



# Continuous Charged Objects

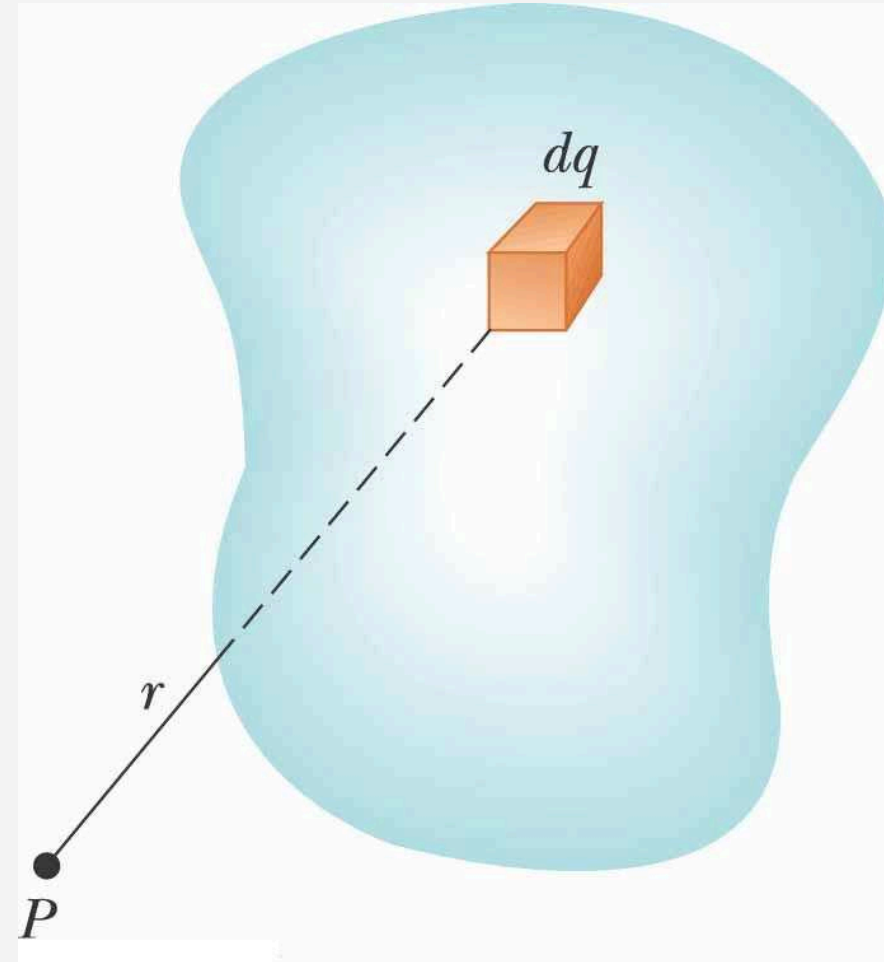
Potentials from Charged Objects Potential Difference and the Electric Field

- If the electric field is not known (or you are not sure how to find the electric field) for a continuous charged object, then we apply the superposition principle for all the charges making up the object in question.

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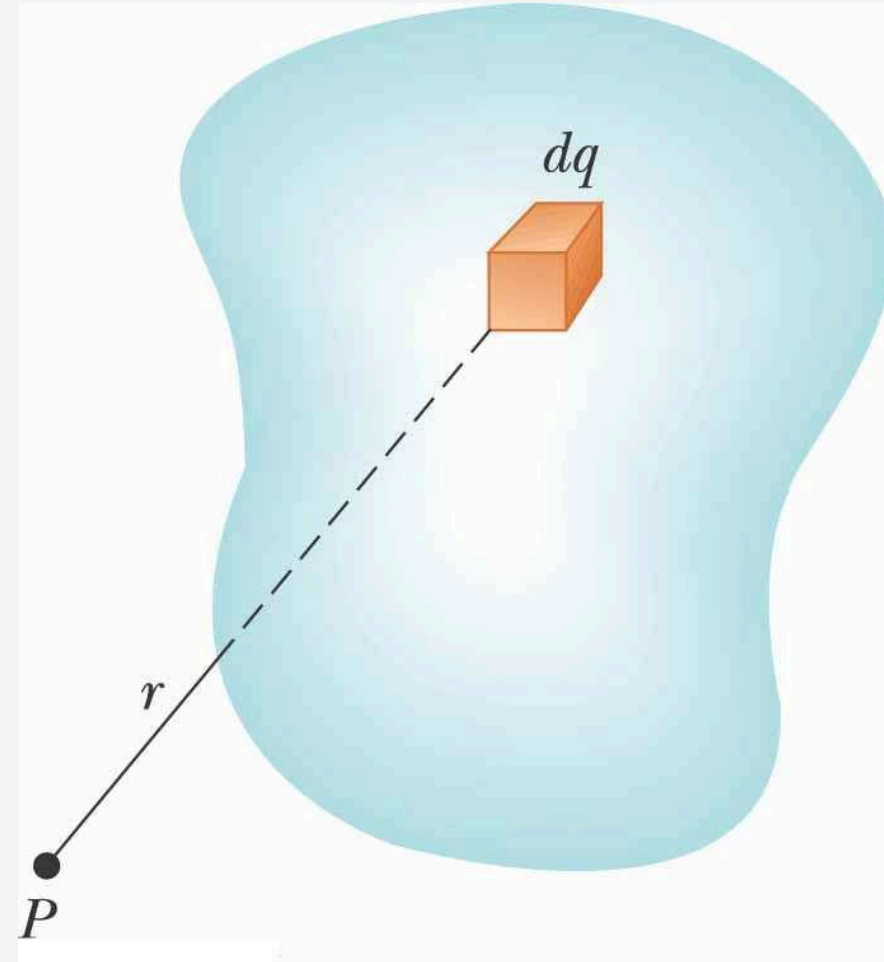
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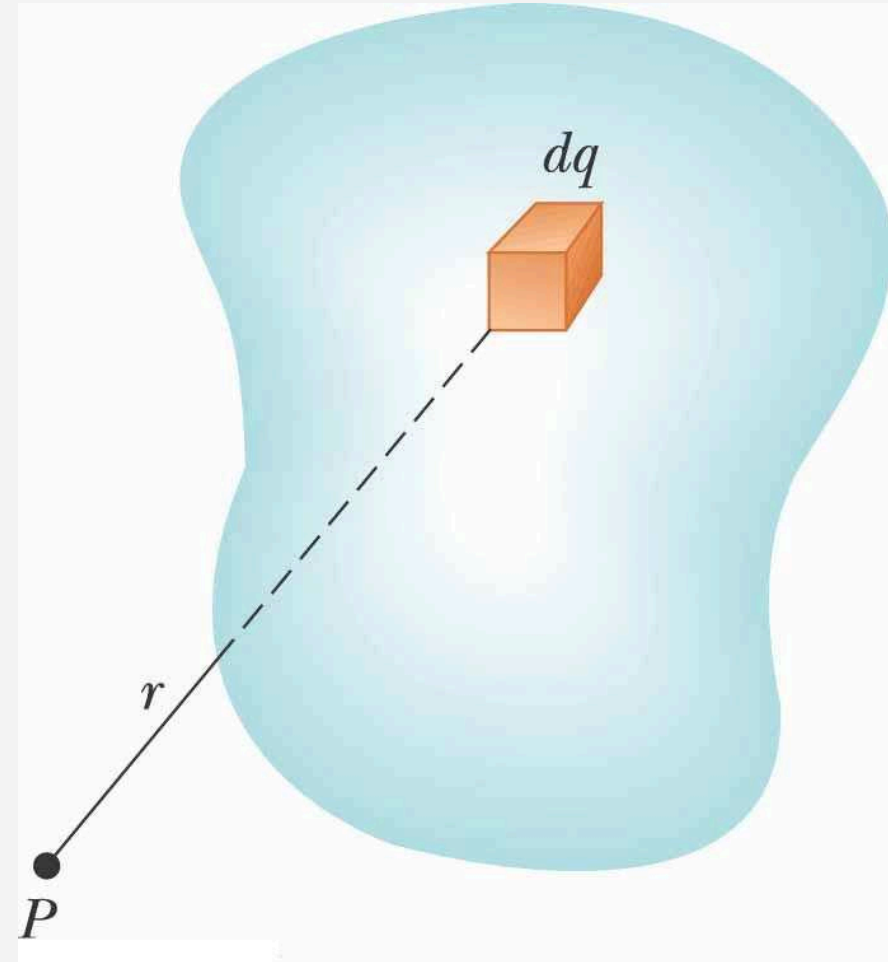


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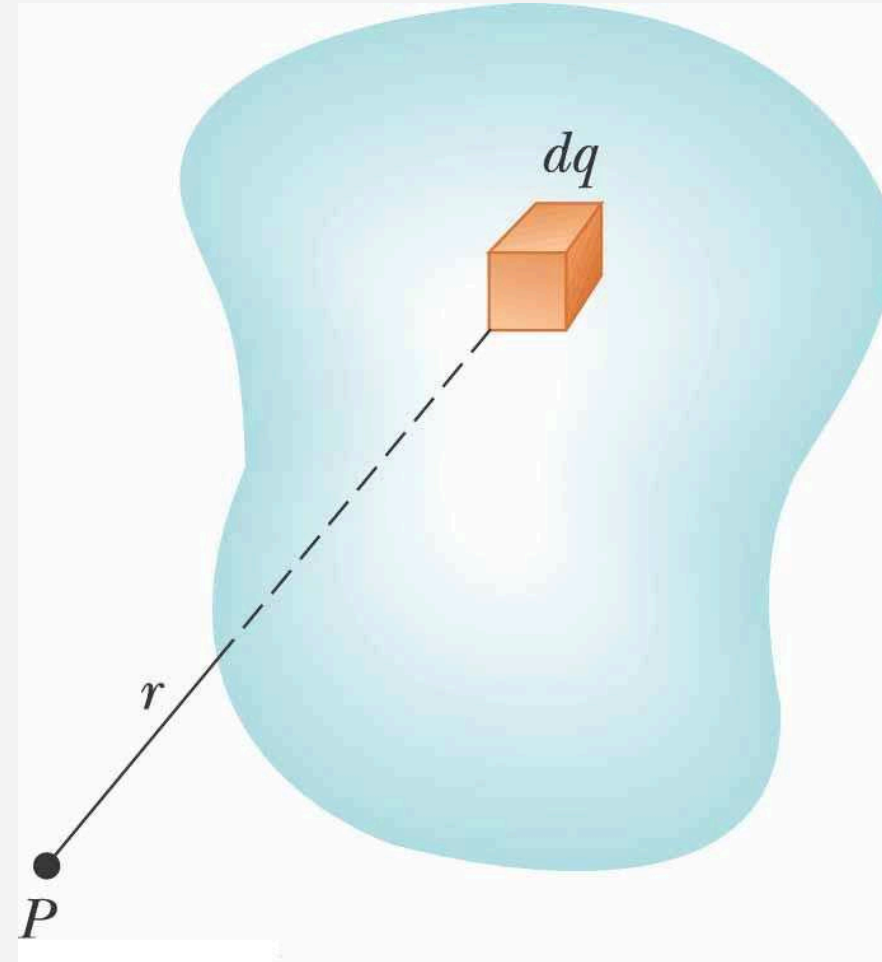
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$$V = \int_{\text{Body}} dV = \int \frac{k dq}{r}$$



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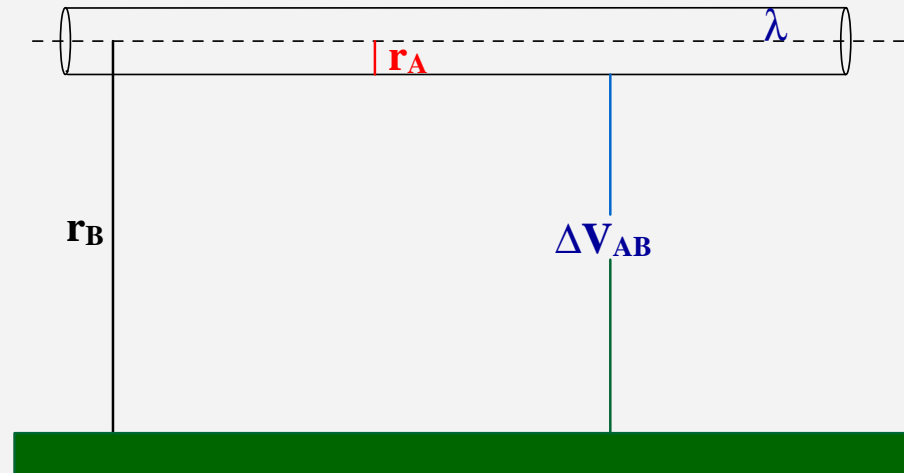
Potentials from Charged Objects Potential Difference and the Electric Field

- Differences in potential energy - and thus in electric potential - have physical significance.
  - ◆ A reference potential is needed!
- Let's work out an example with a few different objects.

# Example - Power Line

Potentials from Charged Objects Potential Difference and the Electric Field

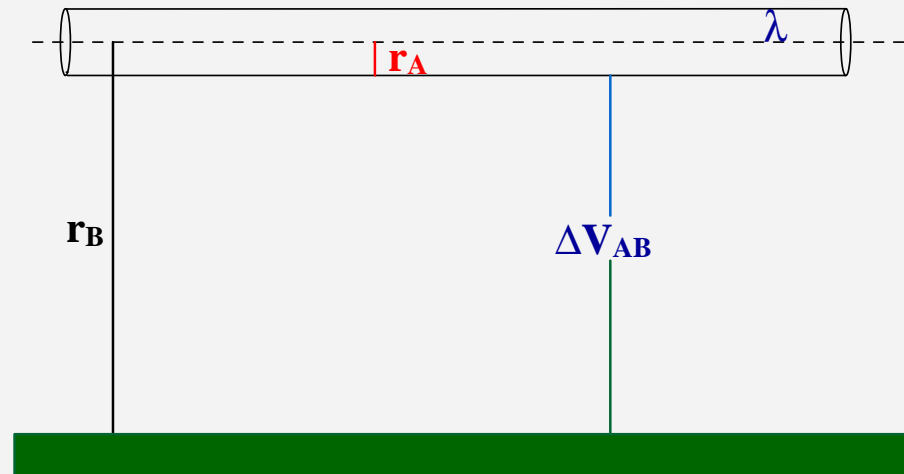
- A long, straight power line is made from wire with radius  $r_A = 1.0$  cm and carries a line charge density  $\lambda = 2.6 \mu\text{C}/\text{m}$  as shown in the figure on the right. Assuming no other charges are present, what is the potential difference between the surface of the wire and the ground, a distance  $r_B = 22$  m below?



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  - Treat the wire as a very long wire, and apply Gauss's law to find the electric field:



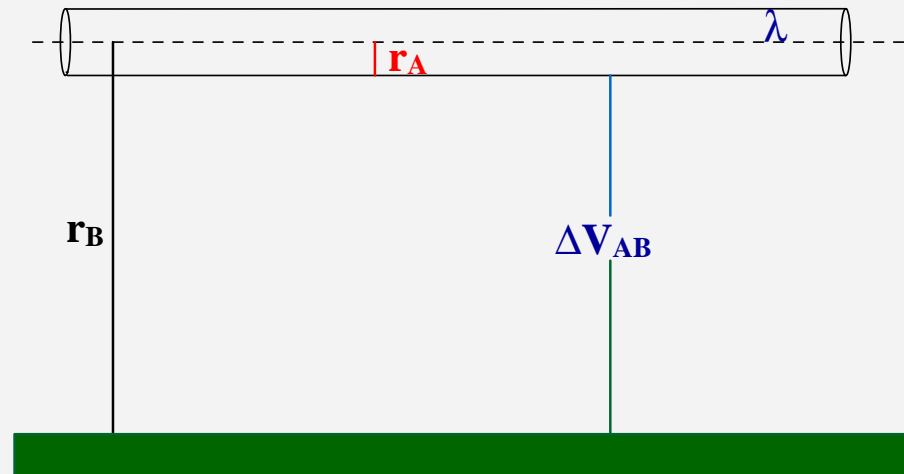
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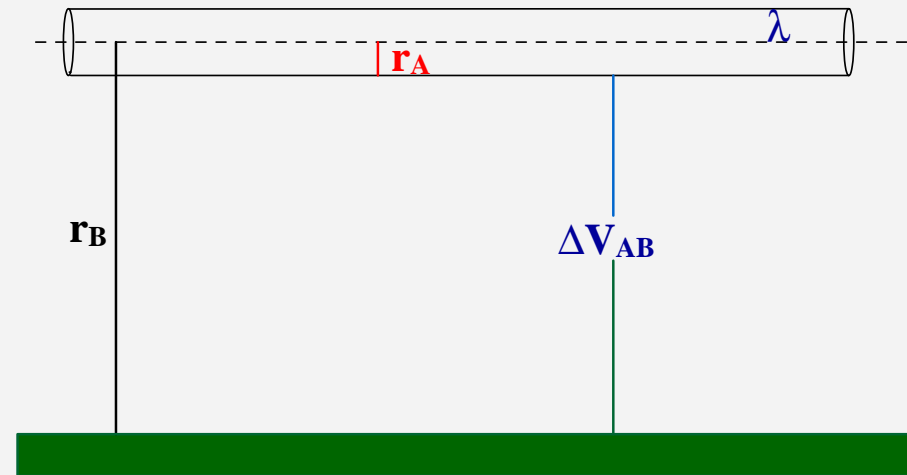
$$\mathbf{E} = \frac{\lambda}{2\pi\epsilon_0 r} \hat{\mathbf{r}} \quad (r > r_A)$$



# Example - Power Line II

Potentials from Charged Objects Potential Difference and the Electric Field

- ii. Use the definition of potential difference.

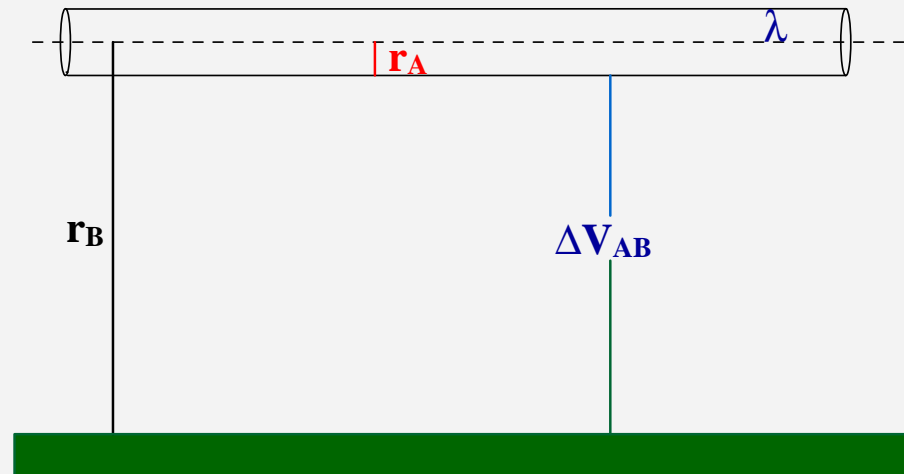


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$$\Delta V_{AB} = - \int_{r_A}^{r_B} \vec{E} \cdot d\vec{l}$$

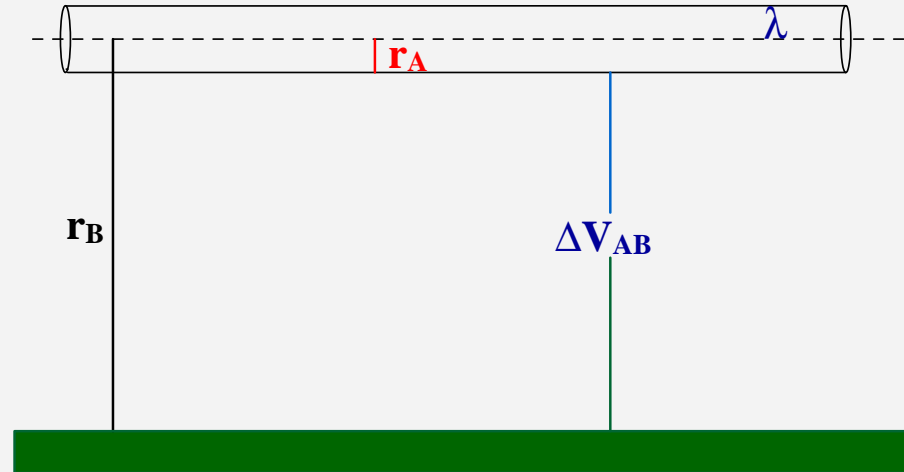


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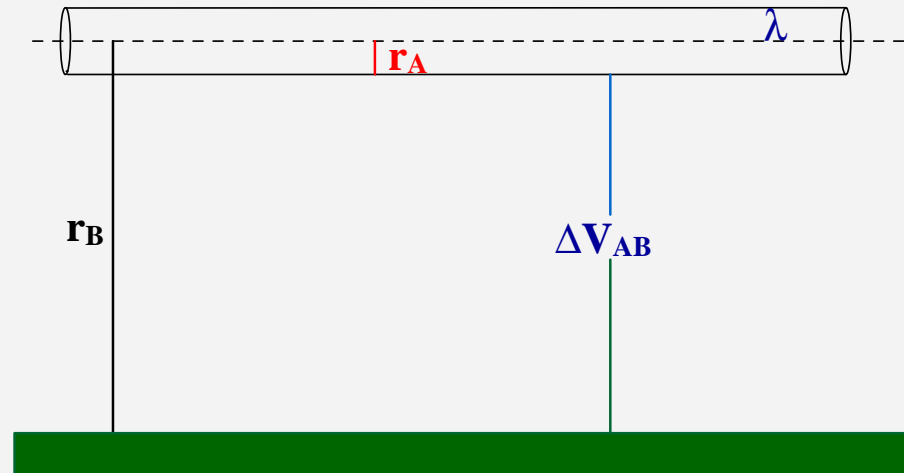
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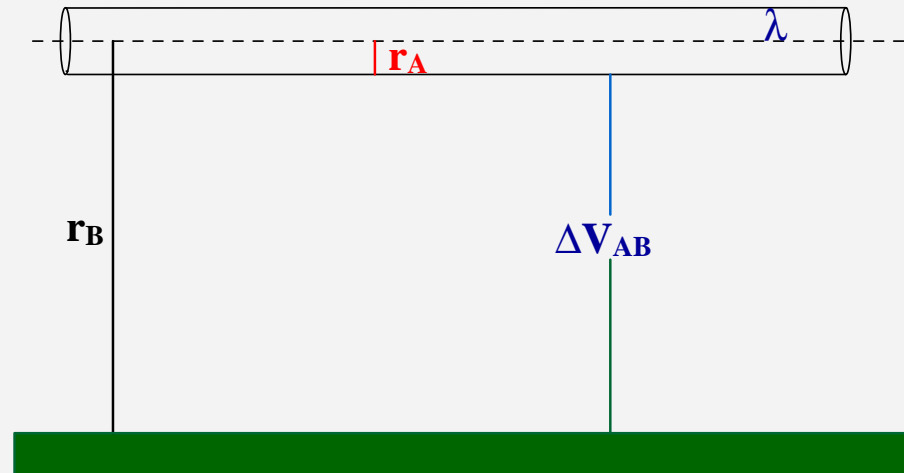


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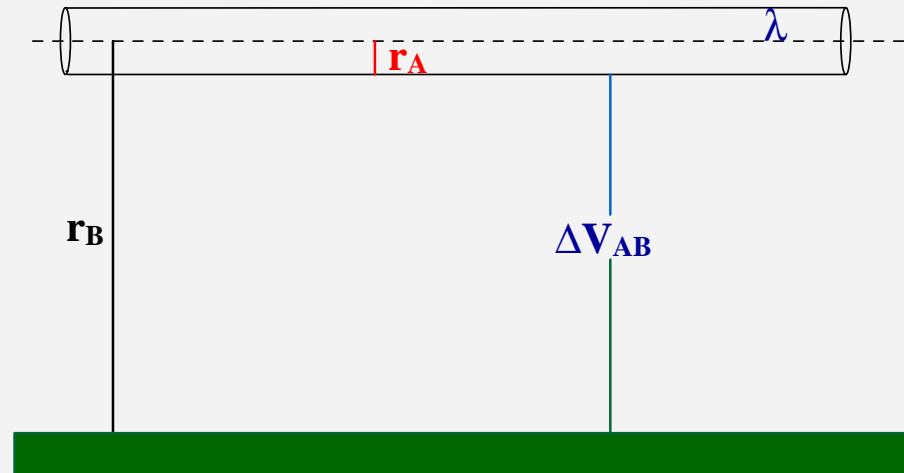


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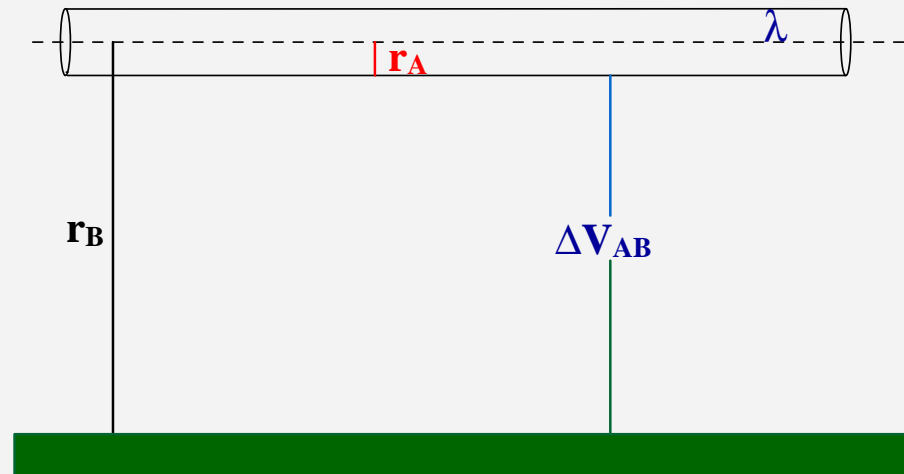
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# Example - Power Line III

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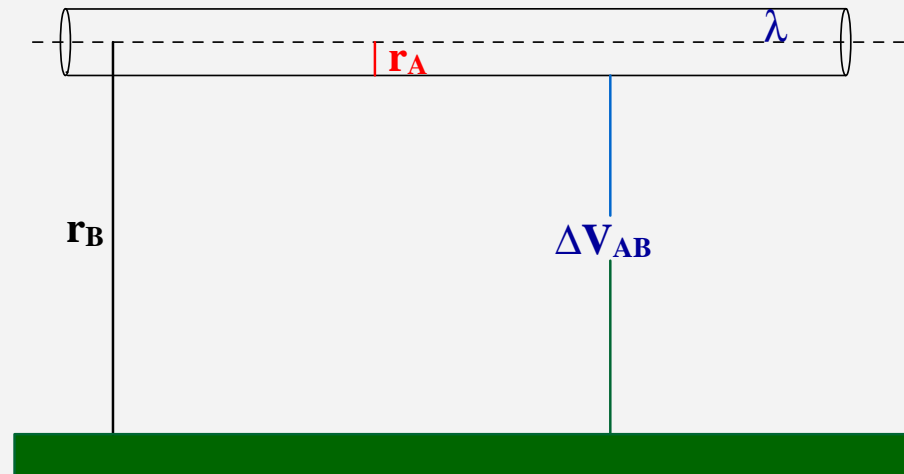
- The potential difference is negative.



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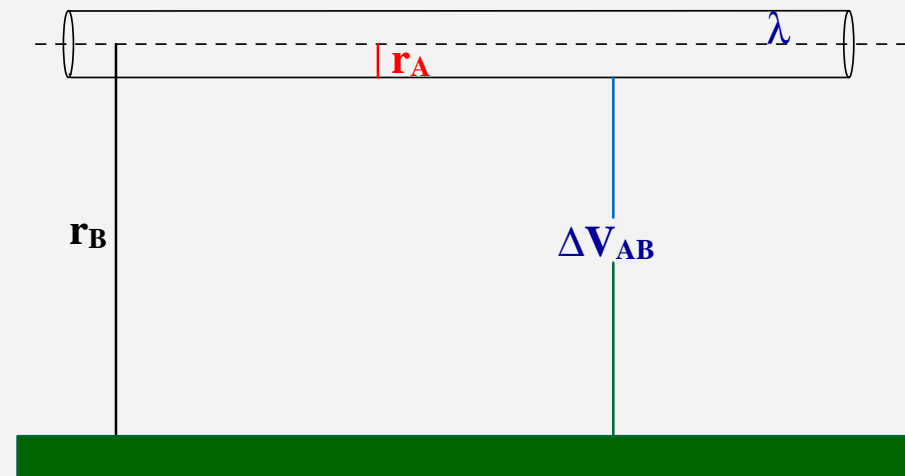
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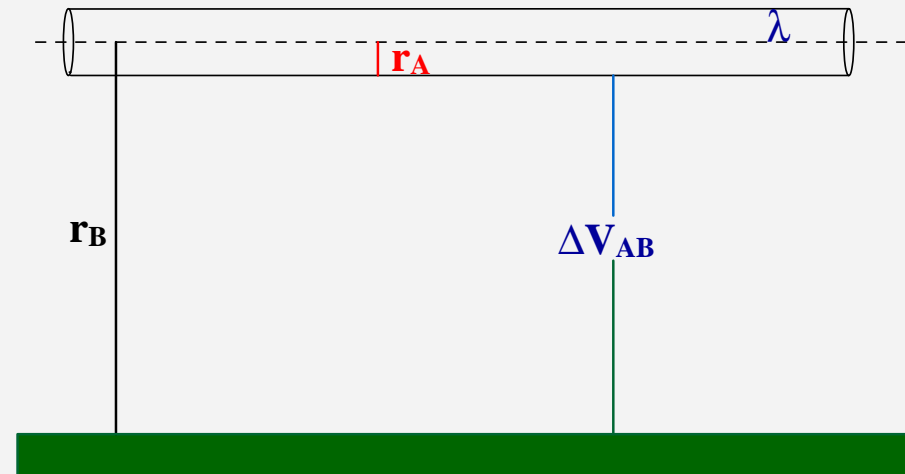
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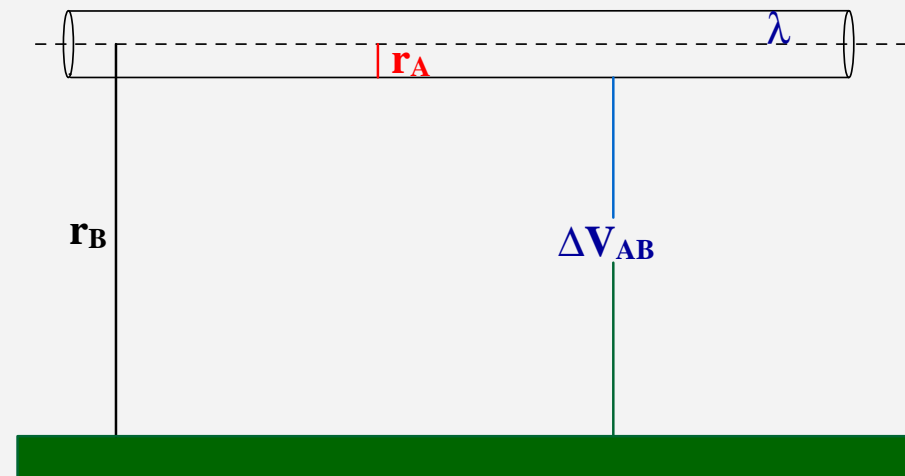
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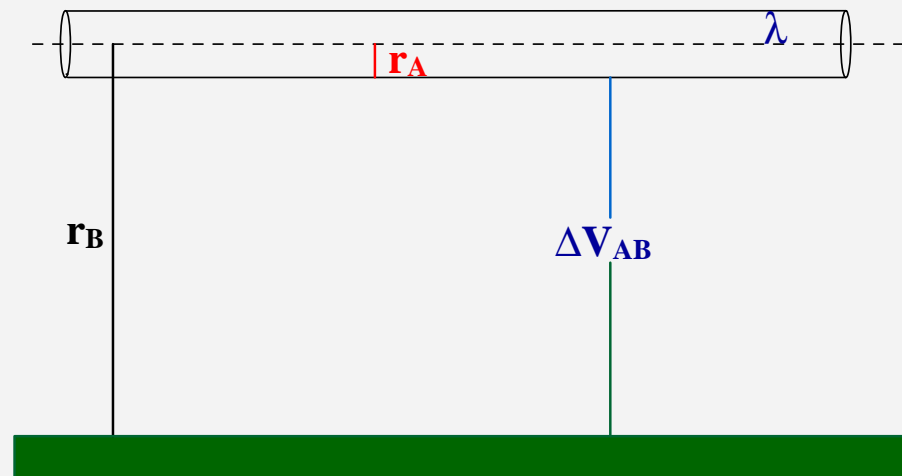


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$$W = q \Delta V \quad \text{with } \Delta V = -360 \text{ kV}$$





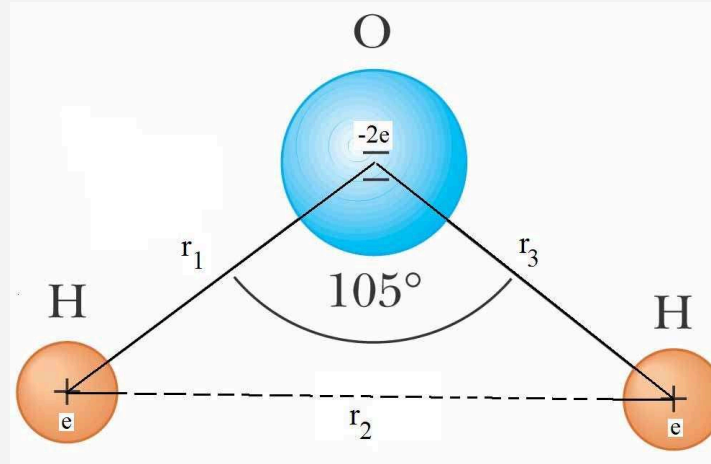
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- Calculate the amount of work (energy) needed to assemble the three charges above (which is a crude approximation to the water molecule).

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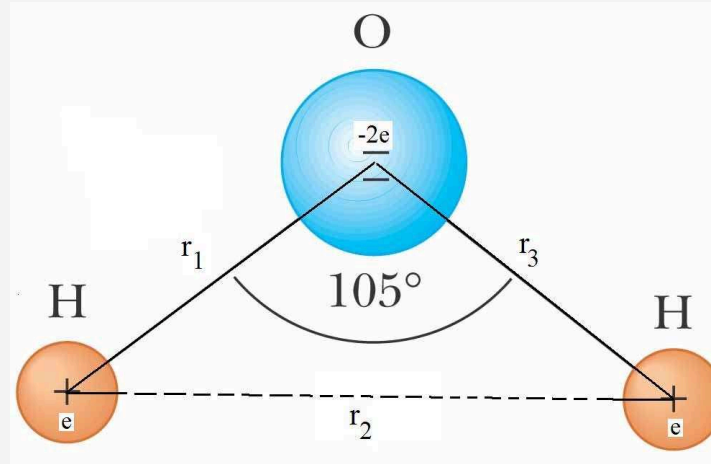
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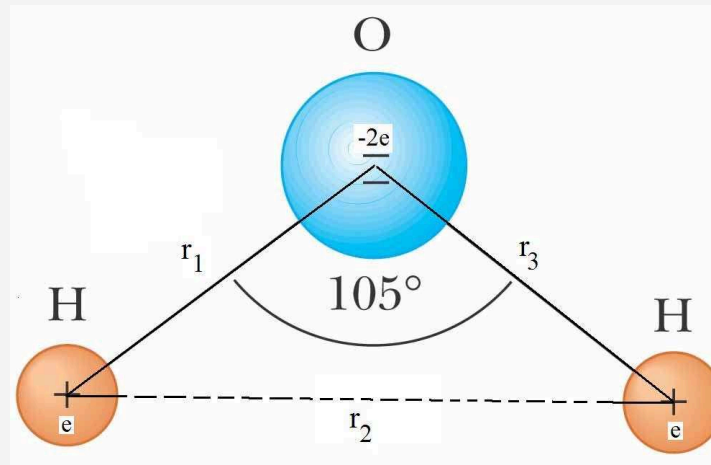
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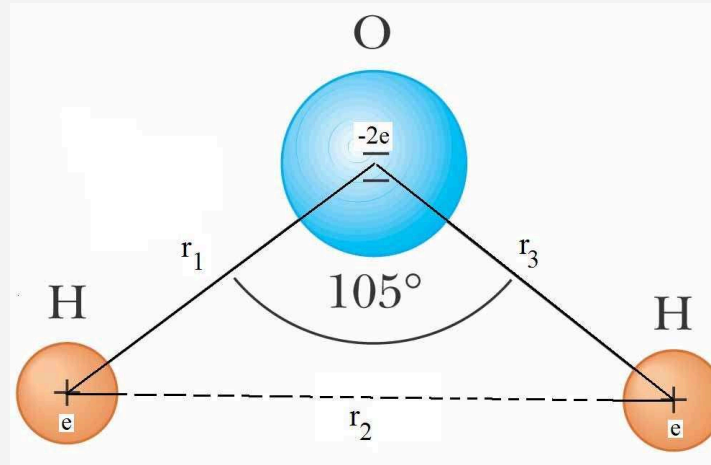
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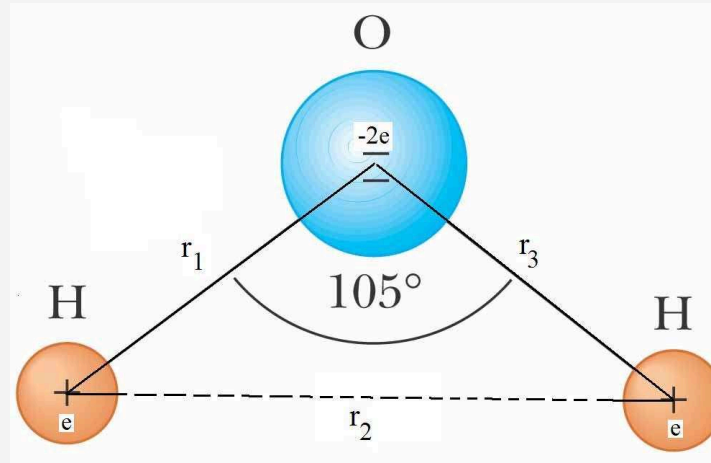
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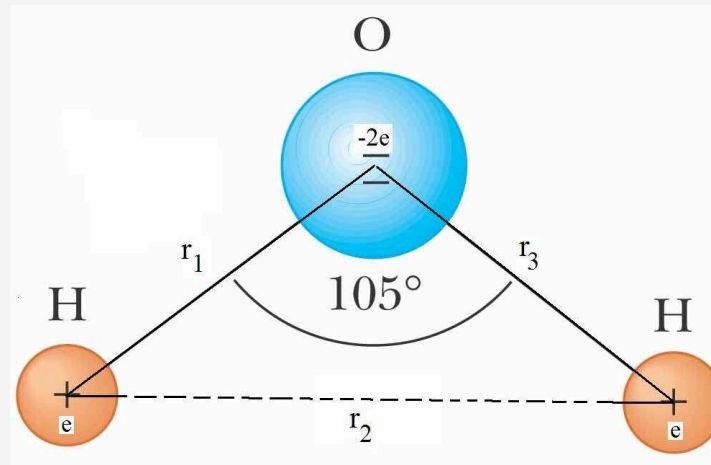
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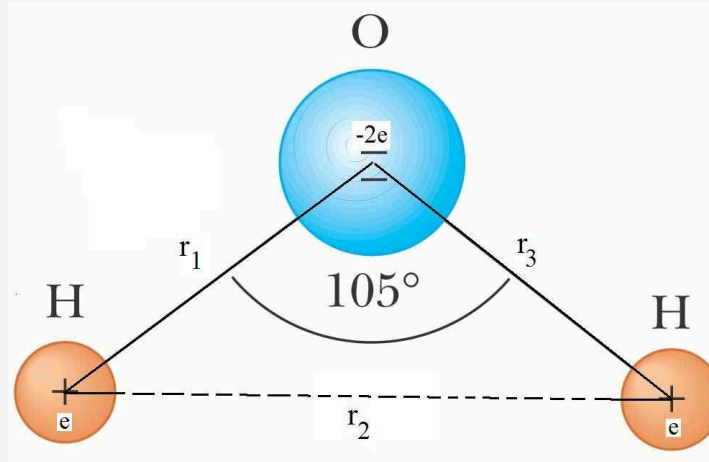
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# Example - Assembling Charges II

Potentials from Charged Objects Potential Difference and the Electric Field

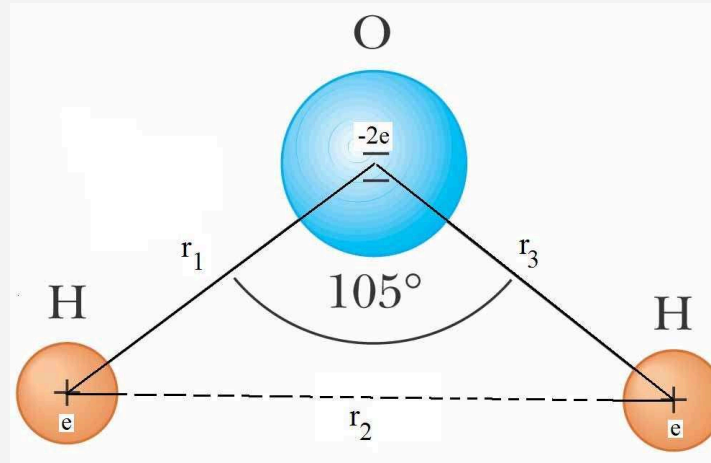


- Assemble the last charge,  $+e$  on the right hand side. This charge interacts with the two potentials,  $V_1$  and  $V_2$ , set up by the two other charges.



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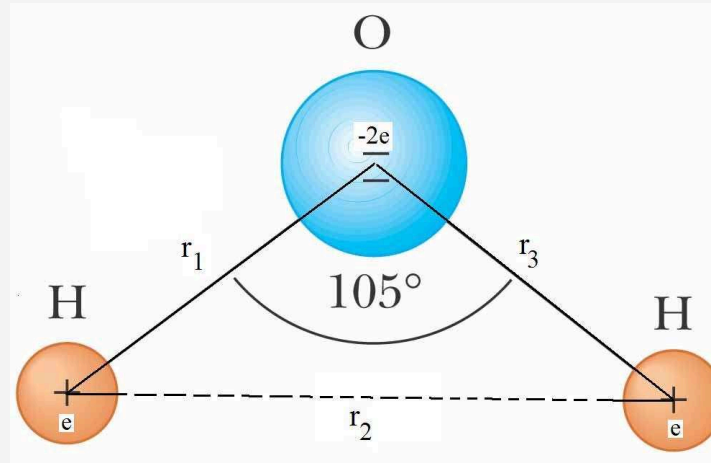
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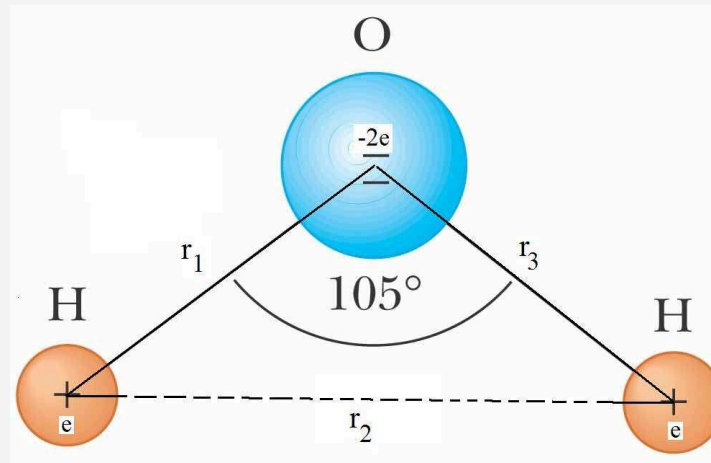
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  - ◆  $V_1 = \frac{ke}{r}$  and  $V_2 = -\frac{2ke}{r}$ .
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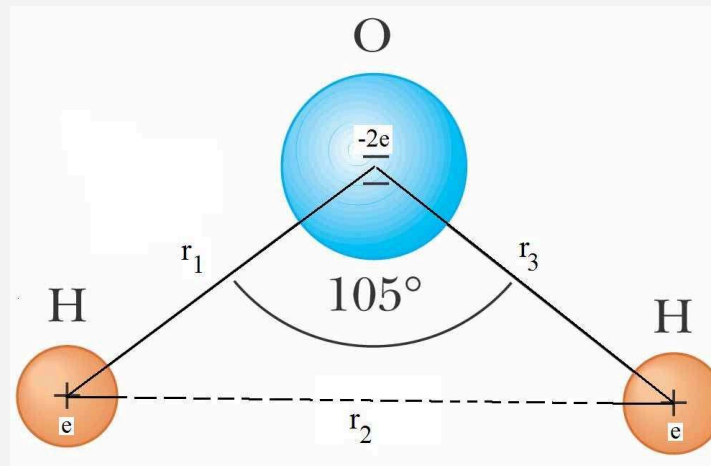
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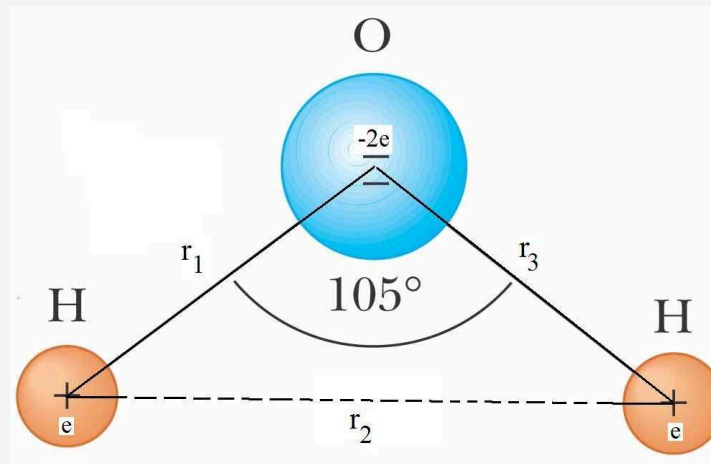
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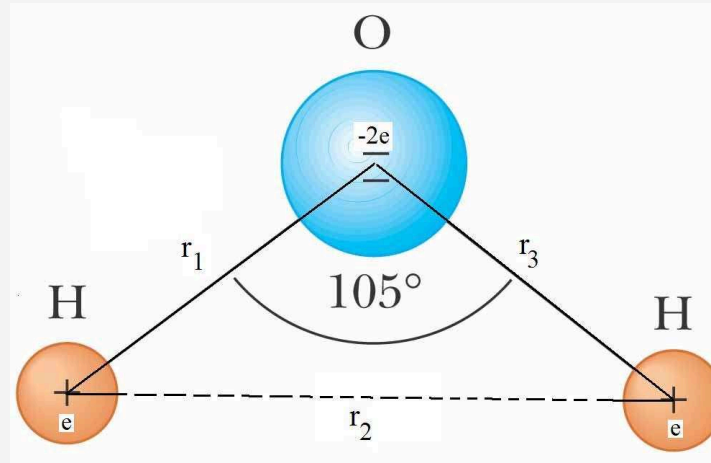
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$$W_{\text{Total}} = W_{eL} + W_{2e} + W_{eR} = 0 - \frac{2k e^2}{r_1} + \frac{k e^2}{r_2} - \frac{2k e^2}{r_3}$$

# Example - Assembling Charges III

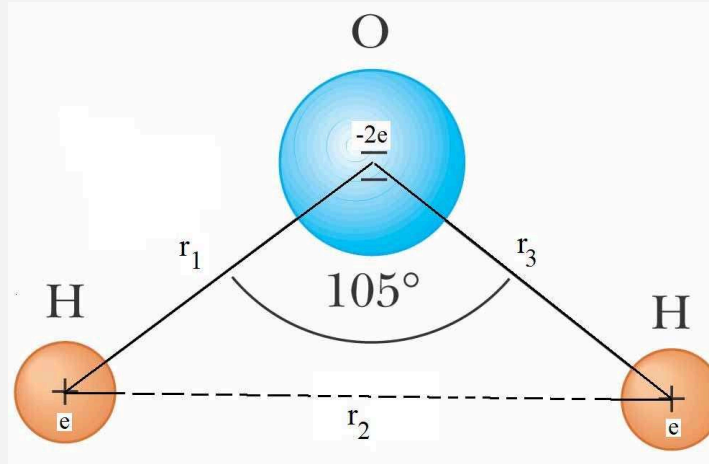
Potentials from Charged Objects Potential Difference and the Electric Field



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# Example - Assembling Charges III

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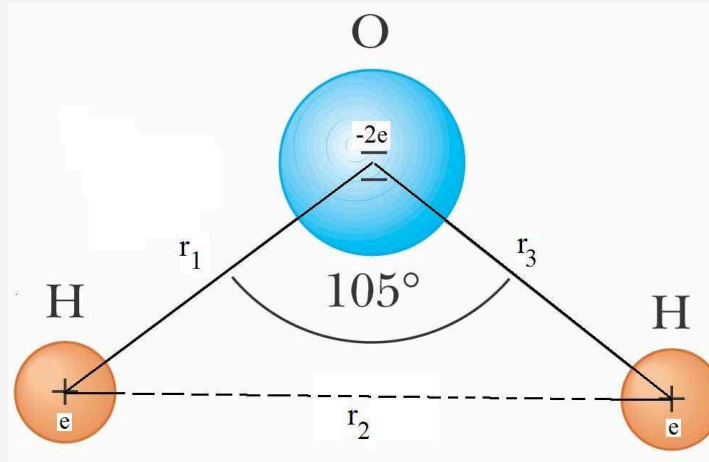


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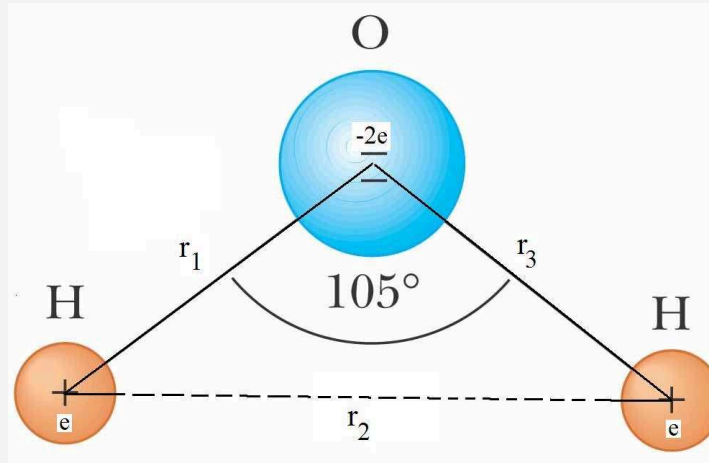
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$$W_{\text{Total}} = ke^2 \left( -\frac{2}{r_1} + \frac{1}{r_2} - \frac{2}{r_3} \right) \quad \text{with } r_1 = r_3 = 0.1 \text{ nm}$$



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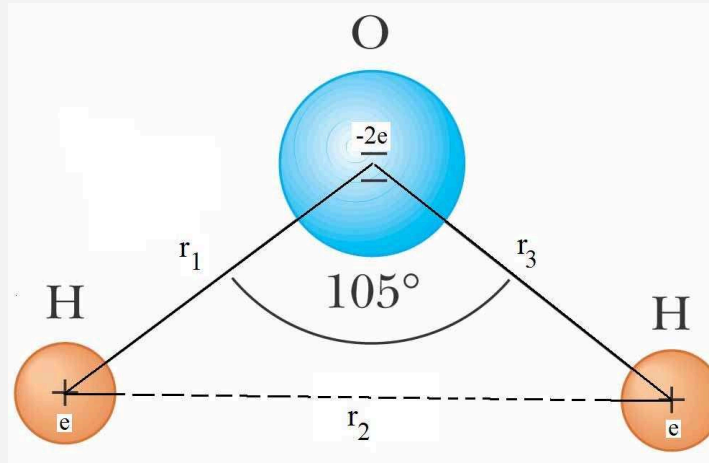
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$$W_{\text{Total}} = -7.76 \times 10^{-18} \text{ J} = -48.5 \text{ eV}$$

# Units

Potentials from Charged Objects Potential Difference and the Electric Field

- On the last slide, we calculated a very small amount of energy in Joules. A more convenient unit of energy when dealing with atoms or molecules is the electron-Volt (eV) which is the amount of energy gained by a charge ( $e$ ) passing through a potential difference of 1 V:

# Units

Potentials from Charged Objects Potential Difference and the Electric Field

- On the last slide, we calculated a very small amount of energy in Joules. A more convenient unit of energy when dealing with atoms or molecules is the electron-Volt (eV) which is the amount of energy gained by a charge ( $e$ ) passing through a potential difference of 1 V:

$$1 \text{ eV} = 1.6 \times 10^{-19} \text{ J}$$

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$$E_l = -\frac{dV}{dl} \quad (\text{where } E_l \text{ is the component of the electric field parallel to } l)$$