PHYS102 - Electric Energy - Capacitors

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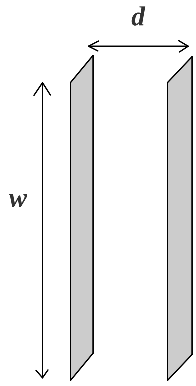
Placing Charges on Conductors

- It takes work to place a total charge Q on an isolated conductor.
- This amount of work is *energy stored* on the conductor.
- It is often very difficult to calculate the stored potential energy.
- Let's look at a system which we can calculate the stored energy.
  - Consider two very large, parallel, conducting plates as shown on the right.
- NOTE: $w \gg d$.



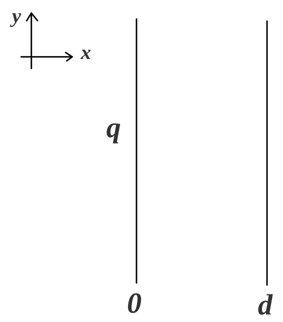
Placing Charges on Conductors II

- First deposit q onto the left plate (this is accomplished by placing a battery across both conducting plates).
- The $\vec{E}$-field is uniform near the center of the plates with magnitude:

$$E = \frac{\sigma}{\varepsilon_0}$$
$$\sigma = \frac{q}{A}$$

- There is a potential difference between the two conductors:

$$\Delta V_{ab} = \int_a^b \vec{E} \cdot d\vec{l}$$

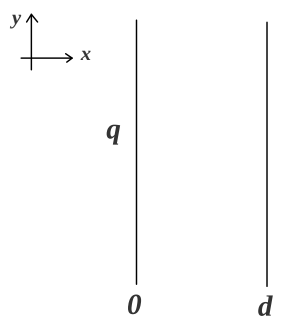


Placing Charges on Conductors III

$$\Delta V_{ab} = \int_a^b \vec{E} \cdot d\vec{l}$$

- So far, there has been no mention of the reference of zero potential.
  - This is because we are concerned with the potential difference between the two plates!

$$\boxed{|\Delta V_{d0}| = \frac{q\,d}{A\,\varepsilon_0}}$$



Placing Charges on Conductors IV

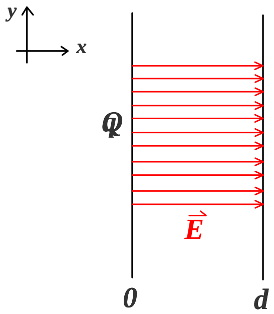
- We need to build Q on the plate, but moving an additional amount of charge dq requires an amount dW of work.
- Find the total amount of work by summing all contributions (dW).

$$\Delta U = W = \int dW = \int_0^Q \Delta V \, dq$$

$$W = \int_0^Q \frac{q\,d}{A\,\varepsilon_0}\, dq$$

$$W = \frac{1}{2}\,\frac{d}{A\,\varepsilon_0}\, Q^2$$

- $\Delta U \propto Q^2$



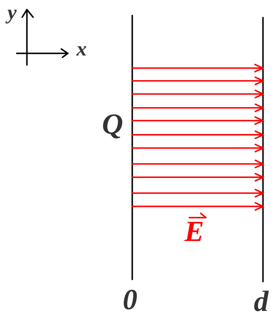
$\vec{E}$-field and Energy

$$W = \frac{1}{2}\,\frac{d}{A\,\varepsilon_0}\, Q^2$$

- This equation is "specific" to a parallel plate assembly.
- This energy stored is related to the $\vec{E}$-field between the plates.
  - We can solve Q in terms of the electric field magnitude.

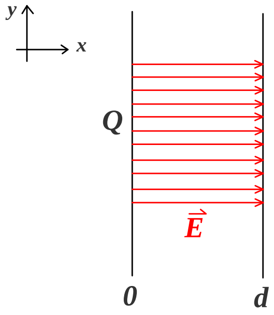
$$E = \frac{Q}{A\,\varepsilon_0} \qquad \text{(Final field strength once } Q \text{ deposited.)}$$

$$\Rightarrow Q^2 = A^2\,\varepsilon_0^2\,E^2$$



$\vec{E}$-field and Energy II

$$\Rightarrow Q^2 = A^2\,\varepsilon_0^2\,E^2$$



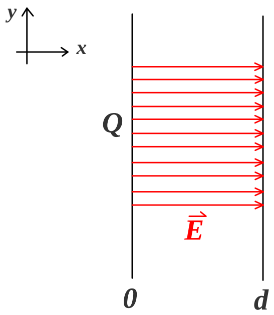
- Rewriting U in terms of electric field strength:

$$U = \frac{d}{2\,A\,\varepsilon_0}\left(A^2\,\varepsilon_0^2\,E^2\right) \Rightarrow U = A\,d\,\frac{1}{2}\,\varepsilon_0\,E^2$$

- This energy stored is related to the square of the $\vec{E}$-field between the plates.
  - The term $A\,d$ is the volume where the electric field is present!

$\vec{E}$-field and Energy III

$$\Rightarrow U = V\,\frac{1}{2}\,\varepsilon_0\,E^2$$ (V is volume where E field present.)



- Rewriting as U/V gives the energy density with units of J/m^3:

$$\Rightarrow \boxed{u_E = \frac{1}{2}\,\varepsilon_0\,E^2}$$

- This result is "General" and only requires knowledge of an electric field.
- This implies that if an electric field is present in space, then there exists stored energy in that space.

Calculating Energy from Electric Field

- If the electric field is known for a region in space (with volume V), one can calculate the stored energy (*IF the electric field is constant!*):

$$U = V\, u_E$$

- If the electric field is not constant (like that for a point particle), then one needs to calculate dU for a very small region of space, dV where the electric field is uniform.

$$dU = u_E\, dV$$
$$U = \int dU = \int \frac{1}{2}\varepsilon_0\, E^2\, dV$$

Calculating Energy from Electric Field II

$$U = \int dU = \int \frac{1}{2}\varepsilon_0\, E^2\, dV$$

- Where the integration limits are over region of space where the electric field exists.
- Let's look at an example.

0.1 Example - Uniform $\vec{E}$-field

Example - Energy and uniform E-field

Consider a typical thundercloud that rises to an altitude of 10 km and has a diameter of 20 km. Assuming an average electric field strength of 10^5 V/m, estimate the total electrostatic energy stored in the cloud.

Solution

The energy density is given by:

$$u_E = \frac{1}{2}\varepsilon_0\, E^2 = \frac{1}{2}(8.85 \times 10^{-12}\text{C}^2/\text{Nm}^2)(10^5\text{V/m})^2$$
$$= 4.4 \times 10^{-2}\text{J/m}^3$$

We assume that the energy density is the same throughout the storm, so:

$$U = \int u_E\, dV = u_E \int dV = u_E\, V = u_E\, \pi\, r^2\, h = \boxed{1.4 \times 10^{11}\text{J}}$$

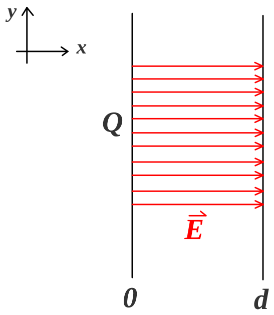
0.2 Parallel plate example

Capacitors

- When using a pair of conductors to store energy, we term the pair of conductors a capacitor. capacitor.
- Capacitors typically used to store short-term*short-term* electrical energy.
- Consider our previous example:

$$|\Delta V| = \frac{Q\,d}{\varepsilon_0\,A} \rightarrow |\Delta V| \propto Q$$

$$|\Delta V| = Q\left(\frac{d}{\varepsilon_0\,A}\right)$$



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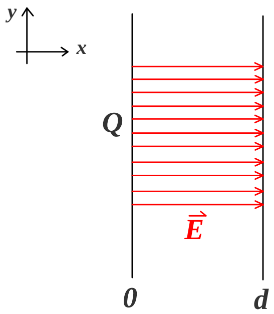
Capacitors II

$$|\Delta V| = Q\left(\frac{d}{\varepsilon_0\,A}\right)$$

- Rewriting to find the amount of charge Q.

$$Q = |\Delta V|\left(\frac{\varepsilon_0\,A}{d}\right) \rightarrow \frac{Q}{|\Delta V|} = \frac{\varepsilon_0\,A}{d}$$

- Notice that the ratio Q/V depends only the geometry of the specific problem.
- The ratio $C \equiv Q/V$ is termed the capacitance.



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0.3 Capacitance

Capacitors II

- C is a measure of the capacity to store charge for a given potential difference across two conductors.
- The unit of capacitance is one "Farad".

$$[C] = \frac{[Q]}{[V]} = 1\text{C} \;/\; 1\text{ V} \equiv 1\text{Farad}$$

- 1 Farad is a very large value, and typical values of capacitance are:pF, nF, and μF.Note: 1 miliFarad is also large.

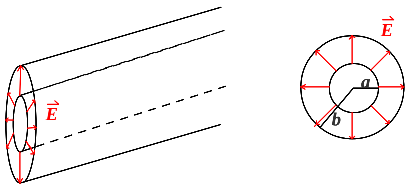
Calculating Capacitance

- Calculate the capacitance for a long coaxial cable of length L. Represent the cable as two concentric cylindrical conductors with radii a and b ($b > a$) as shown on the right.
  - Let the inner conductor carry a charge $+Q$ uniformly distributed over its length.

$$\Delta V_{ba} = -\int_b^a \vec{E} \cdot d\vec{l}$$

$$\vec{E}(a < r < b) = \frac{\lambda}{2\,\pi\,\varepsilon_0\,r}\hat{r}$$

$$\Rightarrow \Delta V_{ba} = \frac{Q}{2\,\pi\,\varepsilon_0\,L}\,\ln(b/a) \to \Delta V_{ba} > 0.✓$$



Calculating Capacitance II

$$\Delta V_{ba} = \frac{Q}{2\,\pi\,\varepsilon_0\,L}\,\ln(b/a)$$

$$\Rightarrow C = Q/V = \frac{2\,\pi\,\varepsilon_0\,L}{\ln(b/a)}$$

- The capacitance of a coaxial cable varies as the length of the cable varies. This is a very important result for many experiments.
- It is convenient to talk about the capacitance per unit length for a coaxial cable.

$$C/L = \frac{2\,\pi\,\varepsilon_0}{\ln(b/a)}$$