

Last Name: _____ First Name: _____

Physics 101 Fall 2004: Test 2—Free Response and Instructions

- Print your LAST and FIRST name on the front of your blue book, on this question sheet, the multiple-choice question sheet and the multiple-choice answer sheet.
- TIME ALLOWED 75 MINUTES
- The test consists of three free-response questions and ten multiple-choice questions.
- The test is graded on a scale of 100 points; the first free-response question accounts for 20 points, the second for 25 points, the third for 25 points and the multiple-choice questions account for 30 points.
- Answer the three free-response questions in your blue book. Answer the multiple-choice questions by marking a dark X in the appropriate column and row in the table on the multiple-choice answer sheet.
- Consult no books or notes of any kind. You may use a hand-held calculator in non-graphing, non-programmed mode.
- Do NOT take test materials outside of the class at any time. Return this question sheet along with your blue book and multiple-choice question sheet.
- Write and sign the Pledge on the front of your blue book.

Show your work for the free-response problems, including neat and clearly labelled figures, in your blue book. It is possible that answers without explanation (even correct answers) will not be given credit.

TAKE $g = 9.8 \text{ m/s}^2$, or 32 ft/s^2 .

Useful moments of inertia: The moment of inertia of a solid disc (mass M and radius R) rotated about an axis perpendicular to the plane of the disc passing through the center of the disc is $\frac{1}{2}MR^2$.

1. (20pts) A uniform solid disk of radius R and mass M is free to rotate on a frictionless pivot through a point on its rim.

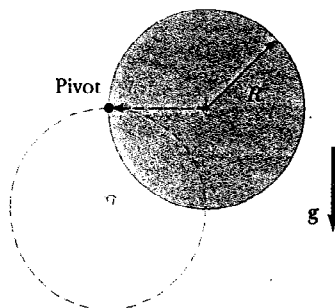


Figure 1: Problem 1

- (a) If the disk is released from rest in the position shown by the solid circle, what is the speed of its center of mass when the disk reaches the position indicated by the dashed circle?
- (b) What is the speed of the lowest point on the disk in the dashed position?
- (c) Repeat part (a) for a uniform hoop.

2. (25pts) A 2.00-kg block situated on a rough incline is connected to a spring of negligible mass having a spring constant of 100 N/m. The block is released from rest when the spring is unstretched, and the pulley is frictionless. The block moves 20.0 cm down the incline before coming to rest. Find the coefficient of kinetic friction between block and incline.

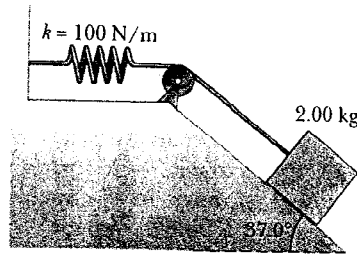


Figure 2: Problem 2

Suppose the incline is frictionless for the system described in problem 2. The block is released from rest with the spring initially unstretched.

- How far does it move down the incline before coming to rest?
- What is its acceleration at its lowest point? Is the acceleration constant?
- Describe the energy transformations that occur during the descent?

3. (25pts) A cannon is rigidly attached to a carriage, which can move along horizontal rails but is connected to a post by a large spring of force constant $k = 2.00 \times 10^4$ N/m. The cannon fires a 200-kg projectile at a velocity of 125 m/s directed 45.0° above the horizontal.

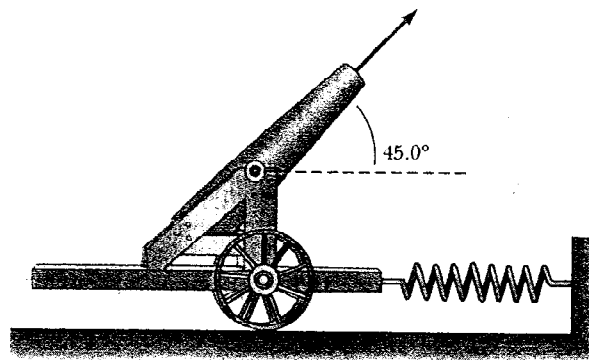
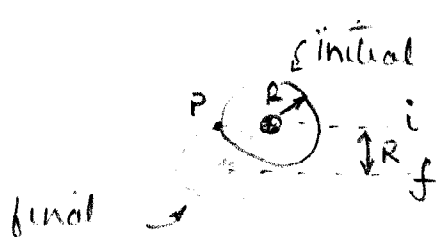


Figure 3: Problem 3

- If the mass of the cannon and its carriage is 5000 kg, find the recoil speed of the cannon.
- Determine the maximum extension of the spring.
- Consider the system to consist of the cannon, carriage, and shell. Is the momentum of this system constant during the firing? Why or why not?

1)



Since only gravitational force does work
conserve total mechanical energy

$$E_{TOTi} = E_{TOTf} \Rightarrow \frac{1}{2} I_P \omega^2 + mgy_{cmi} = E_{kf} + mgy_{cmf}$$

$$E_{kf} = \frac{1}{2} I_P \omega^2 = mg \underbrace{(y_{cmi} - y_{cmf})}_R \quad \dots (i)$$

$$I_P = I_{cm} + mR^2 = \underbrace{\frac{MR^2}{2}}_{\text{parallel axis theorem}} + \frac{3MR^2}{2}$$

$$\frac{1}{2} \cdot \frac{3}{2} mR^2 \omega^2 = mgR$$

$$\therefore \omega^2 = \frac{4}{3} \frac{g}{R} \Rightarrow \omega = 2 \sqrt{\frac{g}{3R}}$$

speed of c.o.m at f $v_T = \omega R = 2 \sqrt{\frac{gR}{3}}$

speed of lowest point of disc $v_{TL} = \omega 2R = 4 \sqrt{\frac{gR}{3}}$

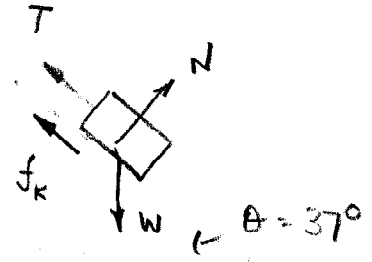
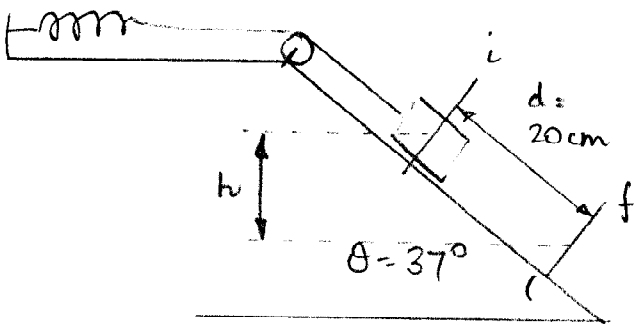
For uniform hoop $I_P = I_{cm} + mR^2 = mR^2 + mR^2 = 2mR^2$

from (i) $\frac{1}{2} \cdot 2mR^2 \cdot \omega^2 = mgR \Rightarrow \omega = \sqrt{\frac{g}{R}}$

speed of c.o.m at f $v_T' = \omega R = \sqrt{gR}$

speed of lowest point on rim $v_{TL}' = \omega 2R = 2\sqrt{gR}$

2)



Applying work-energy theorem for motion $i \rightarrow f$

$$W_{f_k} = \Delta E_{\text{TOT}} = E_{kf} + U_{gf} + U_{Ef} - (E_{ki} + U_{gi} + U_{Ei})$$

$$\int_i^f \vec{f}_k \cdot d\vec{s} = -f_k d = \frac{1}{2} k d^2 + mg(y_f - y_i) = \frac{1}{2} k d^2 - mgh$$

$\uparrow d \sin \theta$

$$-f_k \times 0.2 = \frac{1}{2} \times 100 \times 0.2^2 - 2 \times 9.8 \times 0.2 \times \sin 37$$

$$= 2 - 2.359 = -0.359$$

$$\underline{f_k = 1.796 \text{ N}}$$

but $f_k = \mu_k N$

obtain N by using $\Sigma F_{\perp} = 0$ plane

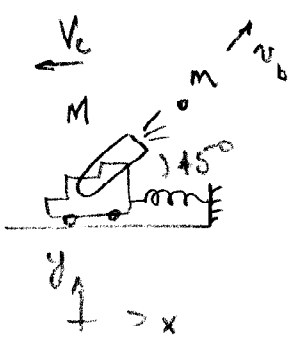
$$\Sigma F_{\perp} = 0 = N - W \cos \theta$$

$$\Rightarrow N = W \cos \theta = mg \cos \theta$$

$$= 15.65 \text{ N}$$

$$\therefore \underline{\mu_k = \frac{f_k}{N} = \frac{1.796}{15.65} = 0.115}$$

3)



At time of firing no forces act in x dirⁿ

$$P_{TOTxi} = P_{TOTxf}$$

$$0 = m v_b \cos 45^\circ - M V_c$$

$$V_c = \frac{m v_b \cos 45}{M} = \frac{200 \times 125 \times \cos 45}{5000} = \underline{3.535 \text{ m/s}}$$

Since no frictional forces act on wheel conserve total mechanical energy

$$E_{ki} + U_{Ei} = E_{kf} + U_{Ef}$$

$$\frac{1}{2} k x^2 = \frac{1}{2} M V_c^2$$

$$x^2 = \frac{M V_c^2}{k} = \frac{5000 \times 3.535^2}{20000} = 3.124$$

$$\Rightarrow \underline{x = 1.77 \text{ m}}$$

Momentum of system is not constant during firing as system experiences a large impulsive upward force during firing from ground!