1) (30 pts)

(A) = 11 pts

- +1 pt $x_{Bomb}(t)$
- +1 pt $x_{shell}(t)$
- +1 pt $y_{Bomb}(t)$
- +1 pt $y_{shell}(t)$
- +2 pt $x_B(t_1) = x_s(t_1)$
- +2 pt $y_B(t_1) = y_s(t_1)$
- +1 pt $\boxed{v_s \approx 416 \text{ m/s}}$
- +1 pt $\boxed{t_1 = 0.626 \text{ s}}$
- +1 pt $\boxed{\Delta y_{hit} \approx 225 \text{ m}}$

(B) = 14 pts

- +4 pts x vs. t curve
  → +2 pts each bomb + shell (straight line w/same slope)
- +4 pts y vs. t
  → 2 pts each bomb + shell
- +2 pts v_x vs. t
  → +1 pt each bomb + shell (flat horizontal line (+))
- +2 pts v_y vs. t
  → +1 pt each bomb + shell
- +1 pt a_x vs. t
  1/2 pt for each bomb + shell (zero)
- +1 pt a_y vs. t
  1/2 pt for each bomb + shell $(-|g|)$

(C) = 5 pts

- +3 pts correct vector addition
- +2 pts $\vec{v}_{SB} = 480$ m/s ↑ (+1 mag. / +1 direction)

1) A.)

Bomb's x & y positions

$$x_f = 240\,m/s\,(\cos 30°)\,t$$
$$y_f = 300\,m - 240\,m/s\,(\sin 30°)\,t - \tfrac{1}{2}gt^2$$

Shell's x & y positions

$$x_f = v_s \cos 60°\, t$$
$$y_f = v_s \sin 60°\, t - \tfrac{1}{2}gt^2$$

At $t = t_1$ the shell "hits" the bomb:

$$\star \quad \underbrace{x_{f_B}(t_1)}_{\text{Bomb}} = \underbrace{x_{f_s}(t_1)}_{\text{Shell}}$$

$$\Rightarrow \quad 240\tfrac{m}{s}(\cos 30°)t_1 = v_s(\cos 60°)t_1$$

$$\Rightarrow \left(\frac{\sqrt{3}}{2}\right) 240\,m/s = v_s\left(\tfrac{1}{2}\right)$$

$$\Rightarrow \boxed{v_s = \sqrt{3}\,(240\,m/s) \simeq 415.7\ m/s}$$

$$\star \quad y_{f_B}(t_1) = y_{f_s}(t_1)$$

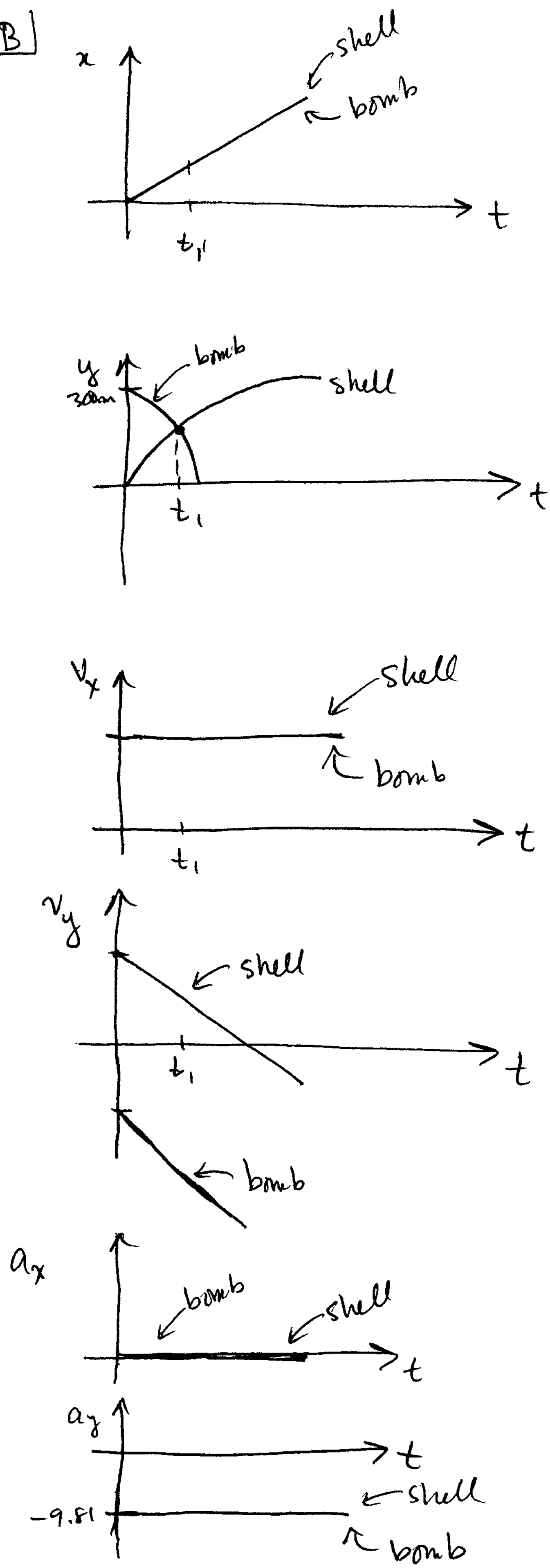
$$300m - 240\sin 30°\, t_1 - \cancel{\tfrac{1}{2}gt_1^2} = v_s \sin 60\, t_1 - \cancel{\tfrac{1}{2}gt_1^2}$$

$$300m = \left(120 + \tfrac{\sqrt{3}}{2}\,415.7\right)t_1 \Rightarrow \boxed{t_1 = 0.626\ s}$$

$$\Rightarrow \boxed{y_{f_s}(t_1) = 300m - 240\sin 30°(0.626s) - \tfrac{1}{2}g(0.626s)^2 = 224.9\ m}$$

1)

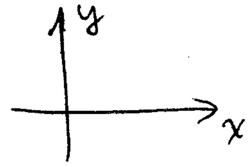
B

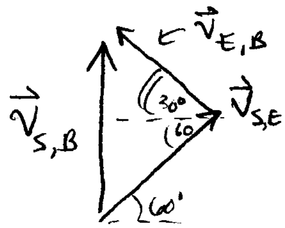


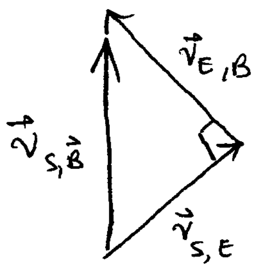
x
shell
bomb
t
t_1
y
300m
bomb
shell
t
t_1
v_x
shell
bomb
t
t_1
v_y
shell
t
t_1
bomb
a_x
bomb
shell
t
a_y
t
-9.81
shell
bomb

1)

c) $\vec{v}_{S,B} = \vec{v}_{S,E} + \vec{v}_{E,B}$







$|\vec{v}_{SB}| = \sqrt{(v_{E,B})^2 + (v_{S,E})^2} = \sqrt{(240)^2 + (416)^2}\ m/s = 480\ m/s$

$$\boxed{\begin{array}{l} \vec{v}_{SB} = 480\ m/s\ \hat{j} \\ \text{or } \vec{v}_{SB} = 480\ m/s \text{ directed up!} \end{array}}$$

#

2) (20 pts)

(A) = 6 pts

+2 pts each correct force

−1 pt each incorrect force

(NOTE: Total score cannot be negative. If so label w/ 0 pts)

(B) = 12 pts

$\sum_x F = ma_x$ (+3 pts)

$\sum_y F = ma_y = 0$ (+3 pts)

+1 pt $a_x = -\frac{v^2}{R}$

+2 pts $v = \sqrt{Rg}$

+2 pts correct geometry leading to $\boxed{R = 1m \sin 60° = \frac{\sqrt{3}}{2} m}$

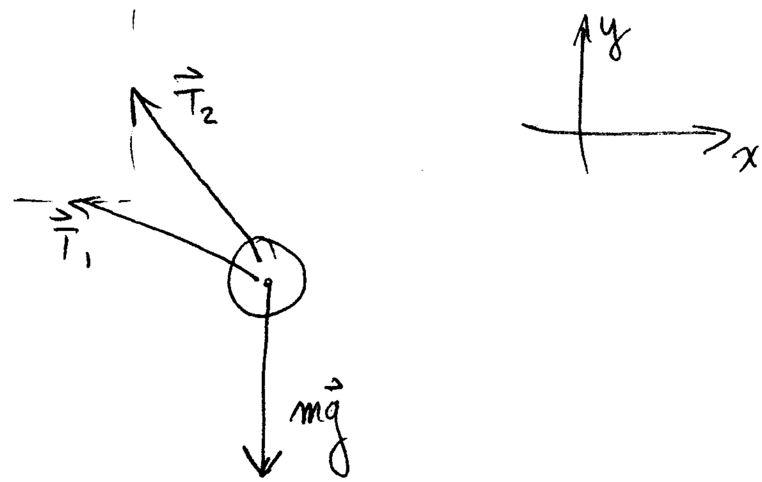
+1 pt $v = 2.9 m/s$

(C) +2 pts

$\boxed{T = 14.36 N}$

2)

A)



B) Assume $|\vec{T}_1| = |\vec{T}_2|$

$$\sum_x F: \quad -T_{1x} - T_{2x} = -ma_x = -\frac{mv^2}{R}$$

$$\sum_y F: \quad T_{1y} + T_{2y} - mg = 0 \qquad (a_y = 0)$$

$T_{1x} = T_1 \sin 60^\circ$; $T_{1y} = T_1 \cos 60$

$T_{2x} = T_2 \cos 60^\circ$; $T_{2y} = T_2 \sin 60^\circ$

$$\Rightarrow -T_1 \sin 60^\circ - T_2 \cos 60^\circ = -mv^2/R$$

$$\Rightarrow T_1(\sin 60^\circ + \cos 60^\circ) = mv^2/R$$

y-component of F: $\quad T_1 \cos 60^\circ + T_2 \sin 60^\circ = mg$

$$\Rightarrow T_1(\cos 60^\circ + \sin 60^\circ) = mg$$

$$\Rightarrow T_1 = \frac{mg}{(\cos 60^\circ + \sin 60^\circ)} = \frac{2mg}{(1+\sqrt{3})} = \left(\frac{2}{1+\sqrt{3}}\right) mg$$

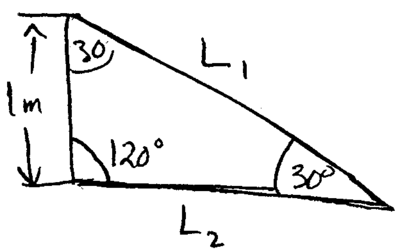
2) B)

$$T_1\left(\sin 60^\circ + \cos 60^\circ\right) = \frac{mv^2}{R}$$

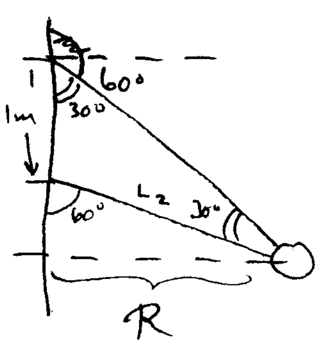
$$\frac{mv^2}{R} = mg\left(\frac{\sin 60^\circ + \cos 60^\circ}{\cos 60^\circ + \sin 60^\circ}\right) \Rightarrow \frac{mv^2}{R} = mg$$

$$\Rightarrow v = \sqrt{Rg} \qquad \text{What is } R\,?$$

Geometry!



Isosceles triangle $\Rightarrow L_2 = 1m$



$$\sin 60^\circ = R/L_2$$

$$\Rightarrow R = \sin 60^\circ = \sqrt{3}/2$$

"R represents radius of circle traced out by ball."

$$\Rightarrow \boxed{v = \sqrt{\left(\frac{\sqrt{3}}{2}\right)g} \simeq 2.91\ m/s}$$

(c) $\boxed{T_1 = T_2 = \left(\frac{2mg}{1+\sqrt{3}}\right) \simeq 14.36\ N}$ #

3) (20 pts)

(A) = 5 pts

$\vec{a} = -9.81 \sin 20°$ parallel to plane!

+3 magnitude ; +2 direction

(B) = 15 pts

+6 pts correct : $2.5\,m = v_0 \cos\theta\, t_1$

→ +2 pts $\Delta x = 2.5\,m$

+2 pts $a_x = 0$

+2 pts $v_{0,x} = v_0 \cos\theta$

+6 pts correct : $0\,m = v_0 \sin\theta\, t_1 - \frac{1}{2} g \sin 20°\, t_1$

→ +2 pts $\Delta y = 0\,m$

+2 pts $a_y = -g \sin 20°$ (or whatever used from part (A))

+2 pts $v_{0,y} = v_0 \sin\theta$

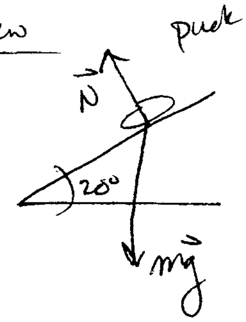
+1 pt $t_1 = \dfrac{2.5\,m}{v_0 \cos\theta}$ ~~of~~ or through Δy equation.

+1 pt correct $\sin 2\theta_1 = 0.932 \Rightarrow \theta_1 = 34.3°$

+1 pt correct $\underline{\theta_2 = 90° - \theta_1 = 55.7°}$

3)

A) Side View



The only acceleration is due to gravity along the incline. Since the puck is on an incline of 20°, the acceleration experienced by the puck is parallel to the incline directed down the incline.

$$\vec{a} = -g \sin 20° \, \hat{e}_{\parallel}$$

$\hat{e}_{\parallel}$ = unit vector parallel to the incline, and pointing up the incline!

B) Δx - denotes (distance) displacement from launch point to target point.
Δy - denotes displacement up the incline!

$$x_f = x_o + v_{ox} t + \tfrac{1}{2} a_x t^2 \qquad (a_x = 0)$$

$$\boxed{x_f - x_o = v_o \cos\theta \, t}$$

$$y_f = y_o + v_{oy} t + \tfrac{1}{2} a_y t^2 \qquad (a_y = -g \sin 20°)$$

$$\boxed{y_f - y_o = v_o \sin\theta \, t - \tfrac{1}{2} g \sin 20° \, t^2}$$

$$x_f - x_o = 2.5\,m = v_o \cos\theta \, t_1 \Rightarrow \boxed{t_1 = \frac{2.5\,m}{v_o \cos\theta}}$$

$$y_f - y_o = 0\,m = v_o \sin\theta \, t_1 - \tfrac{1}{2} g \sin 20° \, t_1^2$$

$$\Rightarrow v_o \sin\theta = \tfrac{1}{2} g \sin 20° \, t_1$$

$$\Rightarrow 2 v_o \sin\theta = g \sin 20° \left(\frac{2.5\,m}{v_o \cos\theta}\right) \Rightarrow 2 v_o^2 \cos\theta \sin\theta = 2.5 g \sin 20°$$

$$\Rightarrow \sin 2\theta = \frac{2.5 g \sin 20°}{v_o^2} = 0.932 \Rightarrow \boxed{\begin{array}{l} 2\theta = 68.6° \Rightarrow \theta_1 = 34.3° \\ \theta_2 = 90 - \theta_1 = 55.7° \end{array}}$$

//