Physics 101 Fall 2006: Test 2—Free Response #1 Solution

1. (**20 pts**) A block of mass m hangs on the end of a light cord and connected to a block of mass M by the pulley arrangement shown in the figure below (Fig. 1). The pulleys have negligible mass and are friction free. Upon release from rest, m begins to accelerate downwards.

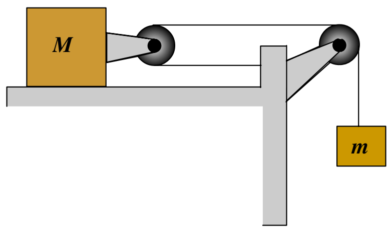

Figure 1: Problem 1

(a) find an expression relating the acceleration of m to the acceleration of M.

First we need to consider the motion of the two masses. The masses move in such a way that the total length of the string remains unchanged (no stretching or compression of the string). Make measurements of the two masses with respect to an inertial reference frame.

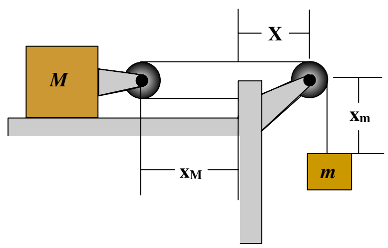

Figure 2: Problem 1

The total length of the string may be written as:

$$L = 2\,x_M + x_m + X \tag{1}$$

$$\frac{dL}{dt} = 2\,v_M + v_m = 0 \tag{2}$$

$$\boxed{a_m = -2\,a_M} \tag{3}$$

(b) Assuming initially that there is no friction between M and the table, use energy considerations to find an expression for the speed of m as a function of the distance h it has fallen. *(Leave your answer in terms of all or some of the following: m, M, g, and h.)*

$$\Delta K_T + \Delta U_T = \Delta E_T = 0 \tag{4}$$

$$\frac{1}{2} M \left(v_{fM}^2 - v_{iM}^2\right) + \frac{1}{2} m \left(v_{fm}^2 - v_{im}^2\right) + m\,g\,(y_{fm} - y_{im}) = 0 \tag{5}$$

$$\text{from (a) } v_{fM} = -\frac{1}{2} v_{fm} \tag{6}$$

$$\boxed{v_{fm} = \sqrt{\frac{2\,m\,g\,h}{(m + \frac{M}{4})}}} \tag{7}$$

(c) Repeat (b) assuming that sliding friction is present between M and the table, the coefficient of sliding friction being μ_k. *(Leave your answer in terms of all or some of the following: m, M, g, μ_k, and h.)*

$$\Delta K_T + \Delta U_T = \Delta E_T = -F_f\,l \tag{8}$$

$$\text{from (a) } l = -\frac{1}{2} h \tag{9}$$

$$\Delta K_T + \Delta U_T = \Delta E_T = -F_f\,h/2 \tag{10}$$

$$F_f = \mu_k\,M\,g \tag{11}$$

$$\frac{1}{2} M \left(v_{fM}^2 - v_{iM}^2\right) + \frac{1}{2} m \left(v_{fm}^2 - v_{im}^2\right) + m\,g\,(y_{fm} - y_{im}) = -\mu_k\,M\,g\,h/2 \tag{12}$$

$$\boxed{v_{fm} = \sqrt{\frac{(2\,m\,g\; - \;\mu_k\,M\,g)\,h}{(m + \frac{M}{4})}}} \tag{13}$$

Physics 101 Fall 2006: Test 2—Free Response #2 Solution

2. (**25 pts**) Two pendulum bobs of differing masses are suspended from strings of equal length as shown in the figure below (Fig. 3). The bob of mass $m_1 = 0.5$-kg is released from rest at a height h. It then hits the second bob of mass m_2 which is initially at rest. The two stick together after the collision. An observant PHYS101 student notices that the composite mass subsequently rises to a maximum height of $h_f = h/9$.

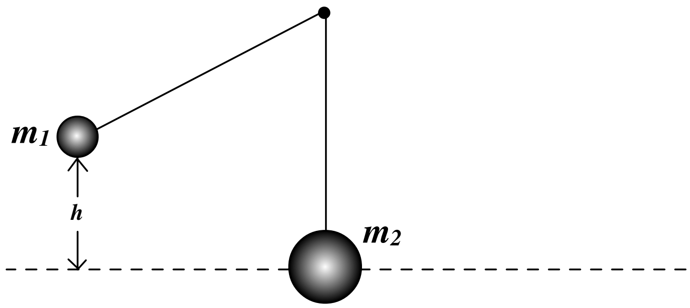

Figure 3: Problem 2.

(a) What is the mass (m_2) of the second pendulum bob?
Since m_1 and m_2 stick together we cannot use Energy Conservation for the entire process. We can; however, use Energy Conservation to find the speed of m_1 right before it collides with m_2.

$$\Delta K_T + \Delta U_T = \Delta E_T = 0 \tag{1}$$

$$\frac{1}{2} m_1 \left(v_{fm_1}^2 - v_{im_1}^2\right) + m_1\, g\, (y_{fm_1} - y_{im_1}) = 0 \tag{2}$$

$$v_{fm_1} = \sqrt{2\, g\, h} \tag{3}$$

The x-component of momentum is conserved immediately before and immediately after the collision:

$$\Delta p_x = 0 \tag{4}$$

$$m_1 \sqrt{2\, g\, h} = (m_1 + m_2)\, V_F \tag{5}$$

$$V_F = \frac{m_1}{(m_1 + m_2)} \sqrt{2\, g\, h} \tag{6}$$

The combined mass system now moves with speed V_F after the collision and this kinetic energy is converted into gravitational potential energy.

$$\Delta K_T + \Delta U_T = \Delta E_T = 0 \tag{7}$$

$$\frac{1}{2} (m_1 + m_2) \left(0 - V_F^2\right) + (m_1 + m_2)\, g\, (h/9 - 0) \tag{8}$$

$$\left(\frac{m_1}{(m_1 + m_2)}\right)^2 = \frac{1}{9} \tag{9}$$

$$\boxed{m_2 = 2\, m_1 = 1kg} \tag{10}$$

(b) How much energy was lost in the collision? *(Leave your answer in terms of g and h.)*

Amount of energy lost in the collision is given by the change in kinetic energy immediately before and immediately after the collision.

$$KE_i = m_1\, g\, h \tag{11}$$

$$KE_f = \frac{1}{2}\,(m_1 + m_2)\, V_F^2 \tag{12}$$

$$KE_f = g\, h \left(\frac{m_1^2}{(m_1 + m_2)}\right) \tag{13}$$

$$\boxed{\Delta K = -\frac{1}{3}\, g\, h} \tag{14}$$

Energy lost in the collision is $\frac{1}{3}\, g\, h$

Physics 101 Fall 2006: Test 2—Free Response #3 - Solution

3. (**25 pts**) Two long barges are moving in the same direction in still water, one with a speed of 9 km/h and the other with a speed of 21 km/h. While they are passing each other, coal is shoveled from the slower barge to the faster barge at a rate of 925 kg/min as illustrated in the figure below (Fig 4). Assume that the shoveling is always perfectly sideways and that the frictional forces between the barges and the water do not depend on the weight of the barges. Determine how much additional force must be provided by the driving engines of each barge as the coal is shoveled if neither is to change speed?

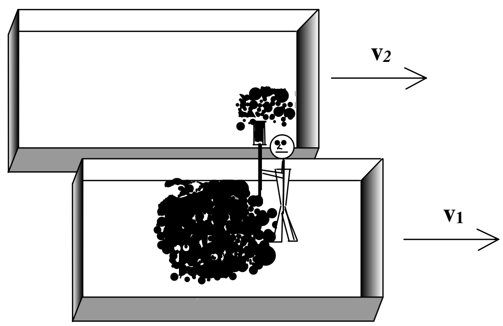

Figure 4: Problem 3.

As the coal is shoveled into the faster moving barge, the coal will accelerate in a short amount of time. By Newton's 3^{rd} law, the force that the coal experiences in the horizontal direction must be the same magnitude of force that the faster barge experiences due to the coal.

$$F_{x_{b,c}} = F_{x_{c,b}} \tag{1}$$

$$\Delta p_{c,b} = -\Delta p_{b,c} \quad \text{where b=barge, c=coal (Netwon's } 3^{rd}\text{-Law)} \tag{2}$$

The change in the coal's momentum is directly reflected in the change in the barges's momentum.

$$\sum_x F = \frac{dp}{dt} \tag{3}$$

$$\frac{dp_{c,b}}{dt} = v_{c,b}\,\frac{dm}{dt} \tag{4}$$

$$\frac{dm}{dt} = 925kg/min = 15.42kg/s \tag{5}$$

$$v_{c,b} = 0 \quad \text{(For coal relative to slow barge.)} \tag{6}$$

$$v_{c,b} = (21-9)km/h = 3.33m/s \quad \text{(For coal relative to faster barge.)} \tag{7}$$

$$\sum_x F = \frac{dp_{c,b}}{dt} = 3.33m/s\ (15.42kg/s) = 51.4N \tag{8}$$

$$\boxed{F_{\text{additional}} = 51.4N} \tag{9}$$

The engine of the barge moving at the larger speed would require an additional force output of 51.4 N in order to maintain a constant speed.

The engine of the barge moving at the smaller speed would require no additional force output to maintain a constant speed.

Last Name: Key First Name: Key

Physics 101 Fall 2006: Test 2—Multiple-Choice Answers

	A	B	C	D	E
1		X			
2		X			
3		X			
4					X
5				X	
6					X
7			X		
8				X	
9				X	
10	X				