Last Name: KEY First Name: KEY

Physics 101 Fall 2006: Final Exam—Free Response and Instructions

- Print your LAST and FIRST name on the front of your blue book, on this question sheet, the multiple-choice question sheet and the multiple-choice answer sheet.
- TIME ALLOWED 180 MINUTES
- The test consists of four free-response questions and sixteen multiple-choice questions.
- The test is graded on a scale of 120 points; the first free-response question accounts for 12 points, the second for 20 points, the third for 20 points, the fourth for 20 and the multiple-choice questions account for 48 points.
- Answer the four free-response questions in your blue book. Answer the multiple-choice questions by marking a dark X in the appropriate column and row in the table on the multiple-choice answer sheet.
- Consult no books or notes of any kind. You may use a hand-held calculator in non-graphing, non-programmed mode.
- Do NOT take test materials outside of the class at any time. Return this question sheet along with your blue book and multiple-choice question sheet.
- Write and sign the Pledge on the front of your blue book.

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*Show your work for the free-response problems, including neat and clearly labeled figures, in your blue book. It is possible that answers without explanation (**even correct answers**) will not be given credit.*

USEFUL INFORMATION

- Throughout the test, when necessary use $g = 9.8\ \mathrm{m/s}^2$.

  Moments of inertia for rotation about an axis passing through the center of mass and perpendicular to the plane containing the object:

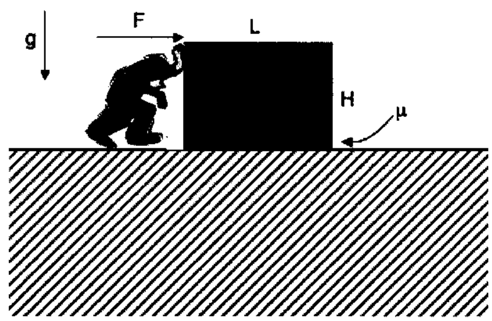
- Solid sphere of mass M and radius R: $I = \frac{2}{5} M R^2$.
- Thin spherical shell of mass M and radius R: $I = \frac{2}{3} M R^2$.
- Solid cylinder or disk of mass M and radius R: $I = \frac{1}{2} M R^2$.
- Hoop or thin cylindrical shell of mass M and radius R: $I = M R^2$.
- Rod of mass M and length L: $I = \frac{1}{12} M L^2$.

1. (**12 pts**) A mover is trying to slide a uniformly filled crate of length L, height H and mass M across a floor. There is static friction with coefficient μ_s between the floor and the crate. The mover exerts a horizontal force F at the upper edge of the crate. If μ_s is large enough, the crate will tip over before it slides.

   (a) Find an expression for the minimum value of the force F required for the box to tip.

   (b) Calculate the minimum value of μ_s required such that the box will tip before sliding begins.

   (Leave your answers in terms of all or some of the following: M, g, L, H).



SOLUTION

(a) Consider torques about the right-hand bottom corner of the box since the system will potentially rotate about this point.
Calling clockwise rotations negative and counter-clockwise rotations positive:

$$-F\,H + M\,g\,\frac{L}{2} \leqslant 0. \tag{1}$$

$$\Rightarrow F\,H \geqslant M\,g\,\frac{L}{2} \tag{2}$$

$$F \geqslant \frac{M\,g\,L}{2\,H} \tag{3}$$

$$\boxed{F_{\min} = \frac{M\,g\,L}{2\,H}} \tag{4}$$

(b) In order to calculate the minimum μ_s, we need to consider all the forces acting on the box.

$$\sum_x F = F_{\min} - f_{\min} = 0 \tag{5}$$

$$\sum_y F = N - M\,g = 0 \tag{6}$$

$$f_{\min} = \mu_{s,\min}\,N = \mu_{s,\min}\,M\,g \tag{7}$$

$$\boxed{\mu_{s,\,\min} = \frac{L}{2\,H}} \tag{8}$$

2. (**20 pts**) A cart of mass M in the shape of a wedge can move freely along a horizontal flat table. A box of mass m is initially near the top of the wedge a distance L from the bottom as indicated in the figure below. Assume there is no friction between M and m. If the system is released from rest in the configuration shown in the figure, what is the speed of mass m just before it leaves the wedge?

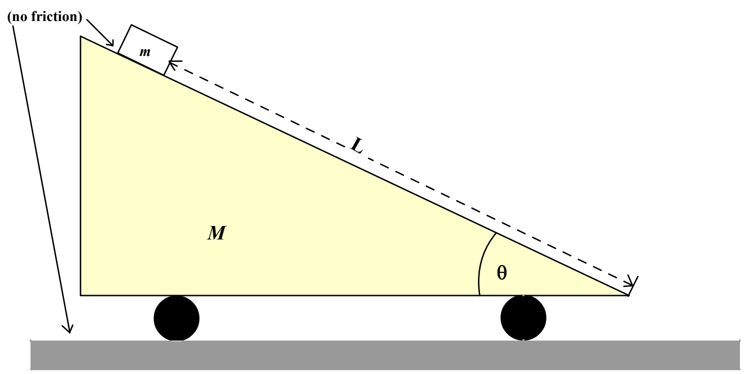


Figure 2: Problem 2

Let's call the final speed of the small cart (m) v_f and the speed of the wedge (M) V. Since there are no external forces acting in the x-direction for the system of the two carts, the x-component of momentum is conserved. (The y-component of momentum is not conserved!).

$$\Delta p_x = 0. \tag{1}$$

$$p_{\mathrm{i,x}} = 0 \tag{2}$$

$$p_{\mathrm{f,x}} = m\,v_f\,\cos\theta\ - M\,V \tag{3}$$

$$\Rightarrow m\,v_f\,\cos\theta = M\,V \tag{4}$$

$$\Rightarrow V = v_f\,\frac{m}{M}\,\cos\theta \tag{5}$$

Since friction is not present in this system, total mechanical energy is conserved.

$$\Delta K_T\ + \Delta U_T = \Delta E_T = 0 \tag{6}$$

$$\frac{1}{2}\,M\,\left(v_{fM}^2 - v_{iM}^2\right) + \frac{1}{2}\,m\,\left(v_{fm}^2 - v_{im}^2\right) + m\,g\,(y_{fm} - y_{im}) = 0 \tag{7}$$

$$\frac{1}{2}\,m\,v_f^2 + \frac{1}{2}\,M\,\left(\frac{m}{M}\,v_f\,\cos\theta\right)^2 - L\,m\,g\,\sin\theta = 0 \tag{8}$$

$$v_f^2\,\left(1 + \frac{m}{M}\,\cos^2\theta\right) = 2\,L\,g\,\sin\theta \tag{9}$$

$$\boxed{v_f = \sqrt{\frac{2\,L\,g\,\sin\theta}{\left(1 + \frac{m}{M}\,\cos^2\theta\right)}}} \tag{10}$$

3. (**20 pts**) A block of mass m is launched horizontally from a compressed spring on a frictionless track that turns upward at a 30° angle, as shown in the figure below. At the end of the track it is launched into the air. Find an expression for its horizontal range x (defined in the figure below), as a function of the distance d by which the spring is initially compressed, the spring constant k, the acceleration due to gravity g, and the height h of the ramp.

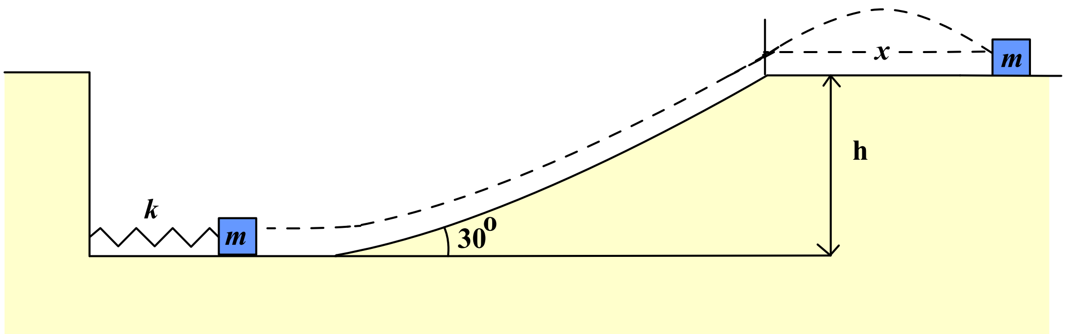


Figure 3: Problem 3

At the end of the ramp, the block will have a speed v_f. There is no friction in the problem so total mechanical energy is conserved.

$$\Delta K_T + \Delta U_T = \Delta E_T = 0 \tag{1}$$

$$\frac{1}{2}\, m\, \left(v_{fm}^2 - v_{im}^2\right) + m\, g\, (y_{fm} - y_{im}) + \frac{1}{2}\, k\, \left(\Delta y_f^2 - \Delta y_i^2\right) = 0 \tag{2}$$

$$\frac{1}{2}\, m\, v_f^2 + m\, g\, h - \frac{1}{2}\, k\, d^2 = 0 \tag{3}$$

$$\Rightarrow v_f = \sqrt{\frac{k\, d^2}{m} - 2\, g\, h} \tag{4}$$

At the end of the ramp, the small mass will behave as a projectile with initial speed v_f launched at an angle of 30° with respect to the horizontal.

$$x_f = x_0 + v_{0,\mathrm{x}}\,\Delta t \tag{5}$$

$$x \equiv x_f - x_0 = v_{0,\mathrm{x}}\,\Delta t \tag{6}$$

$$y_f - y_0 = v_{0,\mathrm{y}}\,\Delta t - \frac{1}{2}\,g\,(\Delta t)^2 \tag{7}$$

$$v_{0,\mathrm{y}} - \frac{1}{2}\,g\,\Delta t = 0 \tag{8}$$

$$\Rightarrow \Delta t = \frac{2\,v_f\,\sin\theta}{g} \tag{9}$$

$$x = \frac{2\,v_f^2\,\cos\theta\,\sin\theta}{g} \tag{10}$$

$$\boxed{x = \sqrt{3}\left(\frac{k\,d^2}{2\,m\,g} - h\right)} \tag{11}$$

4. (**20 pts**) A uniform piece of wire of length $2L$ and mass m is bent at its center into a "V" shape with an angle θ between the legs as shown in the figure below. It is pivoted at the bend, labeled O, over a friction-free peg.

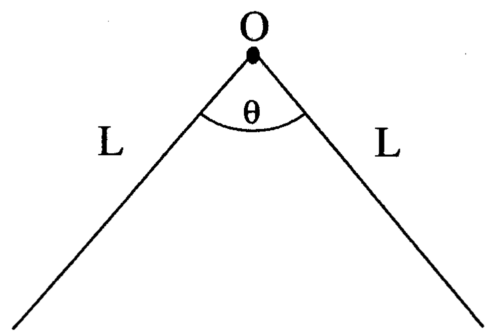


Figure 4: Problem 4

(a) Find the position of the center of mass of the system at equilibrium. (Take the origin of your coordinate system to be the pivot).

To find the center of mass of the system (taking the pivot to be the origin), we need to identify the center of mass of each "leg" of the "V" shape. Since the center of mass for each leg is at its midpoint, the x coordinate for the center of mass is located at 0.

$$\boxed{x_{\text{CofM}} = 0}. \tag{1}$$

$$m\, y_{\text{CofM}} = \frac{m}{2}\left(-\frac{L}{2}\cos(\frac{\theta}{2})\right) + \frac{m}{2}\left(-\frac{L}{2}\cos(\frac{\theta}{2})\right) \tag{2}$$

$$\boxed{y_{\text{CofM}} = -\frac{L}{2}\cos(\frac{\theta}{2})} \tag{3}$$

(b) Show that if the system is displaced through a small angle ϕ and released it will execute simple harmonic motion.
If the system is displaced through a small angle ϕ, then gravity will torque the system.

$$\sum \vec{\tau} = \vec{\mathbf{r}} \times m\,\vec{\mathbf{g}} = -r_\perp\, m\, g\hat{k} \tag{4}$$

$$r_\perp = y_{\text{CofM}}\, \sin\phi \tag{5}$$

$$\Rightarrow \vec{\tau} = -\frac{m\,g\,L}{2}\,\cos(\frac{\theta}{2})\,\sin\phi\,\hat{k} \tag{6}$$

$$I\,\alpha = -\frac{m\,g\,L}{2}\,\cos(\frac{\theta}{2})\,\sin\phi \tag{7}$$

$$\text{for small angles } \sin\phi \approx \phi \tag{8}$$

$$\Rightarrow \alpha \approx -\left(\frac{m\,g\,L}{2\,I}\,\cos(\frac{\theta}{2})\right)\,\phi \tag{9}$$

$$\boxed{\frac{d^2\,\phi}{dt^2} \approx -\left(\frac{m\,g\,L}{2\,I}\,\cos(\frac{\theta}{2})\right)\,\phi} \tag{10}$$

Eq.10 represents the equation for simple harmonic motion in ϕ.

$$\frac{d^2\,\phi}{dt^2} = -\omega^2\,\phi \tag{11}$$

(c) What is the time period of this motion?
The time period of this motion is given by

$$T = \frac{2\,\pi}{\omega} \tag{12}$$

$$\text{where } \omega = \sqrt{\frac{m\,g\,L}{2\,I}\,\cos(\frac{\theta}{2})} \tag{13}$$

$$I = I_{\text{rod1}} + I_{\text{rod2}} \tag{14}$$

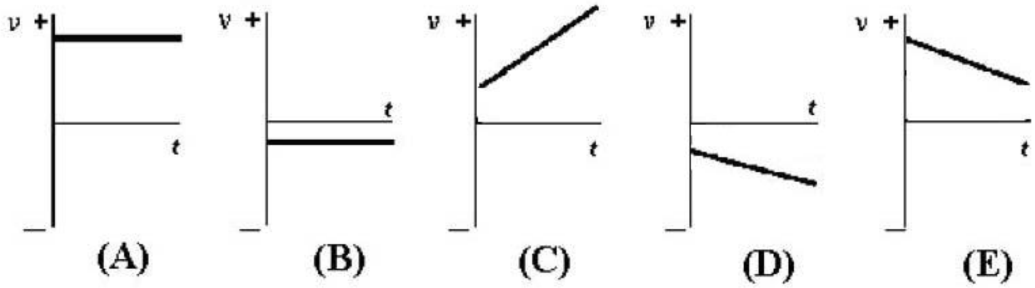
$$I = \frac{1}{3}\,\left(\frac{m}{2}\right)\,L^2 + \frac{1}{3}\,\left(\frac{m}{2}\right)\,L^2 \tag{15}$$

$$I = \frac{1}{3}\,m\,L^2 \tag{16}$$

$$\boxed{T = 2\,\pi\sqrt{\frac{2\,L}{3\,g\,\cos(\frac{\theta}{2})}}} \tag{17}$$

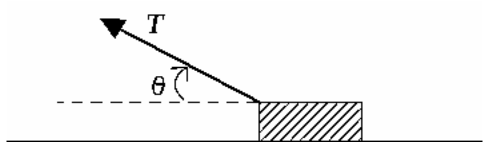
Physics 101 Fall 2006: Final Exam—Multiple-Choice Questions

1. Which graph of v versus t below best describes the motion of a particle with positive velocity and negative acceleration?

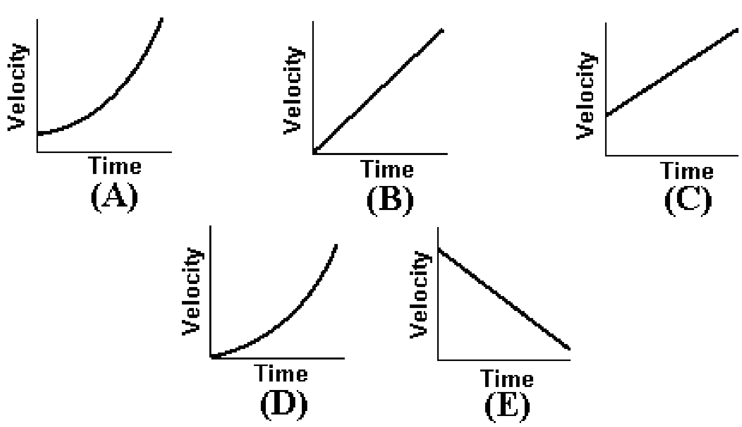


2. A block of mass m is pulled horizontally in the direction shown in the figure below across a rough surface with a constant horizontal acceleration a. The coefficient of kinetic friction between the surface and the block is μ_k. The magnitude of the frictional force is

(A). $\mu_k\, m\, g$.
(B). $T\, \cos\theta - m\, a$.
(C). $\mu_k(T - m\, g)$.
(D). $\mu_k\, T \sin\theta$.
(E). $\mu_k(m\, g + \sin\theta)$.



3. A car accelerates uniformly on a flat, horizontal road from a speed of 10 mi/hr to 30 mi/hr in one minute. Which graph below best describes the motion of the car?



4. A bullet of mass m_1 is fired with a speed V into a block of mass m_2 initially at rest. If the bullet escapes from the block with only a third of its original speed, then the speed of the block is

(A) $\frac{m_1 V}{3 m_2}$.

(B) $\frac{2 m_1 V}{3 m_2}$.

(C) $\frac{m_2 V}{3 m_1}$.

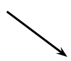
(D) $\frac{2 m_2 V}{3 m_1}$.

(E) $\frac{4 m_2 V}{9 m_1}$.

5. The figure below shows the position of the moon at two different times, about 7 days apart. Which vector best represents the change in the moon's velocity in this time interval?

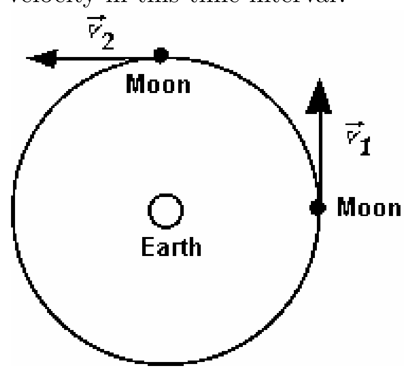
(A)

(B)

(C) 

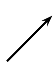
(D) 

(E) 

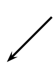


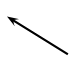
6. A block of mass m is free to slide on a large wedge of mass M which, in turn, is free to slide on the floor. There is **no** friction between any surface. A spring is attached (rigidly) to the floor and to the bottom of the large wedge. Initially the spring is at its equilibrium length. If the system is released from rest which vector most accurately describes the acceleration of the center of mass of the system while the small block m is in contact with M?

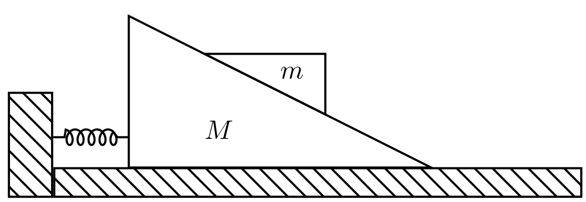
(A) 

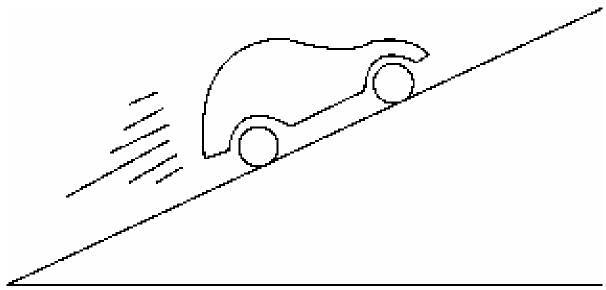
(B) 

(C) 

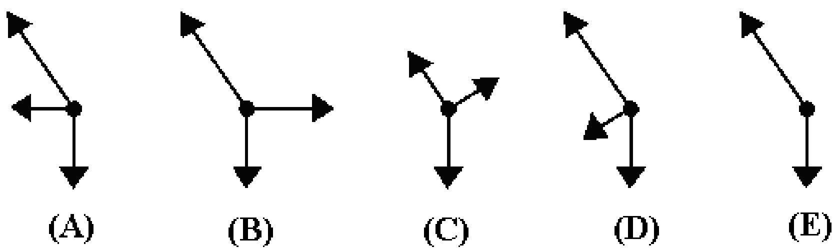
(D) 

(E) 



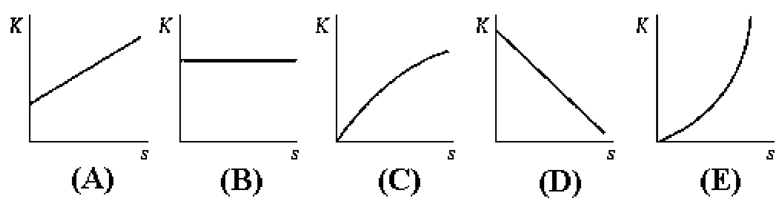


7. Which of the following free-body-diagrams represents the car going uphill at a constant speed (as illustrated in the figure above)?



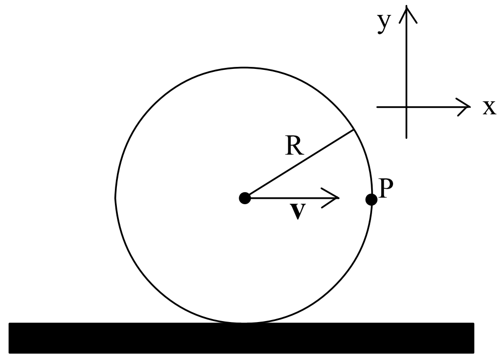
8. You release an object from rest a distance h above the ground. When the effects of air resistance are included, the curve that best represents the kinetic energy of the body as a function of the distance s fallen is

(A). A.
(B). B.
(C). C.
(D). D.
(E). E.



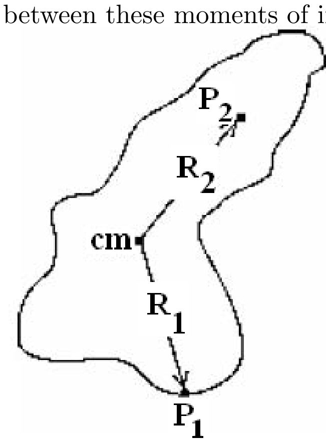
9. A solid ball of radius R rolls without slipping along a flat horizontal surface as indicated in the figure below. If the center of mass of the ball has a speed v with respect a fixed laboratory frame, what is the instantaneous speed of the foremost point P on the ball in this frame.

(A) v.
(B) $\sqrt{2}\,v$.
(C) $2\,v$.
(D) $2\,\sqrt{2}\,v$.
(E) $4\,v$.



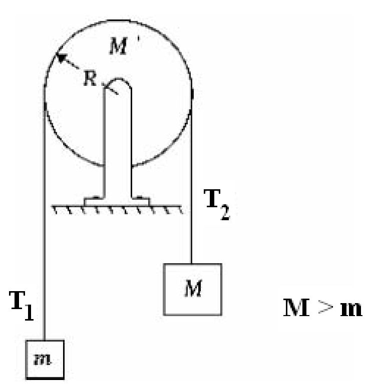
10. In the figure below $R_1 = R_2$ and cm is the center of mass of the irregularly shaped object. The rotational inertia about a perpendicular axis through point P_1 is I_1, the rotational inertia about a perpendicular axis through point P_2 is I_2, and the rotational inertia about a perpendicular axis through point cm is I_{cm}. The relationship between these moments of inertia is

(A) $I_1 = I_2 < I_{cm}$.
(B) $I_1 = I_2 > I_{cm}$.
(C) $I_1 > I_2 > I_{cm}$.
(D) $I_2 > I_1 > I_{cm}$.
(E) $I_1 = I_2 = I_{cm}$.



11. Two masses M and m ($M > m$) are hung over a pulley. Mass M is hung over the right side of the pulley, and mass m is hung over the left side of the pulley. The pulley has a moment of inertia I. Upon release (from rest), the masses accelerate and the pulley begins to rotate. If T_1 is the tension in the cord on the left and T_2 is the tension in the cord on the right of the pulley, then

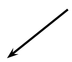
(A) $T_1 = T_2$.
(B) $T_1 > T_2$.
(C) $T_1 < T_2$.
(D) $T_2 = M\,g$.
(E) $T_2 = \frac{M\,g}{m}$.

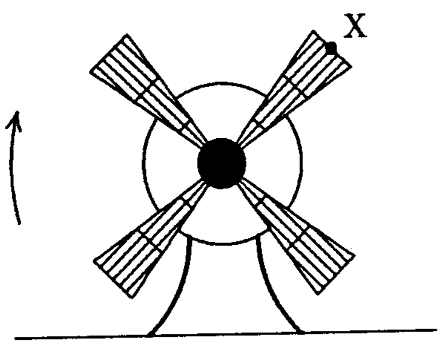


12. A bowling ball and a ping-pong ball are moving towards you each with the same momentum. If you exert the same force to stop each one, for which is the stopping distance greater?

    (A) the bowling ball.
    (B) both require the same amount of time.
    (C) the ping-pong ball.
    (D) need more information.

13. The fan shown below has been turned off and is slowing down. If it is rotating clockwise which of the following vectors can describe the acceleration of point X on the tip of the fan blade shown

    (A)
    (B) 
    (C) 
    (D)
    (E) 

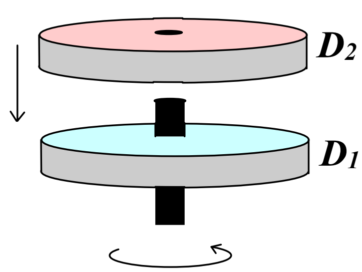


14. To mix a can of paint, a machine shakes the can vertically with simple harmonic motion. The can is shaken with a frequency of $f = 10$ Hz and an amplitude of 3 cm. If the can breaks loose from the machine as it passes (on the way up) through the mid point of its motion which of the following values best specifies the height to which the can will rise.

    (A) 18 cm.
    (B) 25 cm.
    (C) 36 cm.
    (D) 51 cm.
    (E) 72 cm.

15. Disc D_1 is rotating on friction-free bearings with initial kinetic energy E_{ki}. A second identical non-rotating disc D_2 is dropped onto disc D_1. After some time the two rotate with a common angular velocity. The final total kinetic energy (E_{kf}) of the system is

    (A) E_{ki}
    (B) $\frac{E_{ki}}{\sqrt{2}}$
    (C) $\frac{E_{ki}}{2}$
    (D) $\frac{E_{ki}}{2\sqrt{2}}$
    (E) $\frac{E_{ki}}{4}$



16. You are given two carts, A and B. They look identical, and you are told that they are made of the same material. You place A at rest on an air track and give B a constant velocity directed to the right so that it collides elastically with A. After the collision, both carts move to the right, the velocity of B being smaller than what it was before the collision. What do you conclude?

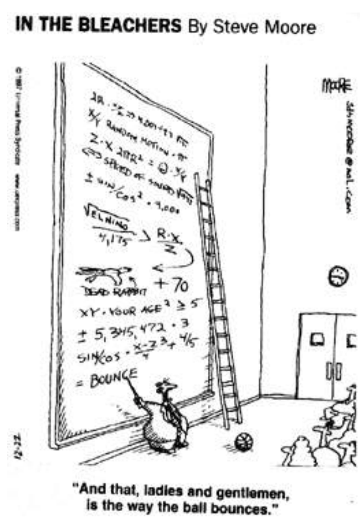
    (A) Cart A is hollow.

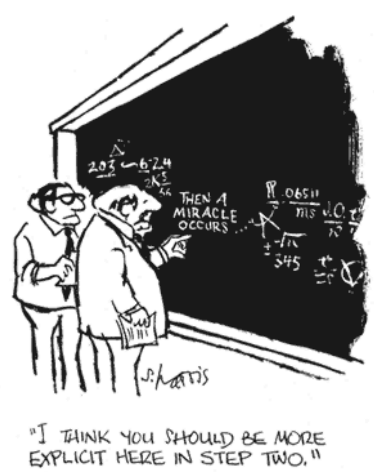
    (B) The two carts are identical.

    (C) Cart B is hollow.

    (D) momentum is not conserved in this collision.

    (E) this question can only be answered if the initial velocity of cart B is given.





Last Name: First Name:

Physics 101 Fall 2006: Final Exam—Multiple-Choice Answers

	A	B	C	D	E
1					X
2		X			
3			X		
4		X			
5					X
6			X		
7			X		
8			X		
9		X			
10		X			
11			X		
12			X		
13		X			
14	X				
15			X		
16	X				