Last Name: ____________________ First Name: ____________________

Physics 101 Fall 2003: Final—Free Response and Instructions
2:00-5:00 PM, 10 December, 2003

- Print your LAST and FIRST name on the front of your blue book, on this question sheet, the multiple-choice question sheet and the multiple-choice answer sheet.
- TIME ALLOWED 180 MINUTES
- The test consists of five free-response questions and ten multiple-choice questions.
- The test is graded on a scale of 150 points; the first free-response question accounts for 15 points, the second for 15 points, the third for 30 points, the fourth for 30 points, and fifth for 30 points and the multiple-choice questions account for 30 points.
- Answer the five free-response questions in your blue book. Answer the multiple-choice questions by marking a dark X in the appropriate column and row in the table on the multiple-choice and matching answer sheet.
- Consult no books or notes of any kind. You may use a hand-held calculator in non-graphing, non-programmed mode.
- Do NOT take test materials outside of the class at any time. Return this question sheet along with your blue book and multiple choice answer sheet.
- Write and sign the Pledge on the front of your blue book.

Show your work for the free-response problems, including neat and clearly labelled figures, in your blue book. Answers without explanation (even correct answers) may not receive full credit.

Throughout the test, when necessary use $g = 9.8\ \text{m/s}^2$.

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1. **[15 pts]** A skeet is projected vertically from the ground with an initial velocity of 30 m/s. A skeet-shooter, located 50 m from the release point, aims his gun and fires at the same instant that the skeet is released as shown in the figure below. The muzzle velocity of the bullet is 300 m/s.

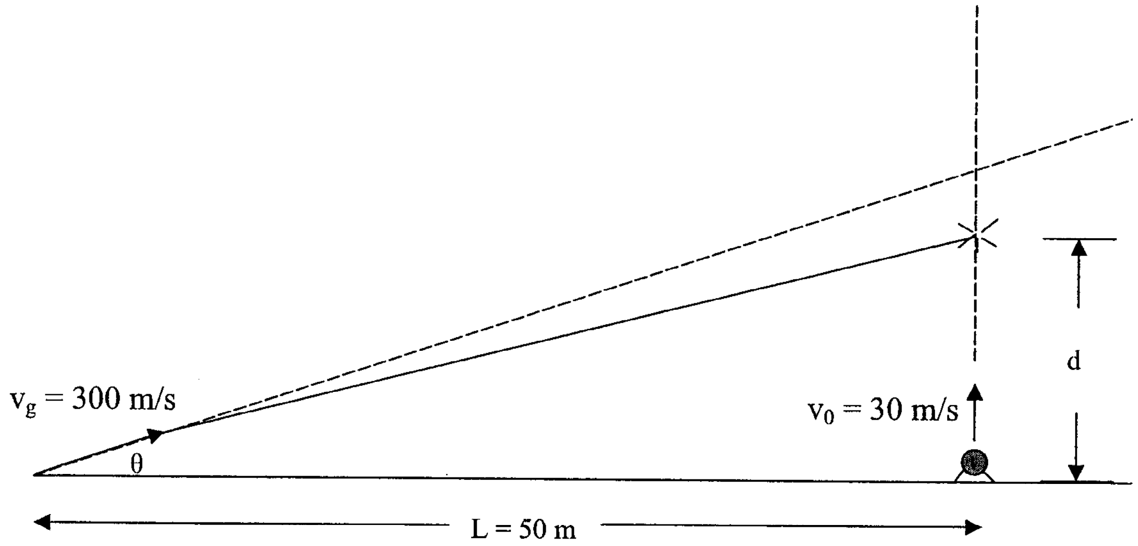


Figure 1: Problem 1

(a) At what angle θ must the shooter aim his gun if he is to score a hit?

(b) How far above the ground (d) will the bullet strike the skeet?

2. **[15 pts]** A uniform ruler of length 1 m and weight 10 N is supported as shown in the diagram by two smooth (i.e., friction free) nails so it is at an angle of 30° to the vertical. One nail, A, is located in a small hole (larger than the diameter of the nail!) 0.1 m from the top of the ruler. The lower end of the ruler rests against a nail B located 0.1 m from the lower end.

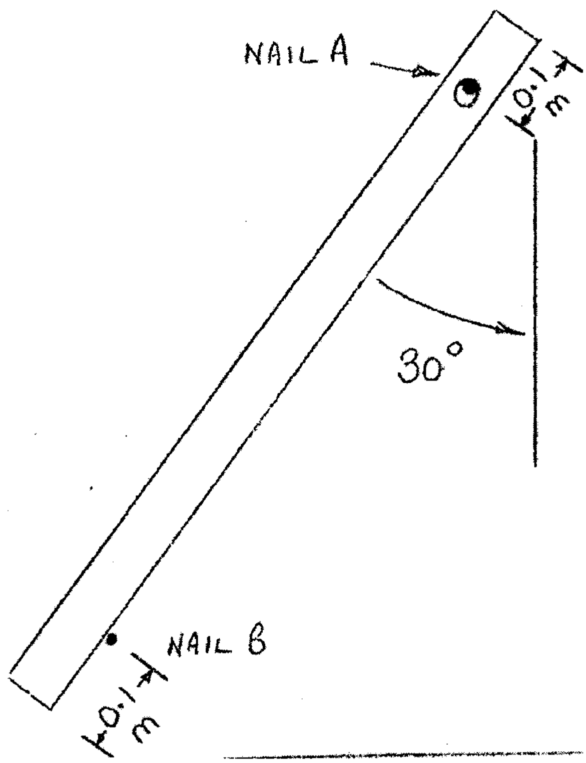


Figure 2: Problem 2

(a) In a *clearly* labelled diagram, show all the forces acting ***upon*** the ruler.

(b) Determine the force (vector!) exerted by nail B on the ruler.

(c) Determine the resultant force (vector!) exerted by nail A on the ruler.

3. [**30 pts**] Jane, whose mass is M_J, needs to swing across a river (of width D) filled with crocodiles in order to save Tarzan from danger as shown in the figure below. However, she must swing into a constant horizontal wind force $\mathbf{F}$ on a vine having length L and initially making an angle θ with the vertical. Assume that $D = 50$ m, $\|\mathbf{F}\| = 110$ N, $L = 40.0$ m, $M_J = 50$ kg, and $\theta = 50.0°$.

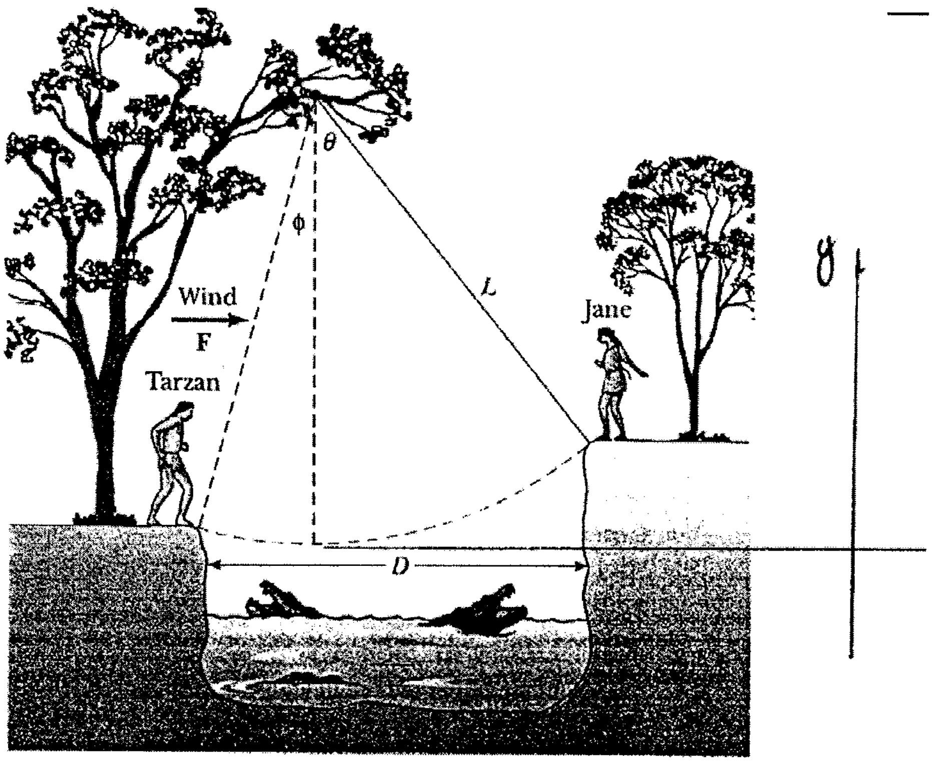


Figure 3: Problem 3

If the lowest point on the swing is taken to define the origin of the y-axis,

(a) Determine the height of Jane when she begins her swing.

(b) Determine the angle ϕ.

(c) Determine the height of Jane when she reaches Tarzan.

Assume now that Jane starts with an initial speed v_i and just makes it over to Tarzan, i.e. her speed is zero the moment she arrives at Tarzan.

(d) How much work is done by the wind force during the swing?

(e) Determine Jane's initial speed v_i.

Once the rescue is complete, Tarzan (who weighs 80.0 kg) and Jane must swing back together across the river.

(e) Assuming that the wind force $\mathbf{F}$ is unchanged, with what minimum speed must they begin their swing?

4. [**30 pts**] A weightless cord is wound around a uniform solid cylinder of mass $M = 100$ gm and radius $R = 5$ cm. The loose end of the cord is attached to a support as shown in the figure. The cylinder is then released from rest so that it simultaneously descends and is set into rotation as the cord unwinds (much like a yo-yo). The moment of inertia of a disk about a perpendicular axis through its center of mass is $I = \frac{1}{2}MR^2$.

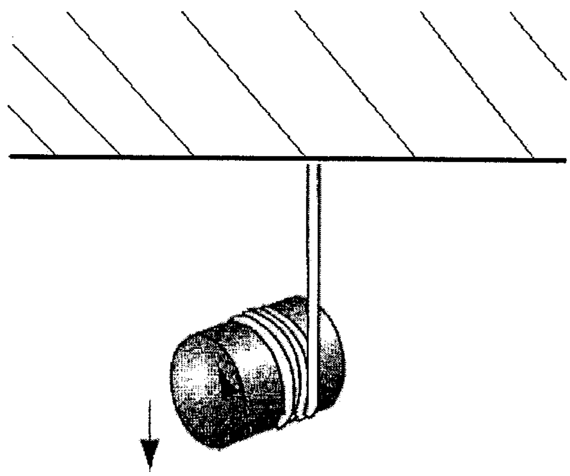


Figure 4: Problem 4

(a) As the cylinder descends, the string will remain vertical. Explain why.

(b) Derive an expression for the angular acceleration of the cylinder in terms of the mass of the cylinder M, R, and the tension T in the cord.

(c) Calculate the downward acceleration of the center of mass of the cylinder.

(d) Calculate the tension T in the cord.

After the cylinder has fallen 10.0 cm

(e) Calculate its rotational kinetic energy.

(f) Calculate its total kinetic energy.

5. **[30 pts]** A cup of mass $m = 0.2$ kg rests on a friction-free horizontal surface and is mounted between two springs of equal spring constant $k_1 = 100$ N/m as shown in the figure below. If the cup of mass m is displaced a distance $A = 0.1$ m and released, it will undergo simple harmonic motion.

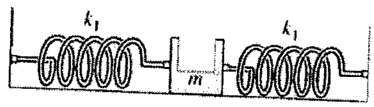


Figure 5: Problem 5

(a) Draw a free-body-diagram for the cup when it is displaced from equilibrium.

(b) What is the period of its oscillation?

Consider now what happens if a piece of putty of mass $m/2$ is dropped into the cup and sticks to it.

If this occurs when the right spring is at its maximum compression

(c) What is now the amplitude of oscillation?

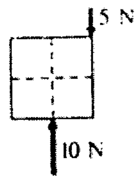
(d) What is now the frequency of oscillation?

If this occurs when the block is moving through its equilibrium position

(e) How much energy was lost in the collision?

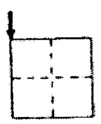
(f) What is now the amplitude of motion?

Multiple-Choice

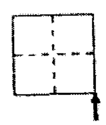


1. A square piece of plywood on a horizontal tabletop is subjected to the two horizontal forces shown above. How should a third force of magnitude 5 N be applied to put the piece of plywood into equilibrium?

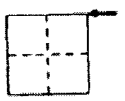
(A)



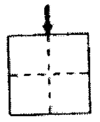
(B)



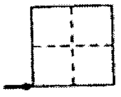
(C)



(D)

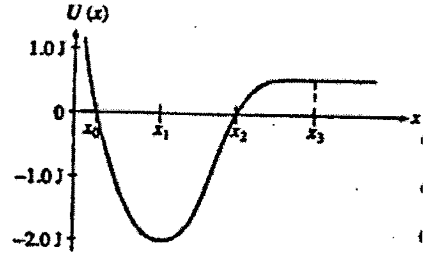


(E)



2. You shoot an arrow with a mass of 0.54 kg from a bow. The bow exerts a force of 125 N for 0.65s. The speed of the arrow as it leaves the bow is

   (a) 0.23 km/s
   (b) 0.10 km/s
   (c) 0.15 km/s
   (d) 0.30 km/s
   (e) 0.27 km/s



3. A conservative force has the potential energy function $U(x)$ as shown by the graph. A particle moving in one dimension under the influence of this force has a kinetic energy of 1.0 J at x_1. Which of the following is a correct statement about the subsequent motion of the particle?

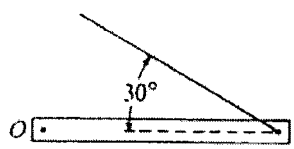
   (a) It oscillates with maximum position x_2 and minimum position x_0.

   (b) It moves to right of x_3 and never returns.

   (c) It moves to the left of x_0 and never returns.

   (d) It just has a zero velocity at either x_0 or x_2 depending on the initial direction of its velocity.

   (e) It cannot reach either x_0 or x_2.



4. A uniform rigid bar of weight W is supported in a horizontal orientation as shown above by a rope that makes a 30° angle with the horizontal and by a pivot at O. The force exerted on the bar by the pivot, is best represented by a vector whose direction is which of the following?

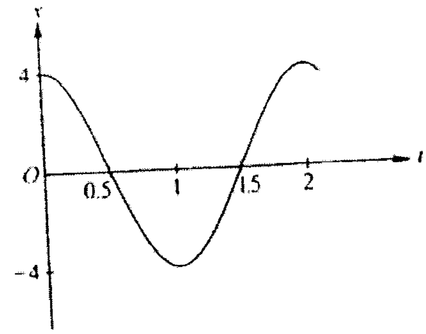
(A)

(B) 

(C)

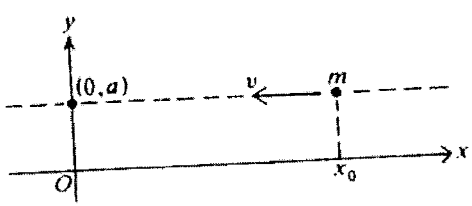
(D) 

(E) 



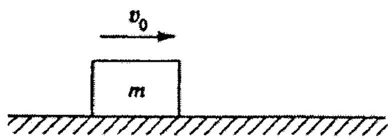
5. A particle moves in simple harmonic motion represented by the graph of displacement versus time above. Which of the following represents the velocity of the particle as a function of time?

   (a) $v(t) = 4 \cos \pi t$
   (b) $v(t) = \pi \cos \pi t$
   (c) $v(t) = -\pi^2 \cos \pi t$
   (d) $v(t) = -4 \sin \pi t$
   (e) $v(t) = -4\pi \sin \pi t$



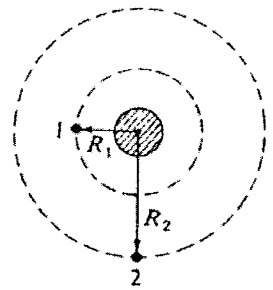
6. A particle of mass m moves in the x-y plane with a constant speed v along the dashed line $y = a$ as shown. When the x-coordinate of the particle is x_0, the magnitude of the angular momentum of the particle with respect to the origin of the system is

   (a) 0
   (b) mva
   (c) mvx_0
   (d) $mv\sqrt{x_0^2 + a^2}$
   (e) $\frac{mva}{\sqrt{x_0^2+a^2}}$



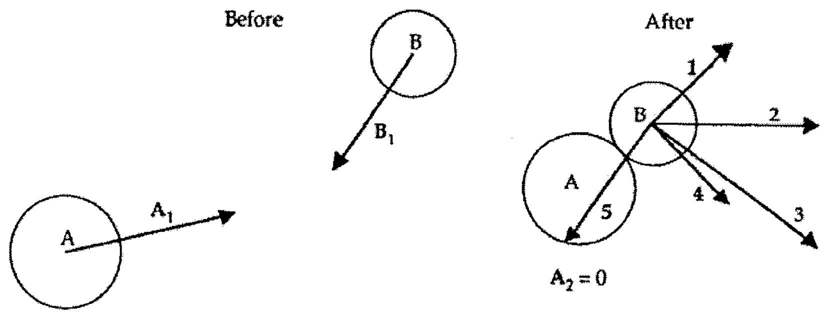
7. An object of mass m is moving with speed v_0 to the right on a horizontal frictionless surface, as shown above, when it explodes into two pieces. Subsequently, one piece of mass $\frac{2}{5}m$ moves with a speed $\frac{v_0}{2}$ to the left. The speed of the other piece of the object is

   (a) $\frac{v_0}{2}$
   (b) $\frac{v_0}{3}$
   (c) $\frac{7v_0}{5}$
   (d) $\frac{3v_0}{2}$
   (e) $2v_0$



8. Two artificial satellites, 1 and 2, orbit the Earth in circular orbits having radii R_1 and R_2, respectively, as shown above. If $R_2 = 2R_1$, the accelerations, a_2 and a_1 of the two satellites are related by which of the following?

   (a) $a_2 = 4a_1$
   (b) $a_2 = 2a_1$
   (c) $a_2 = a_1$
   (d) $a_2 = \frac{a_1}{2}$
   (e) $a_2 = \frac{a_1}{4}$



9. The momentum vectors $\vec{A}_i$ and $\vec{B}_i$ of two bodies are shown before they collide. After they collide, object A is at rest. The vector that most nearly represents the momentum of the body B after the collision is

   (a) $\vec{1}$
   (b) $\vec{2}$
   (c) $\vec{3}$
   (d) $\vec{4}$
   (e) $\vec{5}$

10. If one only knows the constant resultant force acting on an object and the time during which this force acts, one can determine the

    (a) change in momentum of the object.
    (b) change in velocity of the object.
    (c) change in kinetic energy of the object.
    (d) mass of the object.
    (e) acceleration of the object.

(1)

(a) For bullet to hit skeet we require

$V_{by} = V_{s,y}$

$\vec{V}_b \equiv$ velocity of bullet

$\vec{V}_s \equiv$ velocity of skeet

$\Rightarrow V_b \sin\theta = V_s$

$\Rightarrow \sin\theta = V_s / V_b = \frac{30}{300} = \frac{1}{10}$

$\Rightarrow \theta = \sin^{-1}(1/10) = \underline{5.74°}$ #

Proof: $y_b(t_1) = y_s(t_1)$ $\quad y_{0b} = y_{0s} = 0$

$V_b \sin\theta\, t_1 - \frac{1}{2} g t_1^2 = V_s t_1 - \frac{1}{2} g t_1^2$

$\Rightarrow \underline{V_b \sin\theta = V_s}$ #

(b) Require $y_b(t_1) = y_s(t_1)$

$t_1 = ?$

We know $V_b \cos\theta\, t_1 = L$

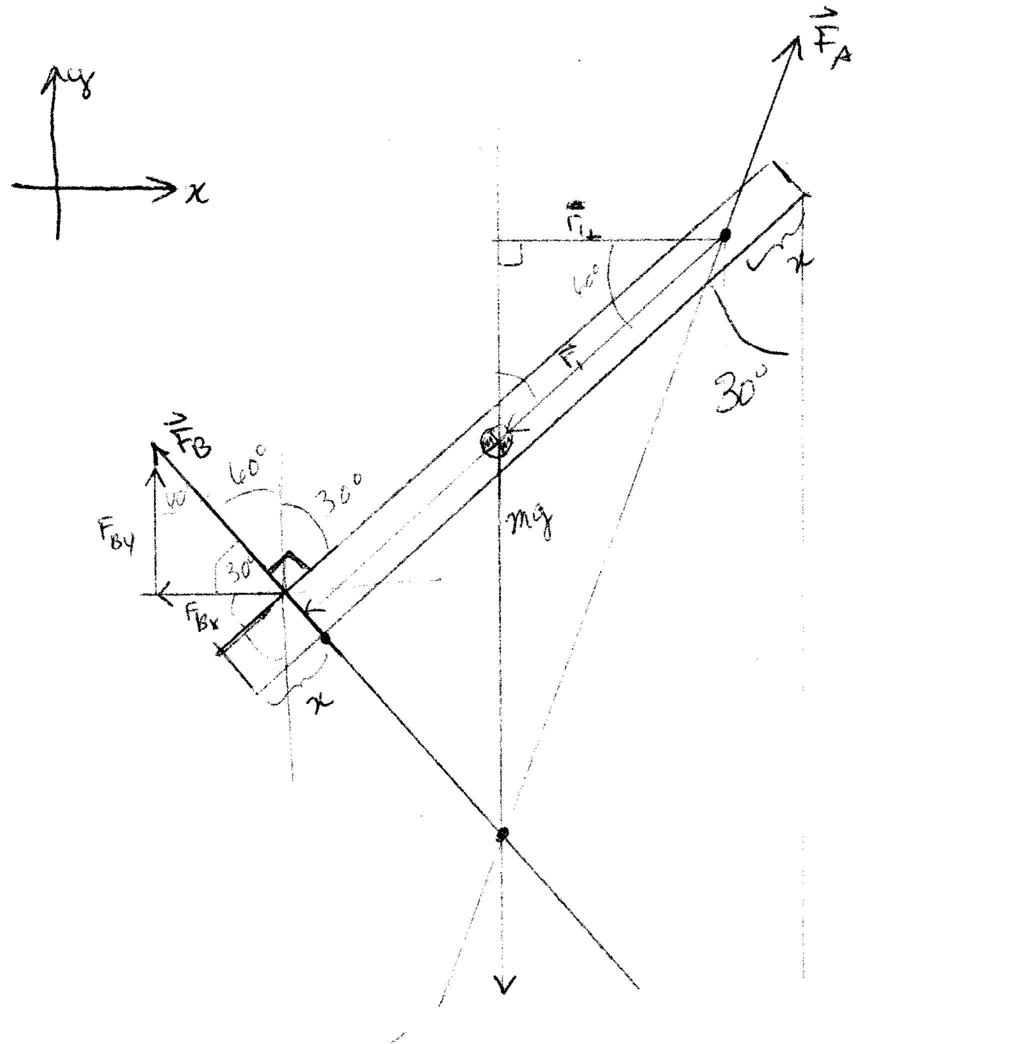
$\Rightarrow t_1 = \frac{L}{V_b \cos\theta} = 0.168\,s$

.138

$\Rightarrow y_s(t_1) = 30\,m/s\,(0.168\,s) - \frac{1}{2} g (0.168\,s)^2 =$

$\underline{y_s(t_1) = 4.902\,m}$ #

(2)



$$\sum F_x: \quad F_{A,x} - F_{B,x} = 0 \quad \Rightarrow \quad F_{A,x} = F_{B,x}$$

$$\sum F_y: \quad F_{Ay} + F_{BY} - mg = 0$$

$$\sum \tau_{A,z}: \quad r_{1\perp} mg - r_2 F_B = 0 \Rightarrow F_B = \frac{r_{1\perp} mg}{r_2}$$

$$r_1 = \tfrac{1}{2}m - x \quad \Rightarrow \quad r_{1\perp} = (\cos 60^\circ)\, r_1$$

$$r_2 = 1m - 2x$$

$$\Rightarrow F_B = \frac{r_1 \cos 60\, mg}{r_2} = \frac{mg\,(\frac{1}{2} - 0.1)\cos 60}{(1 - 0.2)} = 2.5\text{N} \ \#$$

$$\vec{F}_B = 2.5\text{N}\,(-\sin 60\,\hat{i} + \cos 60\,\hat{j})$$

(b) $\vec{F}_B = [-2.17\,\hat{i} + 1.25\,\hat{j}]\,N$

(c) $\vec{F}_A = F_{A,x}\,\hat{i} + F_{A,y}\,\hat{j}$

$F_{Ax} = 2.17\,\hat{i}$

$F_{A,y} = mg - F_{B,y} = 10N - 1.25N = 8.75N$

(c): $\vec{F}_A = (2.17\,\hat{i} + 8.75\,\hat{j})\,N$ #

Alternatives $|\vec{F}_B| =$

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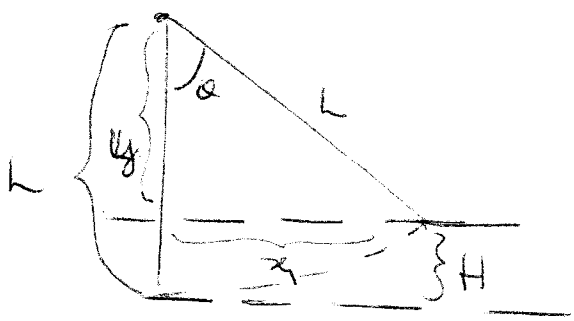
(3)



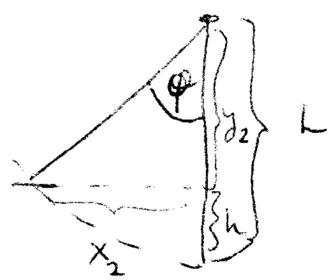
$L = 40\,m$
$D = 50\,m$

$\theta = 50°$

$y + H = L \quad ; \quad \cos\theta = y/L \Rightarrow L\cos\theta = L - H$

(a) so $H = L(1-\cos\theta) = \underline{14.29\ m}$ #

(b) $\varphi = ?$



$\sin\varphi = x_2/L$

But $D = x_1 + x_2$

so $\underline{x_2 = D - x_1}$

$\sin\varphi = \dfrac{D - x_1}{L}$

& $x_1 = L\sin\theta$

$\sin\varphi = \dfrac{D - L\sin\theta}{L} = 0.48396 \Rightarrow \underline{\varphi = 28.94°}$ #

(c) $\cos\varphi = y_2/L \Rightarrow \quad h = L(1-\cos\varphi) = \underline{4.99 \sim 5m}$ #

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③ (d) $W_F = \vec{F}\cdot\vec{D} = -FD = 110N(50m) = -5,500\,J$ #

(e) Work-Energy Thm

$\Delta K_T + \Delta U_T = W_F$

$\frac{1}{2}m_J(v_f^2 - v_i^2) + m_J(y_f - y_i)g = W_F$

$v_f = 0;\quad y_f = h,\ y_i = H$

$m_J = 50\,kg$

$h = 5m$

$H = 14.3m$

$-\frac{1}{2}m_J v_i^2 + m_J(h-H)g = -5,500\,J$

$-\frac{1}{2}m_J v_i^2 = -5500\,J - m_J(h-H)g$

$v_i = \sqrt{\frac{2(5500\,J)}{m_J} + 2g(h-H)}$

$v_i = \sqrt{220 \quad -182.47}\ m/s = \sqrt{37.53}\ m/s = 6.13\ m/s$ #

(f) $W_{F_f} = -W_{F_e} \Rightarrow W_F = +5500\,J$

$\frac{1}{2}M_T(v_f^2 - v_i^2) + M_T g(y_f - y_i) = 5500\,J$

$v_f = 0$

$\Rightarrow \frac{1}{2}M_T v_i^2 + M_T g(h-H) = -5500\,J$

$v_i = \sqrt{\frac{2M_T g(H-h)}{M_T} - \frac{2(5500\,J)}{M_T}}$

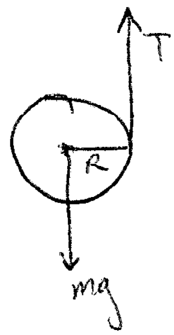
$M_J + M_T = M_T$

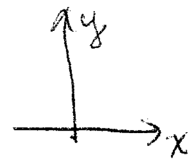
$M_T = (50 + 80)\,kg$

$v_i = \sqrt{2g(14.3m - 5m) - \frac{2(5500)J}{130\,kg}} = \sqrt{97.85}\ m/s = 9.89\ m/s$ #

182.466   84.62

(4)





(a) Only y-forces

(b) $\sum \tau_z = RT = I\alpha \Rightarrow \alpha = \frac{RT}{I}$

$I = \frac{1}{2} m R^2$

$\Rightarrow \alpha = \frac{2RT}{mR^2} = \frac{2T}{mR}$ #

(c) $\sum F_y : T - mg = -ma$ $\qquad$ $a = a_{cm}$

$\Rightarrow a_{cm} = \frac{mg - T}{m}$

But $T = \frac{I\alpha}{R}$; $\alpha = \frac{a_{cm}}{R} \Rightarrow T = \frac{I a_{cm}}{R^2} = \frac{\frac{1}{2} m R^2 a_{cm}}{R^2}$

$\Rightarrow T = \frac{m a_{cm}}{2}$

$\Rightarrow a_{cm} = \frac{mg}{m} - \frac{m a_{cm}}{2m} \Rightarrow a_{cm}(1 + 1/2) = g$

$\Rightarrow a_{cm} = \frac{2}{3} g$ #

(d) $T = \frac{m\, a_{cm}}{2} = \frac{m}{2}\left(\frac{2}{3} g\right) = \frac{1}{3} mg$ #

(4)(e) After 10cm:

We know $v_0 = 0$; $v_f = ?$; $a = \frac{2}{3}g$ "constant"

$$\Rightarrow v_f^2 = v_0^2 + 2a(\Delta y)$$

$$v_f^2 = -2\left(\frac{2}{3}g\right)(y_f - y_i) = \frac{4}{3}g(0.1m)$$

$$v_f = \sqrt{\frac{4}{3}g(0.1m)}$$

$$v_f = 1.14 \text{ m/s} \Rightarrow \omega_f = \frac{v_f}{R} = \frac{1.14 \text{ m/s}}{0.05m}$$

$$\underline{\omega_f = 22.8 \text{ rad/s}}$$

$$\Rightarrow KE_f = \frac{1}{2}I\omega_f^2 \qquad KE_i = 0.$$

$$\Rightarrow KE_f = \frac{1}{2}\left(\frac{1}{2}mr^2\right)\omega_f^2 = \frac{1}{4}mR^2\frac{v_f^2}{R^2} = \frac{1}{4}mv_f^2$$

$$KE_{f(rot)} = \frac{1}{4}(.129)J = \underline{0.033 J} \#$$

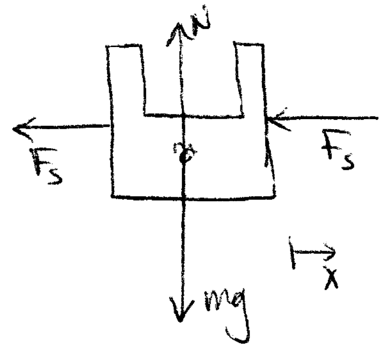
(f)

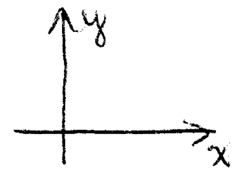
$$KE \text{ Total} = \frac{1}{2}I\omega_f^2 + \frac{1}{2}mv_f^2 \qquad \underline{KE_i = 0}$$

$$= \frac{1}{4}mv_f^2 + \frac{1}{2}mv_f^2 = \frac{3}{4}mv^2 = \frac{3}{4}(0.129)J = \underline{0.0974 J} \#$$

(5)

(a)





(b) $\sum F_x: \; ma_x = -2F_s = -2kx$

$$\Rightarrow a_x = -\frac{2k}{m}x \quad \Rightarrow \quad \omega^2 = \frac{2k}{m}$$

$$\omega = \sqrt{\frac{2k}{m}} \quad \Rightarrow \quad T = 1/f = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{m}{2k}}$$ #

$$T = 2\pi\sqrt{\frac{0.2\,kg}{2(100\,N/m)}} = 0.199\,s$$ #

(c) During max. compression! $\underline{v=0}$

$$\Delta K_T + \Delta U_T = 0 \qquad A_1 = \text{old Amplitude}$$

$$0 + \tfrac{1}{2}k(A_2^2 - A_1^2) = 0 \Rightarrow \underline{A_1 = A_2}$$

c) $\underline{A_2 = 0.1\,m}$

(d) $$\omega = \sqrt{\frac{2k}{(m + \frac{1}{2}m)}} = \sqrt{\frac{2k}{\frac{3}{2}m}} = \sqrt{\frac{4k}{3m}} = \underline{25.82\ rad/s}$$ #

5(e) if moving through equilibrium $v_i \neq 0$; $a = 0$ "No ext force"

$\Rightarrow \Delta p = 0$

$p_i = mv$

$p_f = (m + \frac{1}{2}m)v' = \frac{3}{2}mv'$

$\Rightarrow mv = \frac{3}{2}mv' \Rightarrow v' = \frac{2}{3}v$

$KE_i = \frac{1}{2}mv^2$; $KE_f = \frac{1}{2}(m + \frac{1}{2}m)v'^2$

$\Delta K = \frac{1}{2}(\frac{3}{2}m)(\frac{2}{3}v)^2 - \frac{1}{2}mv^2 = \frac{3}{4}m(\frac{4}{9})v^2 - \frac{1}{2}mv^2$

$\Delta K = \frac{1}{3}mv^2 - \frac{1}{2}mv^2 = \frac{-1}{6}mv^2$

But $v = \omega_0 A = \sqrt{\frac{2k}{m}}(0.1m) = \sqrt{\frac{2(100N/m)}{0.2kg}}(0.1m)$

$v = 3.16$ m/s

$\Rightarrow \Delta K = -\frac{1}{6}(0.2 kg)(3.16 m/s)^2 = -0.33 J$ #

(f) After $\frac{m}{2}$ is stuck to m:

$\Delta KE_T = \frac{1}{2}(\frac{3}{2}m)(v_f^2 - v_i^2) = -\frac{3}{4}mv_i^2 = -\frac{3}{4}m\left(\frac{2}{3}v\right)^2$

$\Delta KE_T = -\frac{3}{4}\left(\frac{4}{9}\right)mv^2 = -\frac{1}{3}mv^2$

$\Delta U_T = \frac{1}{2}k_1(A_2^2 - 0^2) + \frac{1}{2}k_1(A^2 - 0^2)$

$\Rightarrow -\frac{1}{3}mv^2 + \frac{2}{2}kA_2^2 = 0 \Rightarrow \frac{2}{2}kA_2^2 = \frac{1}{3}mv^2 = \frac{1}{3}m\omega_0^2A_0^2$

$\Rightarrow A_2^2 = \frac{1}{3}\frac{m}{k}\left(\frac{2k}{m}\right)A_0^2 = \frac{2}{3}A_0^2 \Rightarrow A_2 = \sqrt{\frac{2}{3}}A_0$

Last Name: Key First Name:

Physics 101 Fall 2003: Final—Multiple-Choice Answers
2-5 PM, 10 December, 2003

For each question, mark a single X in the box corresponding to the correct answer.

	A	B	C	D	E
1	X				
2			X		
3					X
4		X			
5					X
6		X			
7					X
8					X
9				X	
10	X				