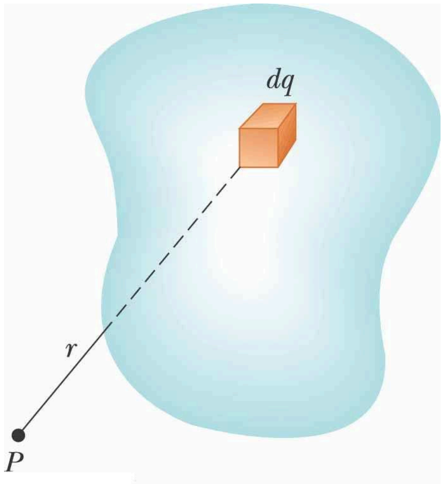
Potentials from Charged Objects

Continuous Charged Objects

- If the electric field is not known (or you are not sure how to find the electric field) for a continuous charged object, then we apply the superposition principle for all the charges making up the object in question.
- Breaking up the charged object into small portions of charge dq:

$$dV = \frac{k\,dq}{r}$$

$$V = \int_{\text{Body}} dV = \int \frac{k\,dq}{r}$$



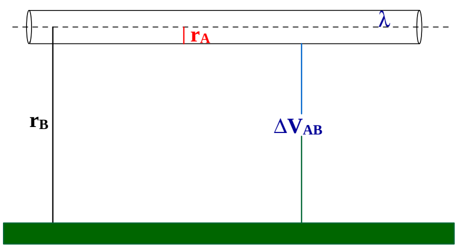
Reminder

- Differences in potential energy - and thus in electric potential - have physical significance.
  - A reference potential is needed!
- Let's work out an example with a few different objects.

Example - Power Line

- A long, straight power line is made from wire with radius $r_A = 1.0$ cm and carries a line charge density $\lambda = 2.6\,\mu C/m$ as shown in the figure on the right. Assuming no other charges are present, what is the potential difference between the surface of the wire and the ground, a distance $r_B = 22$ m below?
  i. Treat the wire as a very long wire, and apply Gauss's law to find the electric field:

$$\mathbf{E} = \frac{\lambda}{2\pi\epsilon_0 r}\hat{\mathbf{r}} \quad (r > r_A)$$



Example - Power Line II

ii. Use the definition of potential difference.

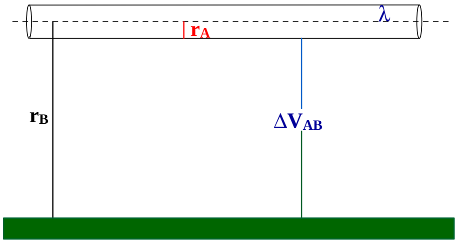
$$\Delta V_{AB} = -\int_{r_A}^{r_B} \vec{E} \cdot d\vec{l}$$

$$\Delta V_{AB} = -\int_{r_A}^{r_B} \frac{\lambda}{2\,\pi\,\varepsilon_0\,r}\hat{\mathbf{r}} \cdot \hat{\mathbf{r}}dr$$

$$\Delta V_{AB} = -\frac{\lambda}{2\,\pi\,\varepsilon_0}\int_{r_A}^{r_B} \frac{dr}{r}$$

$$= \frac{\lambda}{2\,\pi\,\varepsilon_0}\ln\left(\frac{r_A}{r_B}\right)$$

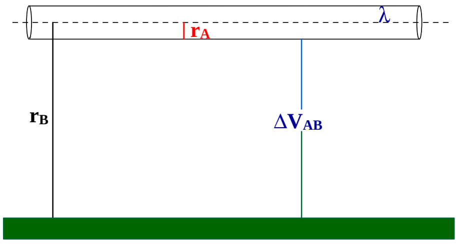
$$\Delta V = -360\text{kV}$$



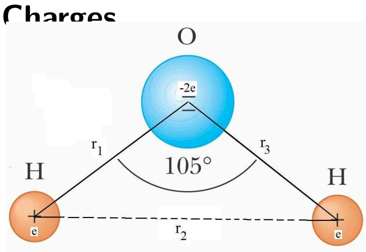
Example - Power Line III

- The potential difference is negative.WHY?
  - Moving a positive charge q from r_A to r_B, would require a force opposite to the displacement.
  - Moving a negative charge $-q$ from r_A to r_B, would require a force in the direction of the displacement.
  - This is all summed up with the equation:

$W = q\,\Delta V$ with $\Delta V = -360$ kV

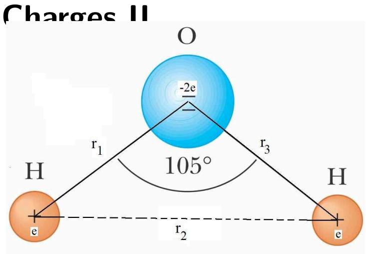


Example - Assembling Charges



- Calculate the amount of work (energy) needed to assemble the three charges above (which is a crude approximation to the water molecule).
- Let's assemble the bottom left charge $(+e)$ first.
  - Since there is no electric field initially, $W_{eL} = 0$.
  - This charge sets up a electric potential, $V_1 = \frac{k\,e}{r}$
- Assemble the $-2\,e$ charge next. This charge interacts with the potential generated by e, V_1.
  - $W_{2e} = -2\,e\,V_1(r_1) = -2\,e\,\frac{k\,e}{r_1} = -\frac{2\,k\,e^2}{r_1}$

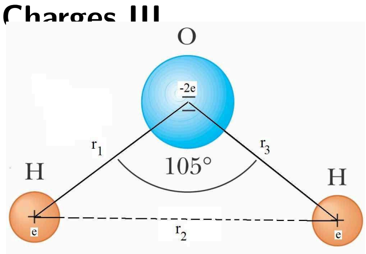
Example - Assembling Charges II



- Assemble the last charge, $+e$ on the right hand side. This charge interacts with the two potentials, V_1 and V_2, set up by the two other charges.
  - $V_1 = \frac{k\,e}{r}$ and $V_2 = -\frac{2\,k\,e}{r}$.
- $W_{eR} = e\,(V_1(r_2) + V_2(r_3)) = \frac{k\,e^2}{r_2} - \frac{2\,k\,e^2}{r_3}$
- The total work required to assemble the charges in the configuration above is the sum of the above energies:

$$W_{\text{Total}} = W_{eL} + W_{2e} + W_{eR} = 0 - \frac{2\,k\,e^2}{r_1} + \frac{k\,e^2}{r_2} - \frac{2\,k\,e^2}{r_3}$$

Example - Assembling Charges III



- Sum all the energies to find the total work required to assemble the charges.

$$W_{\text{Total}} = W_{eL} + W_{2e} + W_{eR} = 0 - \frac{2\,k\,e^2}{r_1} + \frac{k\,e^2}{r_2} - \frac{2\,k\,e^2}{r_3}$$

$$W_{\text{Total}} = k\,e^2\left(-\frac{2}{r_1} + \frac{1}{r_2} - \frac{2}{r_3}\right) \quad \text{with } r_1 = r_3 = 0.1 \text{ nm}$$

$$W_{\text{Total}} = -7.76 \times 10^{-18} \text{ J} = -48.5 \text{ eV}$$

Units

- On the last slide, we calculated a very small amount of energy in Joules. A more convenient unit of energy when dealing with atoms or molecules is the electron-Volt (eV) which is the amount of energy gained by a charge (e) passing through a potential difference of 1 V:

$$1 \text{ eV} = 1.6 \times 10^{-19} \text{ J}$$

Potential Difference and $\vec{E}$-field

- Since the electric field and the potential difference are related by:

$$dV = -\vec{E} \cdot d\vec{l} = -E_l \, dl$$

$$E_{l\textcolor{red}{l}} = -\frac{dV}{dl\textcolor{red}{l}}$$ (where E_l is the component of the electric field parallel to l)