between the two surfaces of a capacitor.

$$U = \int dU = \int_0^Q V\,dq = \int_0^Q q/C\,dq \quad \text{(where V = q/C.)}$$
$$U = \frac{Q^2}{2\,C} = \frac{Q\,V}{2} = \frac{C\,V^2}{2}$$

- The three equations above are equivalent since $C \equiv Q/V$.

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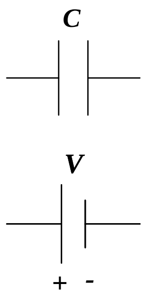
Symbols slide 8

0.4 Circuit Diagrams

Circuit Symbols

- Typically, one uses a combination of capacitors in an electric circuit (to store electric energy).
- We need to define symbols which will represent capacitors and batteries (very common in everyday electric circuits).
- The figure on the right (two parallel lines of equal length) will be used to represent a capacitor.
- The next figure on the right will be used to represent a battery (Notice the long and short sides).
  - The long side of the battery represents the side of a battery which is at a higher potential (i.e., $+$).
  - The short side of the battery represents the end of the batter which is lower in potential (i.e., -).



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0.5 Capacitors only

Circuits

- We begin circuits with a circuit diagram (a map of how we want to connect our components).
- Assumptions:
  - The components in our circuit will be connected by perfectly conducting wires.
  - The battery is an ideal battery unless otherwise stated.
    - The purpose of a battery is to maintain constant potential difference between the two ends of the battery at all costs.
- Let's begin.

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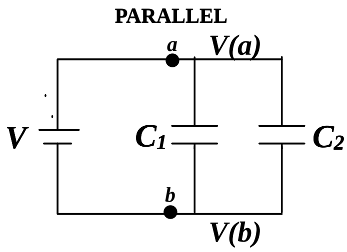
0.6 Parallel Circuit

Capacitors in Parallel

- When placing two or more components as shown in the figure on the right, we term the assembly a *PARALLEL* circuit.
- The top of the capacitors are connected to the top of the battery.
- The bottom of the capacitors are connected to the bottom of the battery.
- The difference in potential between the top and bottom of the battery is V.
- This means that the potential difference across C_1 and C_2 is the same and is given by the V.



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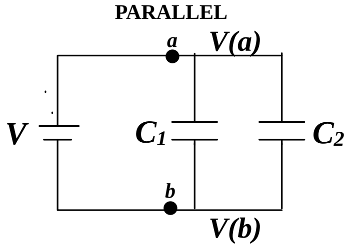
Capacitors in Parallel II

- This means that the potential difference across C_1 and C_2 is the same and is given by the V.

$$\Delta V_{ba} = V(a) - V(b) = V \quad \text{(The potential difference of the battery)}$$
$$\Rightarrow Q_1 = C_1 V \quad \& \quad Q_2 = C_2 V$$



- The total charge over the two capacitors must be the charge supplied by the battery!

$$Q_T = Q_1 + Q_2$$
$$C_T V_T = C_1 V_1 + C_2 V_2$$
$$C_T V_T = V(C_1 + C_2)$$

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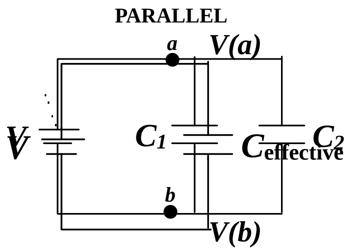
Capacitors in Parallel III

$$C_T V_T = V(C_1 + C_2) \Rightarrow C_T V = V(C_1 + C_2)$$
$$C_T = (C_1 + C_2)$$

- This means that the entire circuit may be represented as a single capacitor ($C_{\text{effective}}$) as shown on the right
- where $C_{\text{effective}} = C_1 + C_2$.
- If we generalize for capacitors connected in parallel:

$$C_{\text{effective}} = C_1 + C_2 + C_3 + \cdots$$



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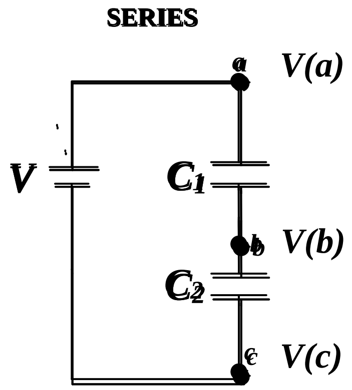
0.7 Series Circuit

Capacitors in Series

- When placing two or more components as shown in the figure on the right, we term the assembly a *SERIES* circuit.
- The capacitors are connected "back-to-back".
- The bottom of one plate is connected *by the same wire* to the top of the other plate.
- If we label points along the circuit as in the diagram, we can discuss the potential difference across the points.
  - ◆ The difference in potential between points c and a is the potential difference, V, of the battery.



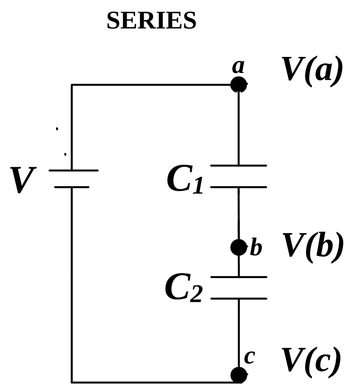
Capacitors in Series II

- The difference in potential between points c and a is the potential difference, V, of the battery.

$$\Delta V_{ca} = V(a) - V(c) = V$$
$$\Delta V_{ba} = V(a) - V(b)$$
$$\Delta V_{cb} = V(b) - V(c)$$
$$\Delta V_{ca} = \Delta V_{ba} + \Delta V_{cb}$$

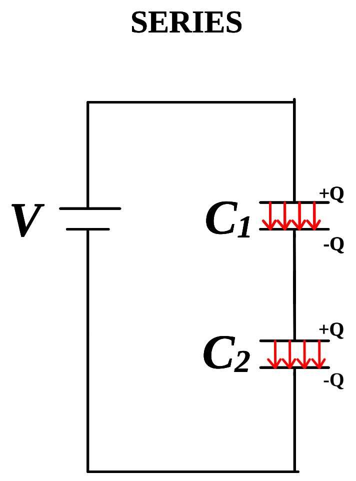
- ΔV_{ba} is the potential difference across capacitor C_1, and ΔV_{cb} is the potential difference across capacitor C_2.

$$V = V_1 + V_2$$



Capacitors in Series III

- There is a potential difference across C_1.
- The electric field in C_1 does something to the wire connecting C_1 with C_2 (What is it?).
- The same magnitude of charge will develop on the top plate of C_2 (due to conservation of charge).
- This charge will create an electric field (but we already knew that - from the previous slide) which will induce a $-Q$ on the bottom plate.
- MORAL OF THE STORY: Capacitors in series have the same amount of charge!!



Capacitors in Series IV

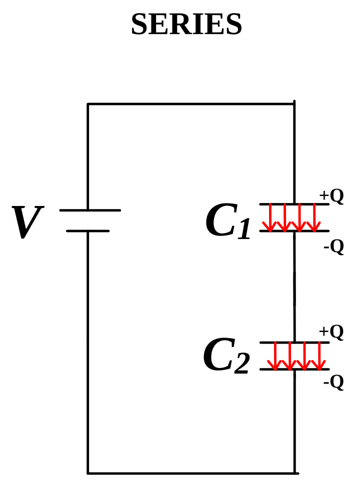
- Starting with:

$$V = V_1 + V_2 = \frac{Q_1}{C_1} + \frac{Q_2}{C_2}$$

$$V = Q\,(\frac{1}{C_1} + \frac{1}{C_2})$$

$$V/Q = (\frac{1}{C_1} + \frac{1}{C_2})$$

$$1/C = (\frac{1}{C_1} + \frac{1}{C_2})$$

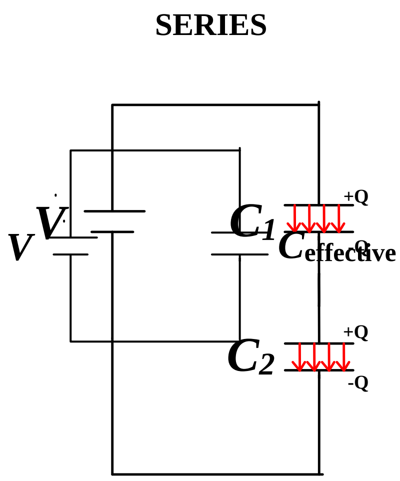


Capacitors in Series V

$$1/C = (\frac{1}{C_1} + \frac{1}{C_2})$$

- This means that the circuit on the right can be represented by the following circuit.
- Where $1/C_{\text{effective}} = 1/C_1 + 1/C_2$:
- If we generalize for capacitors connected in series:

$$\frac{1}{C_{\text{effective}}} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} + \cdots$$



SUMMARY

- Parallel circuits:
  - Elements in parallel reside at the same potential.
  - For capacitors in parallel, you need to add the capacitance of each capacitor in order to find the effective capacitance.
- Series circuits:
  - Capacitors in series have identical charges.
  - For capacitors in series, you need to add the *reciprocal* of the capacitance of each capacitor in order to find the *reciprocal* of the effective capacitance.

Dielectrics

Let's move to the chalk board.