PHYS102 - Electric Energy - Capacitors

Dr. Suess

February 14, 2007

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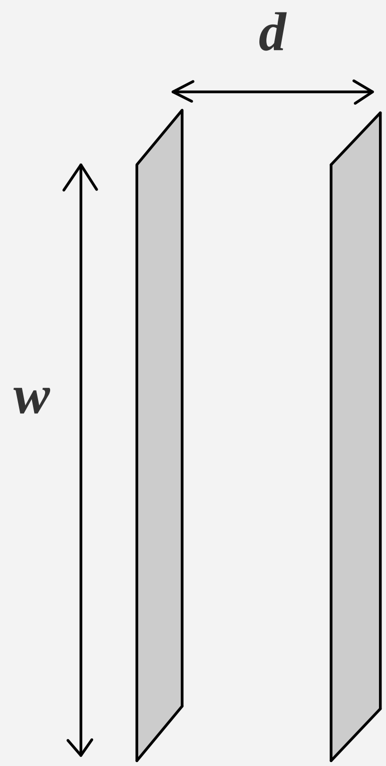

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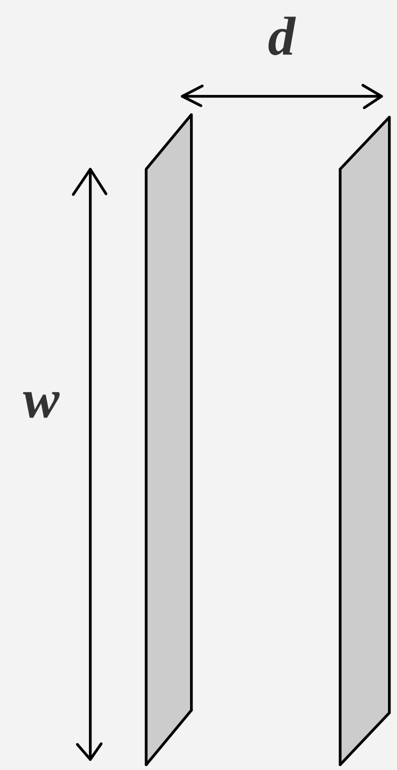
    - NOTE: $w \gg d$.



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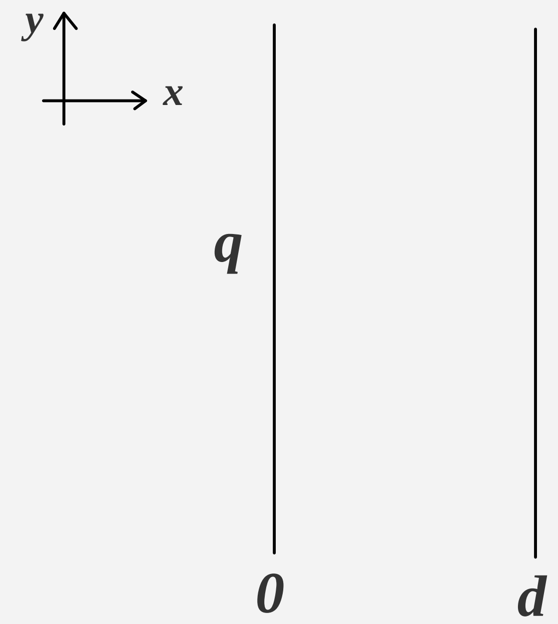

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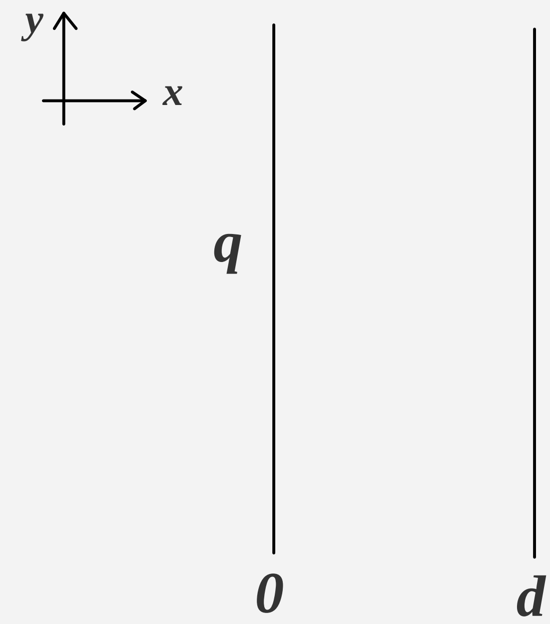
- The $\vec{E}$-field is uniform near the center of the plates with magnitude:



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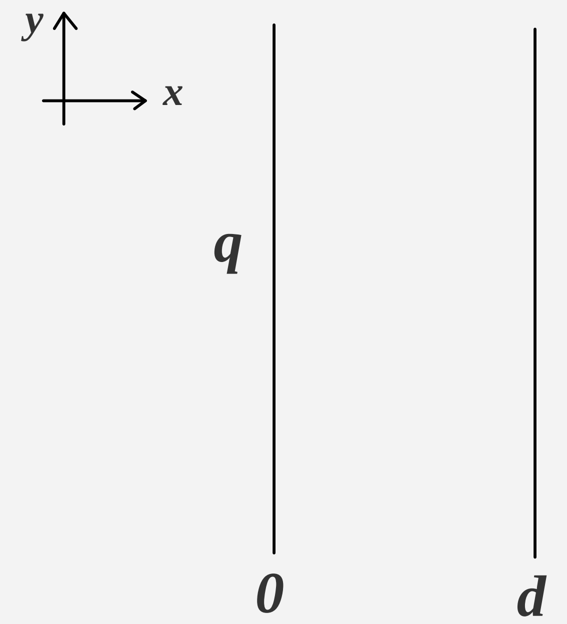

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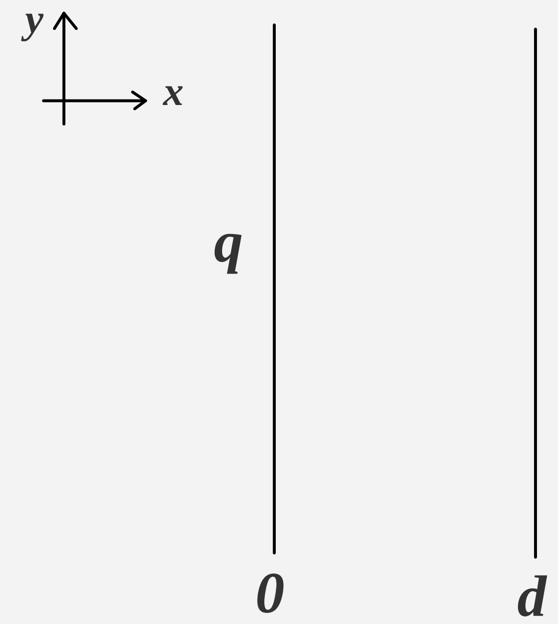

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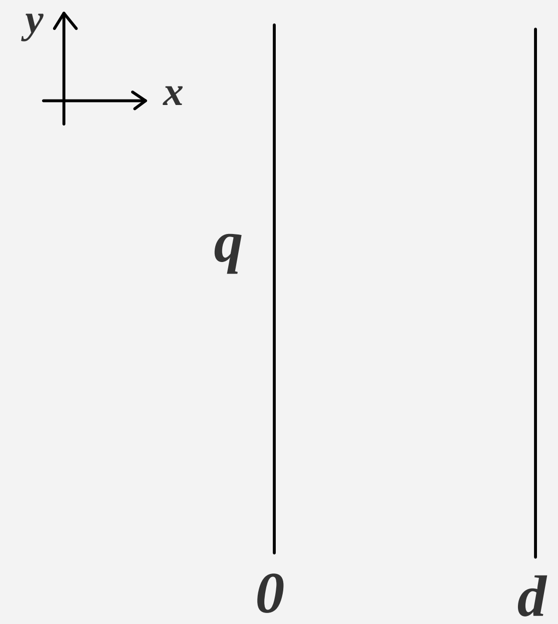

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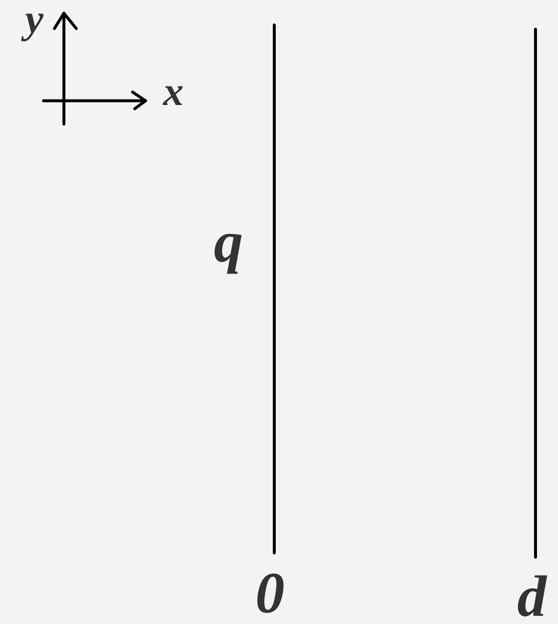

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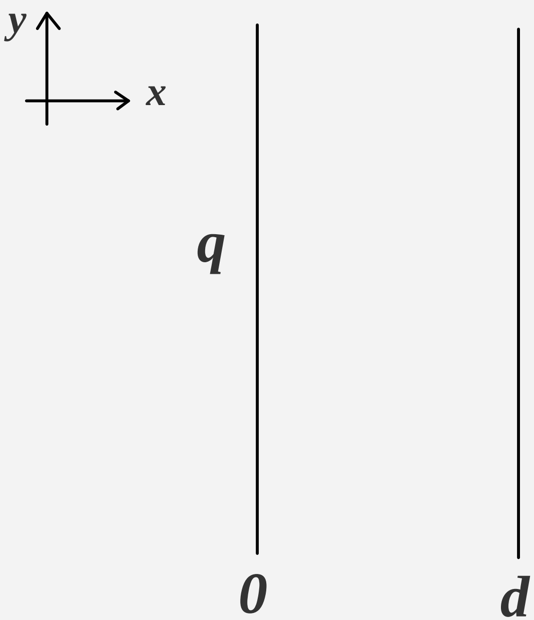

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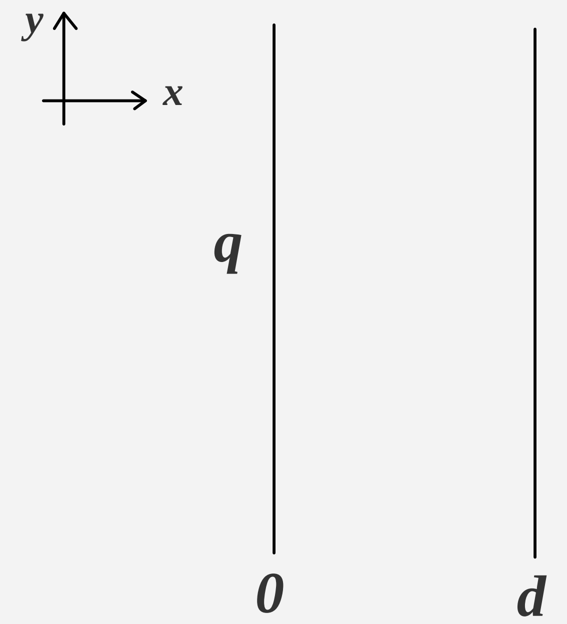

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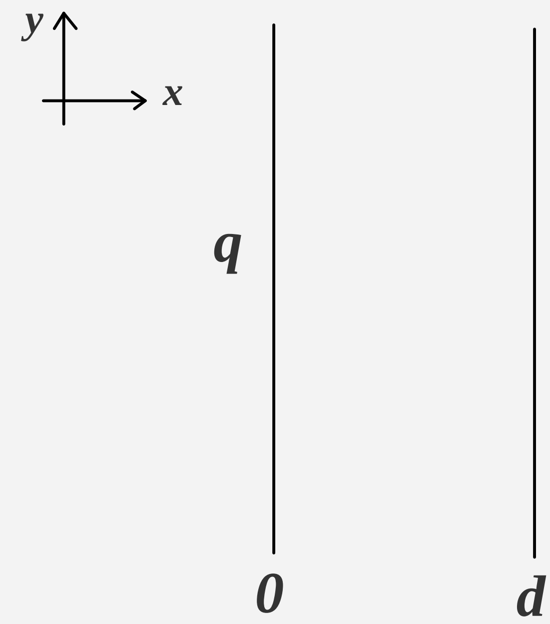

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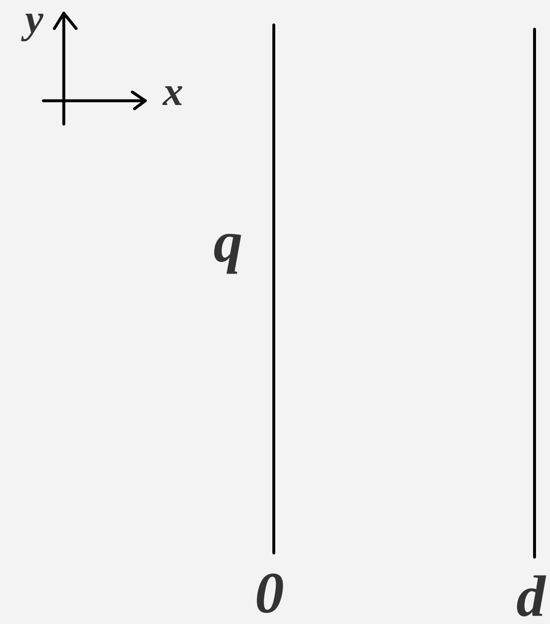

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$$\boxed{|\Delta V_{d0}| = \frac{q\,d}{A\,\varepsilon_0}}$$

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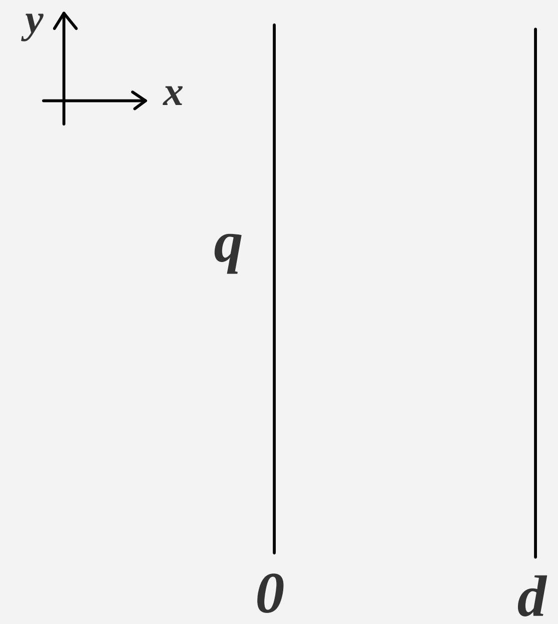

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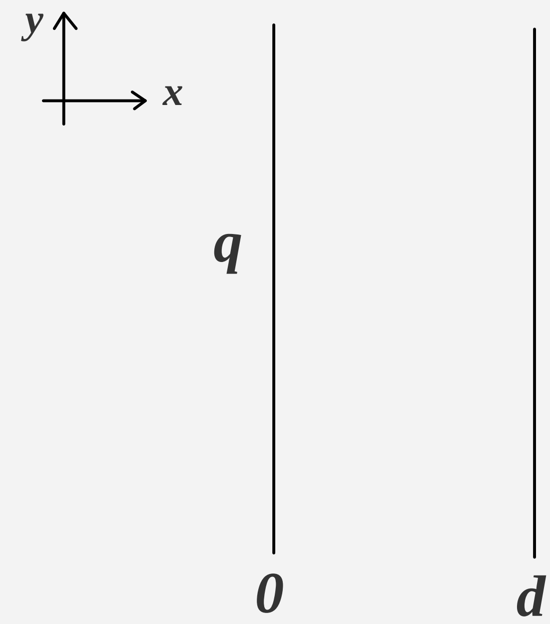

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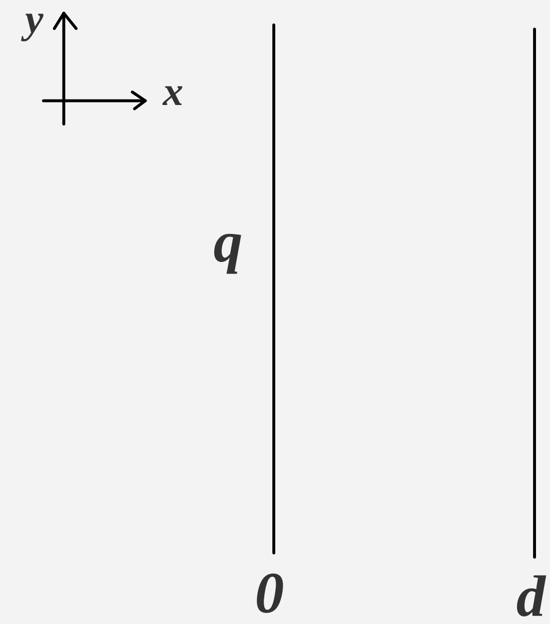

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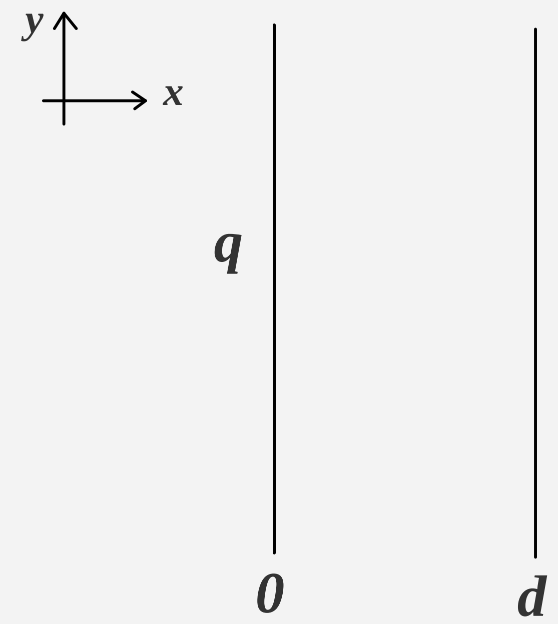

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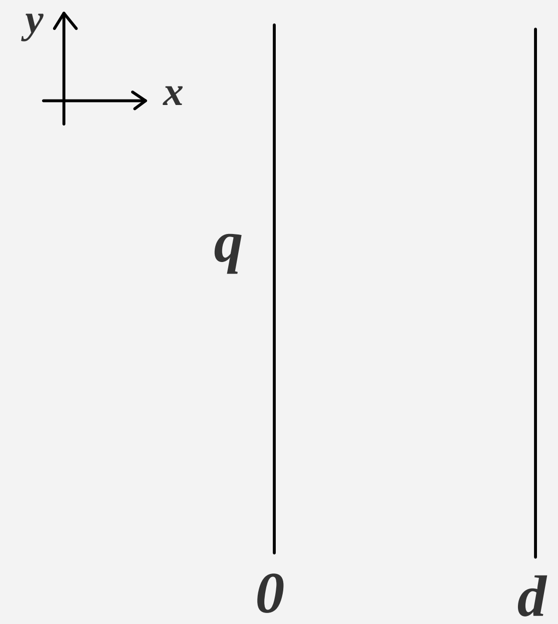

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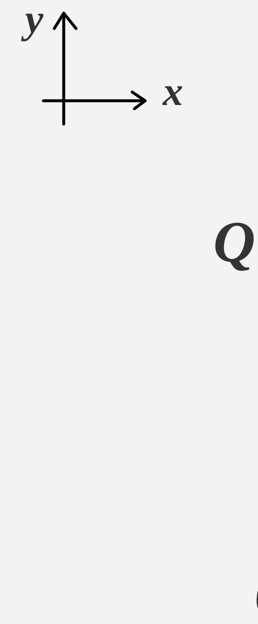
- $\Delta U \propto Q^2$



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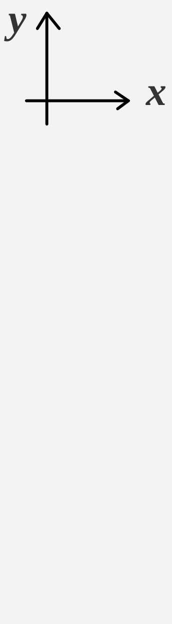
$$W = \frac{1}{2}\,\frac{d}{A\,\varepsilon_0}\,Q^2$$

- This equation is "specific" to a parallel plate assembly.



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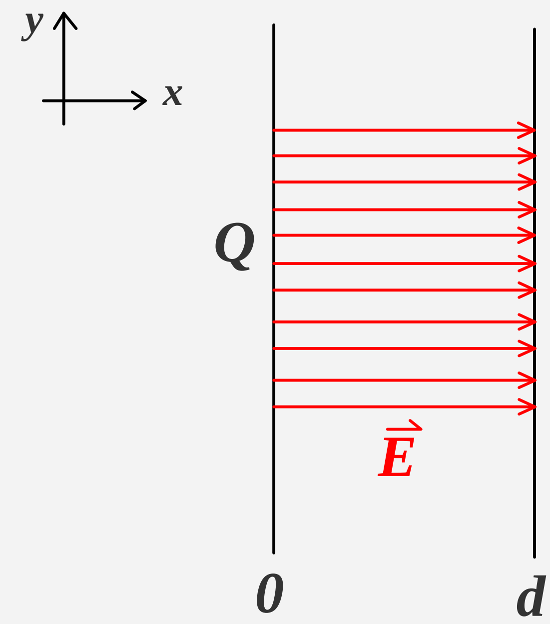

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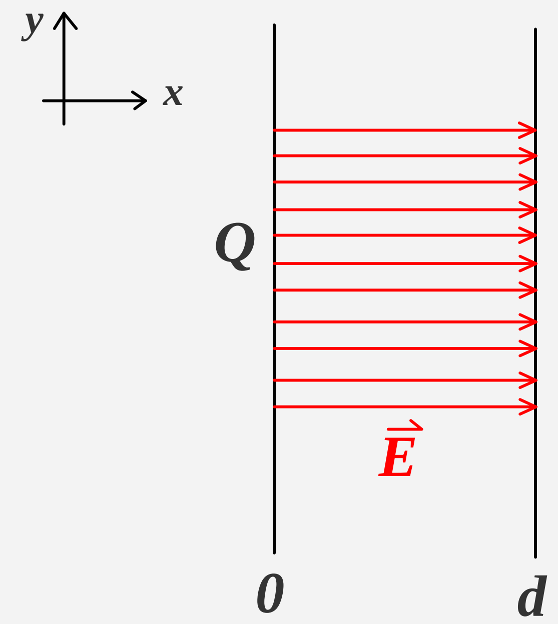

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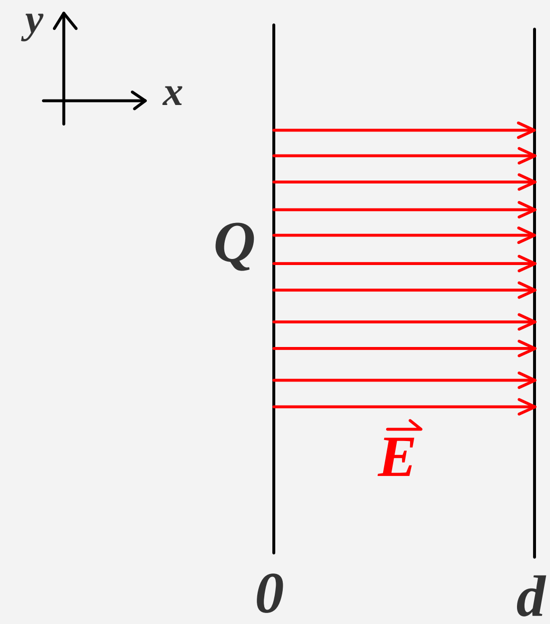

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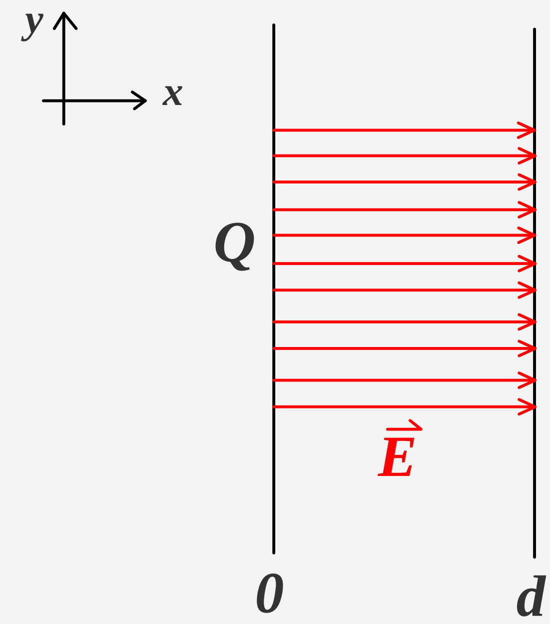

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- Rewriting U in terms of electric field strength:

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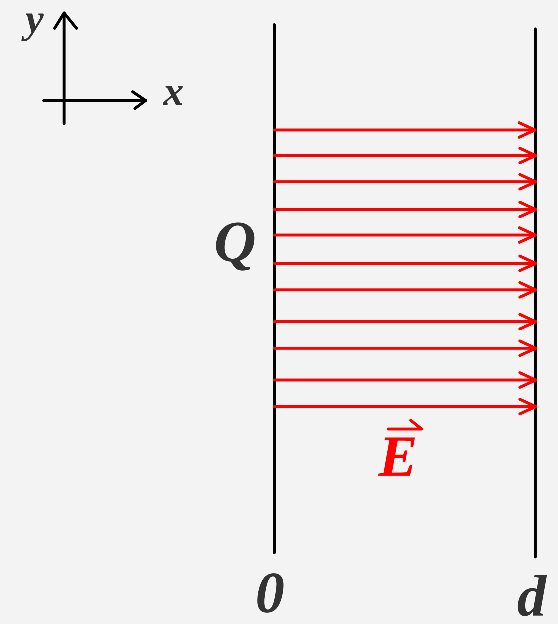

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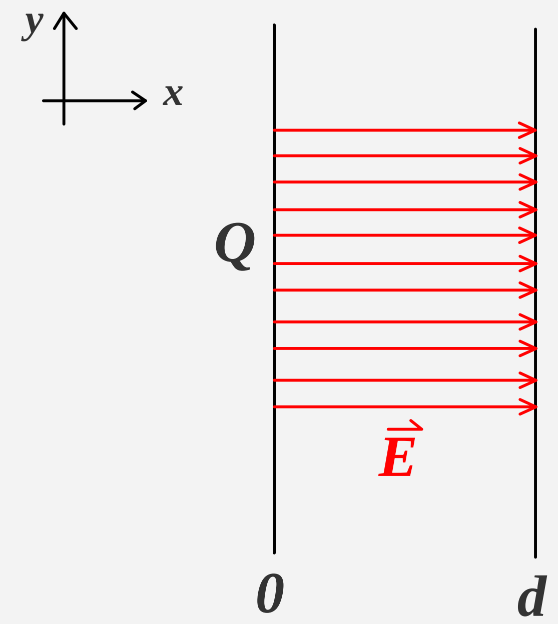

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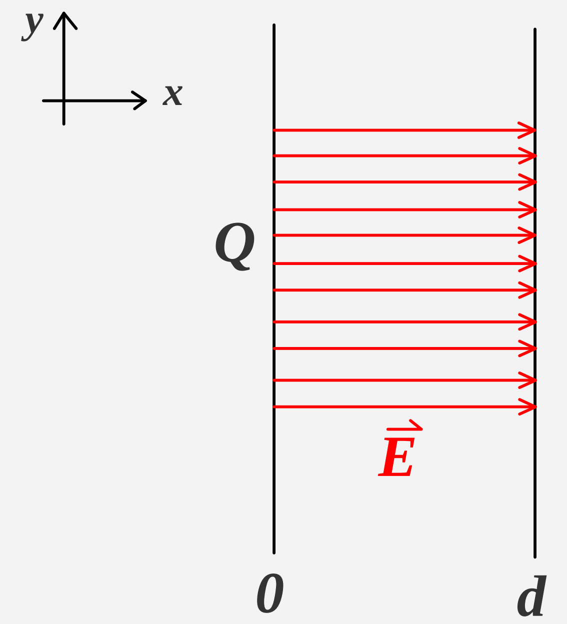

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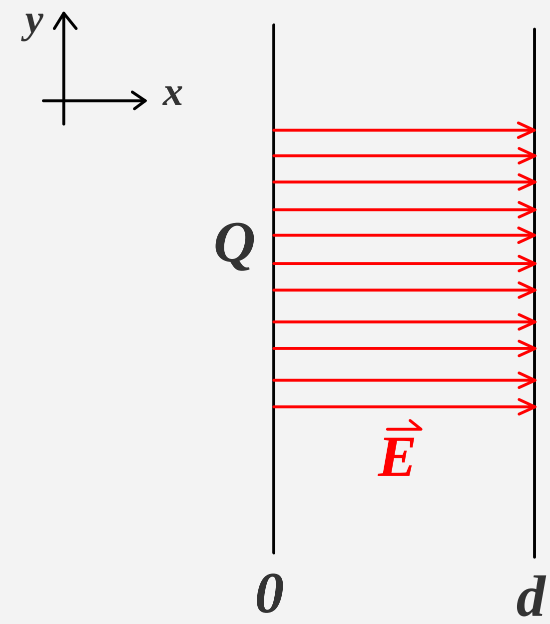

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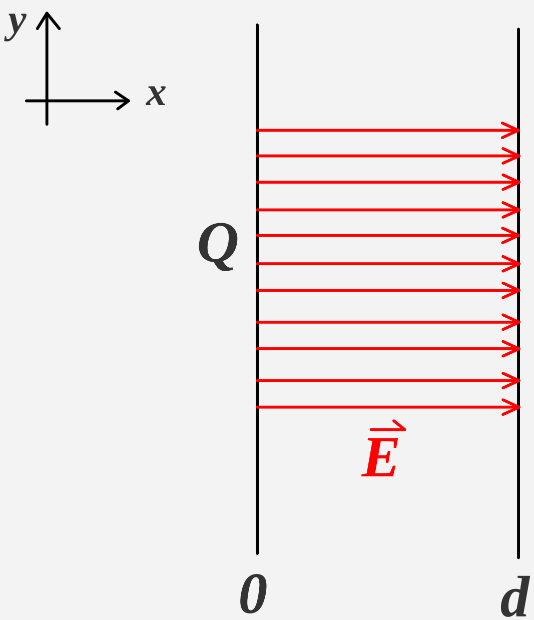

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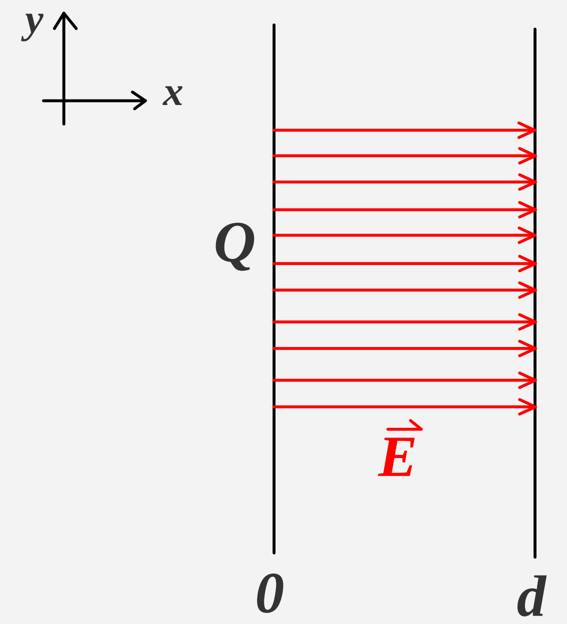

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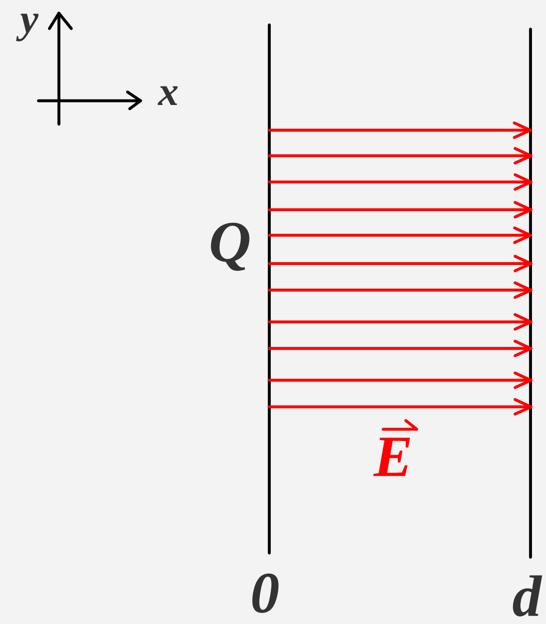

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Calculating Energy from Electric Field II

$$U = \int dU = \int \frac{1}{2}\,\varepsilon_0\,E^2\,dV$$

- Where the integration limits are over region of space where the electric field exists.
- Let's look at an example.

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Example - Energy and uniform E-field

Consider a typical thundercloud that rises to an altitude of 10 km and has a diameter of 20 km. Assuming an average electric field strength of 10^5 V/m, estimate the total electrostatic energy stored in the cloud.

Solution

The energy density is given by:

$$u_E = \frac{1}{2}\varepsilon_0 E^2 = \frac{1}{2}(8.85 \times 10^{-12}\mathsf{C}^2/\mathsf{Nm}^2)(10^5\mathsf{V/m})^2$$
$$= 4.4 \times 10^{-2}\mathsf{J/m}^3$$

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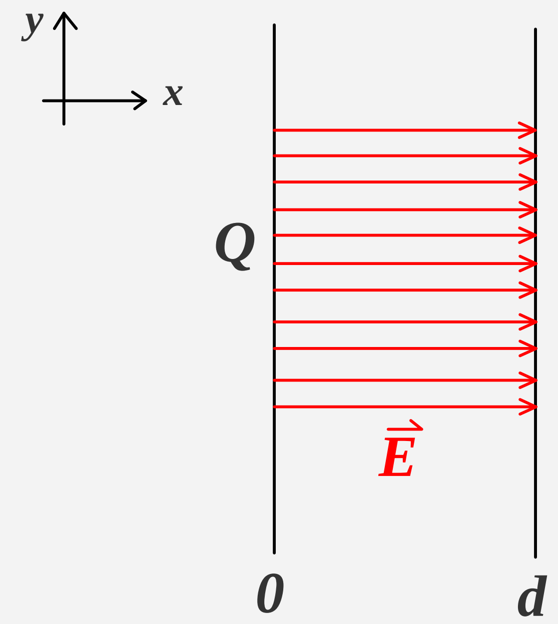

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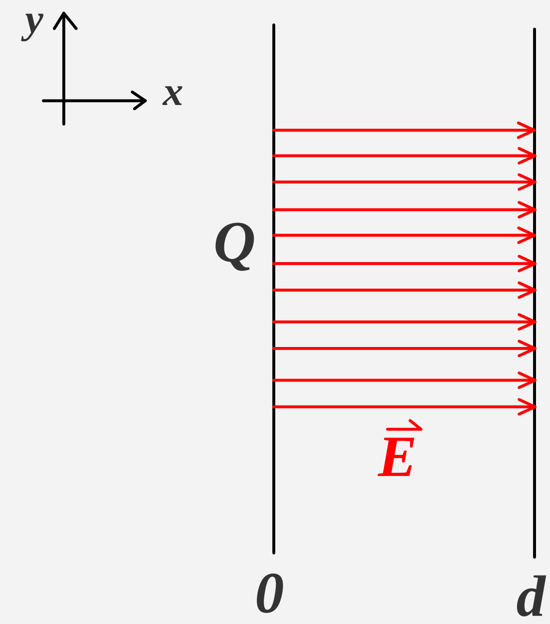

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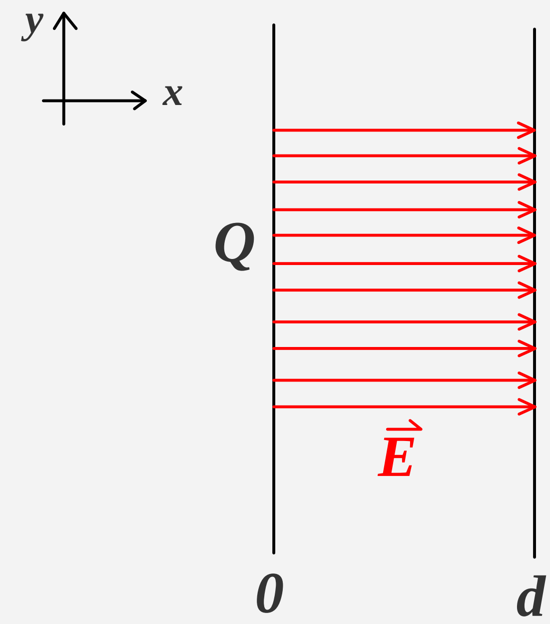

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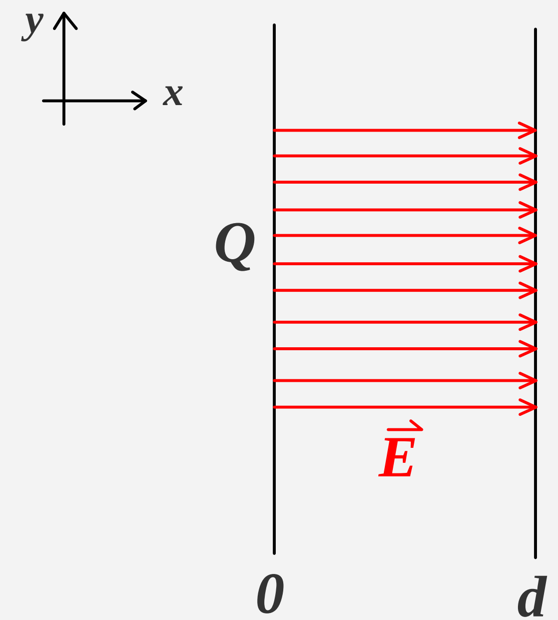

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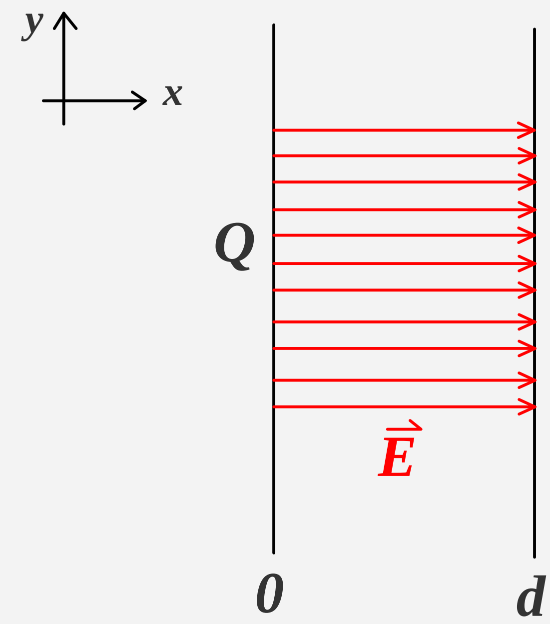

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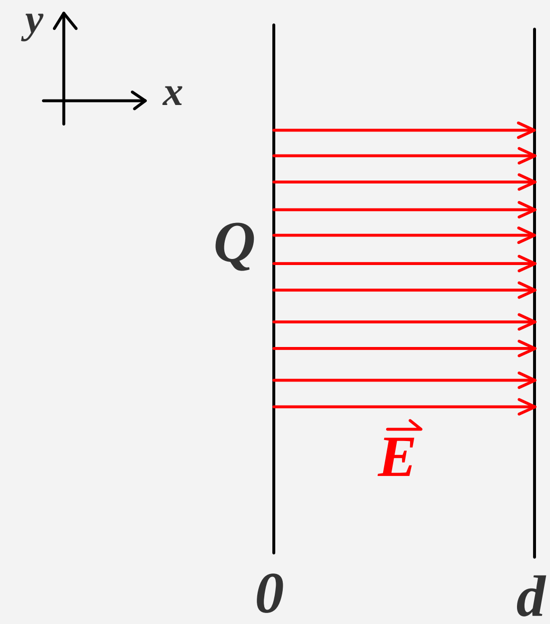

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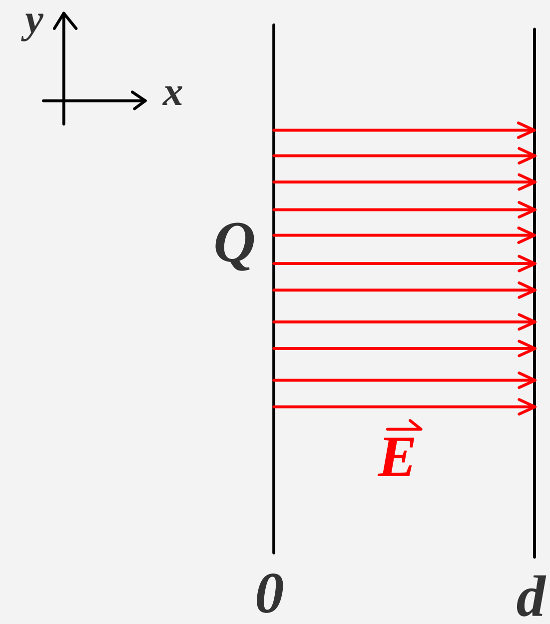

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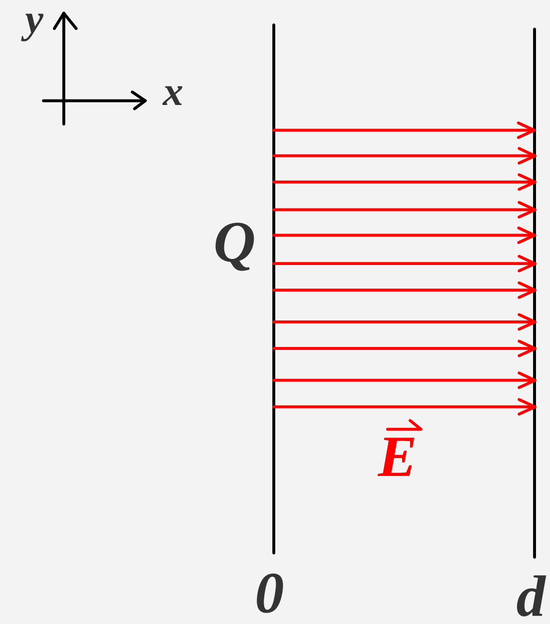

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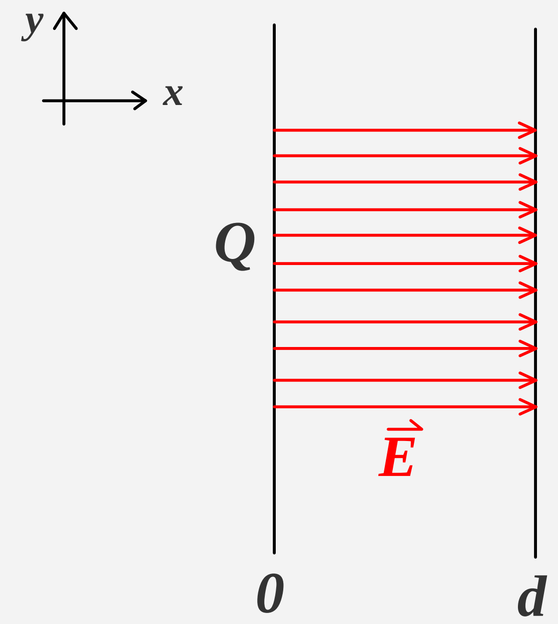

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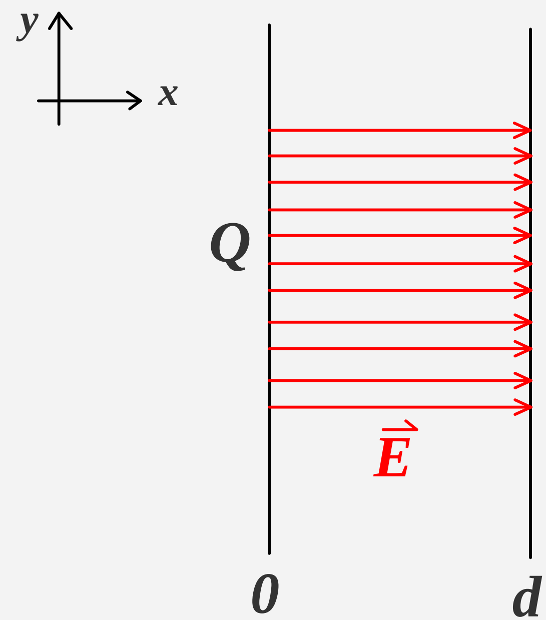

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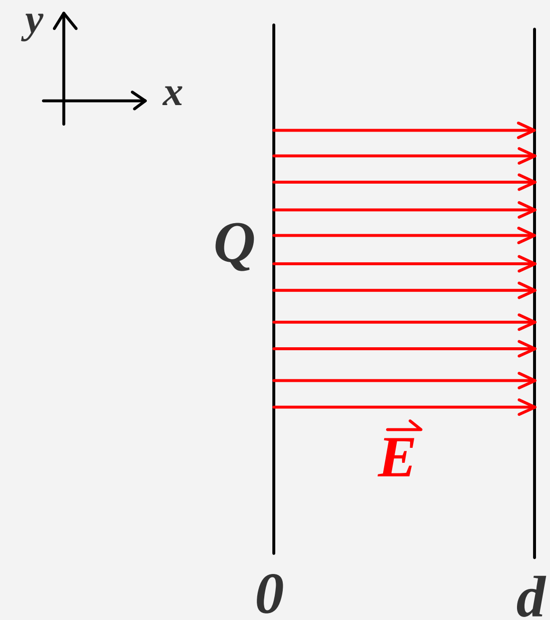

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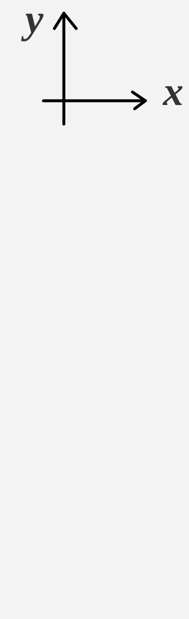

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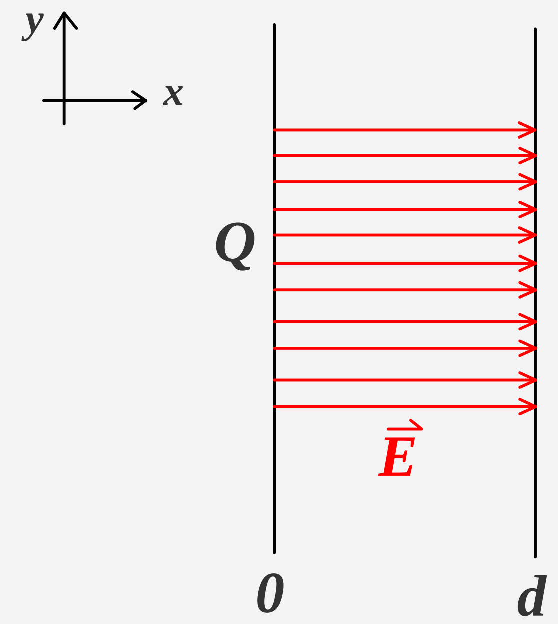

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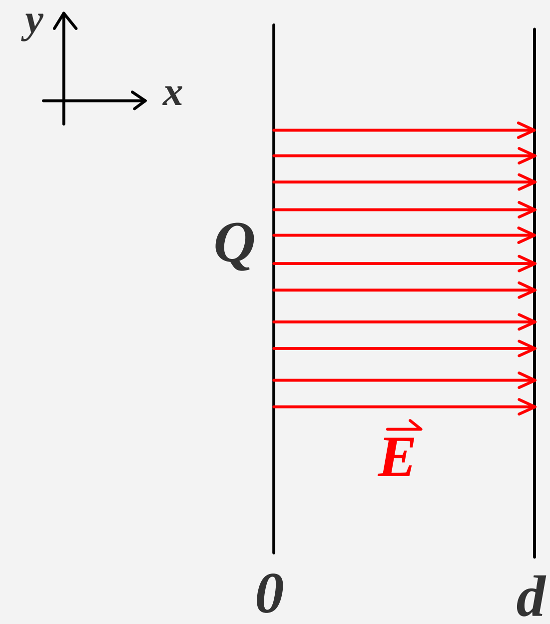

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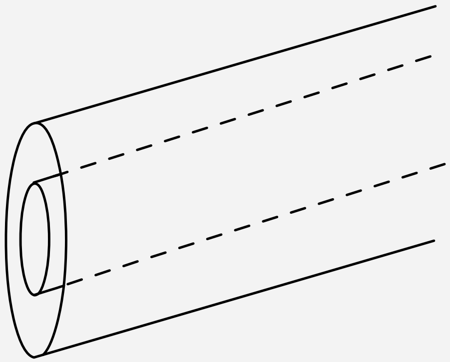

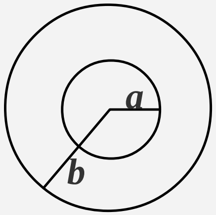

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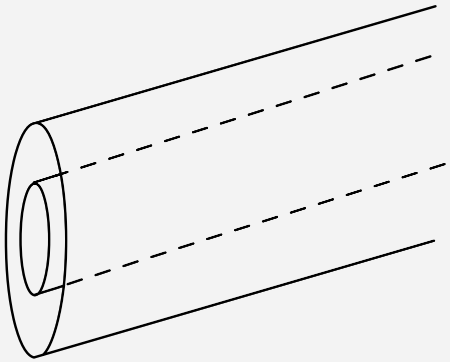

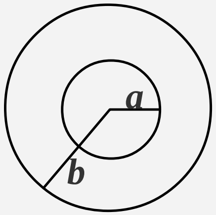

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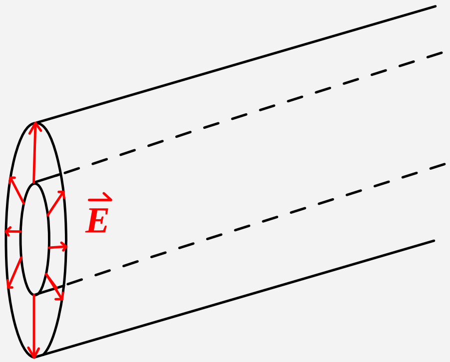

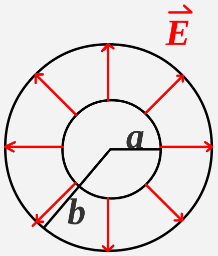

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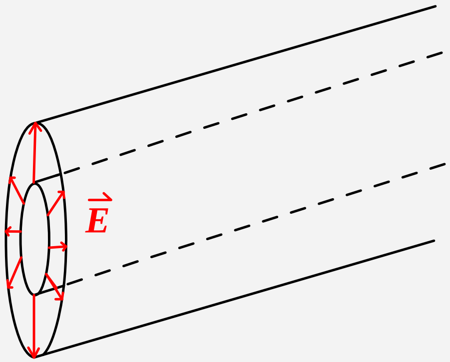

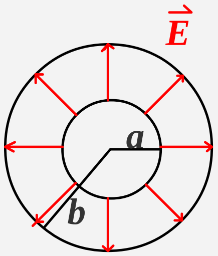

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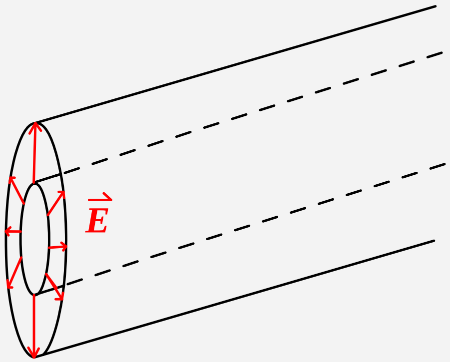

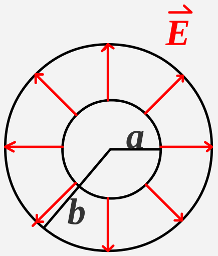

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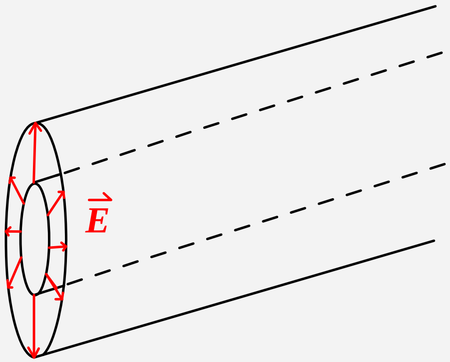

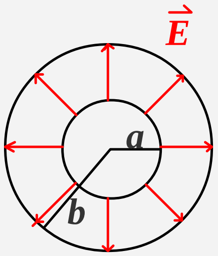

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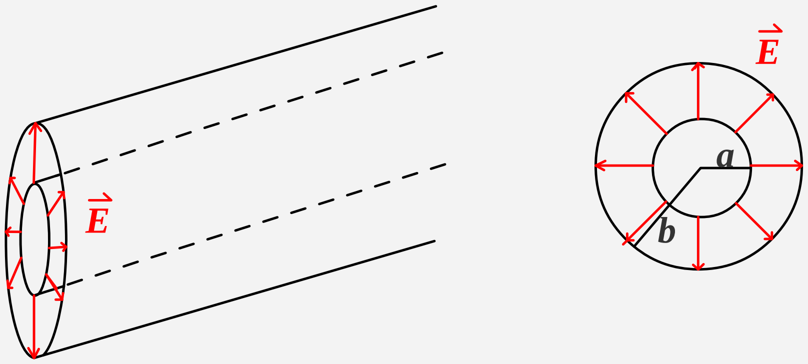

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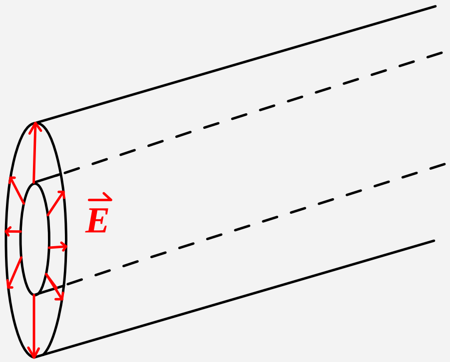

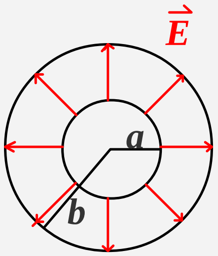

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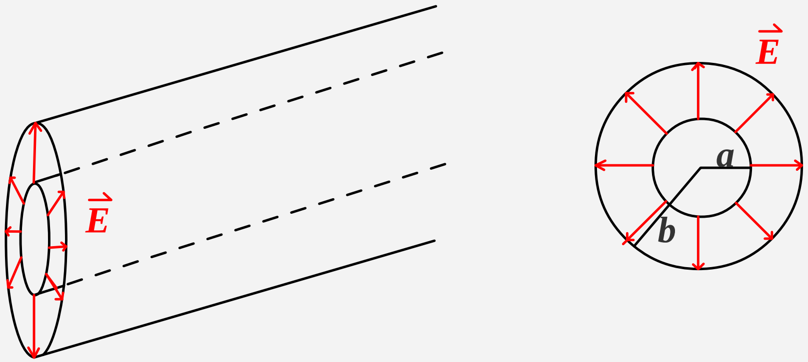

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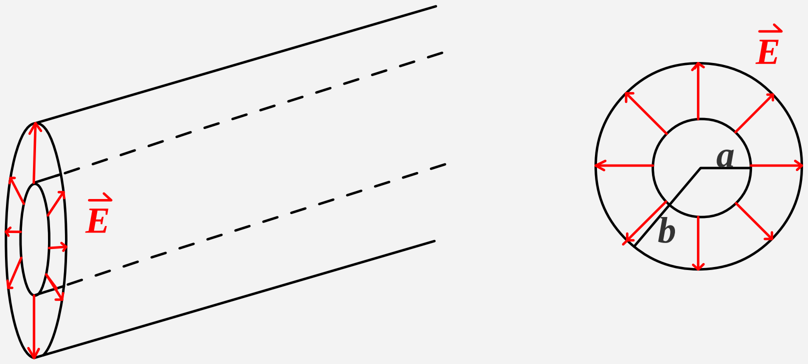

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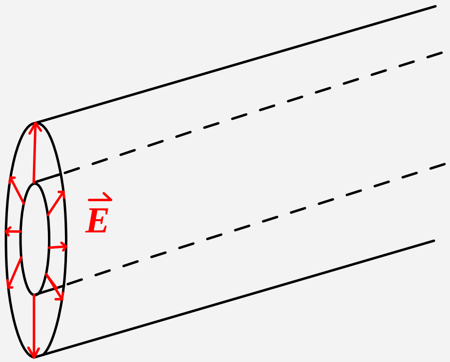

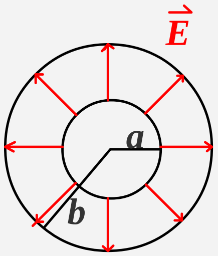

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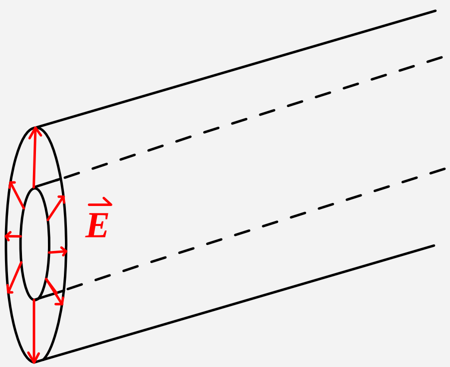

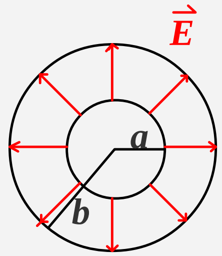

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