Last Name: ______________________ First Name: ______________________

Physics 101 Fall 2002: Test 2—Free Response and Instructions

- Print your LAST and FIRST name on the front of your blue book, on this question sheet, the multiple-choice question sheet and the multiple-choice answer sheet.
- TIME ALLOWED 90 MINUTES
- The test consists of two free-response questions and ten multiple-choice questions.
- The test is graded on a scale of 100 points; the first free-response question accounts for 40 points, the second for 30 points and the multiple-choice questions account for 30 points.
- Answer the two free-response questions in your blue book. Answer the multiple-choice questions by marking a dark X in the appropriate column and row in the table on the multiple-choice answer sheet.
- Consult no books or notes of any kind. You may use a hand-held calculator in non-graphing, non-programmed mode.
- Do NOT take test materials outside of the class at any time. Return this question sheet along with your blue book and multiple-choice question sheet.
- Write and sign the Pledge on the front of your blue book.

Show your work for the free-response problems, including neat and clearly labelled figures, in your blue book. Answers without explanation (even correct answers) will not be given credit.

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1. Consider figure 1 which depicts a block of mass m on a frictionless quarter-circular track of mass M and radius R. The block is released from rest.

   First, assume that the track is fixed to the ground.

   10 (a) Find the speed of the block after it has slid from position A to position B. Express your answer in terms of some or all of m, R, g and θ.

   10 (b) Find the magnitudes of the normal and tangential acceleration of the block at position B. Express your answer in terms of some or all of m, R, g and θ.

   5 (c) Find the speed of the block once it reaches the horizontal surface at position C. Express your answer in terms of some or all of m, R, g and θ.

   Now, consider that the track is sitting on a frictionless surface and is free to move.

   10 (d) Find the velocities of the block and track when the block reaches position C. Express your answer in terms of some or all of m, M, R, g and θ.

   5 (e) Find the work done on the block by the track. Express your answer in terms of some or all of m, M, R, g and θ.

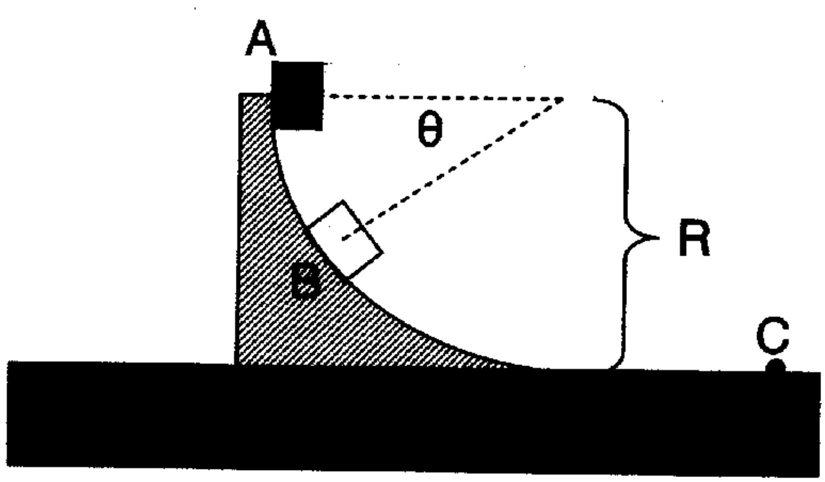

Figure 1

2. A tank full of candy is bringing joy to children but an angry dentist with a machine gun is trying to stop the tank. See figure 2. The machine gun fires bullets of mass $m_b = 55$ g horizontally at a speed of $v_b = 725$ m/s and a rate of $R = 250$ bullets per second. The tank has a mass of $m_t = 2450$ kg and is moving towards the dentist at a speed of $v_t = 15$ m/s. The bullets bounce back from the tank at an angle of $\theta = 60°$ above the horizontal. The collision is inelastic; when the bullets bounce, they lose 36% of their kinetic energy. The tank's velocity remains unchanged.

6 (a) What is the change in speed of the bullets when they bounce off the tank?

9 (b) What is the change in momentum of the bullets when they bounce off the tank? (Remember: Momentum is a vector.)

8 (c) What is the the average force delivered to the tank by the collision with the stream of bullets?

7 (d) What additional power must be supplied by the tank's engine when the tank is being struck by the bullets to maintain a constant speed v_t?

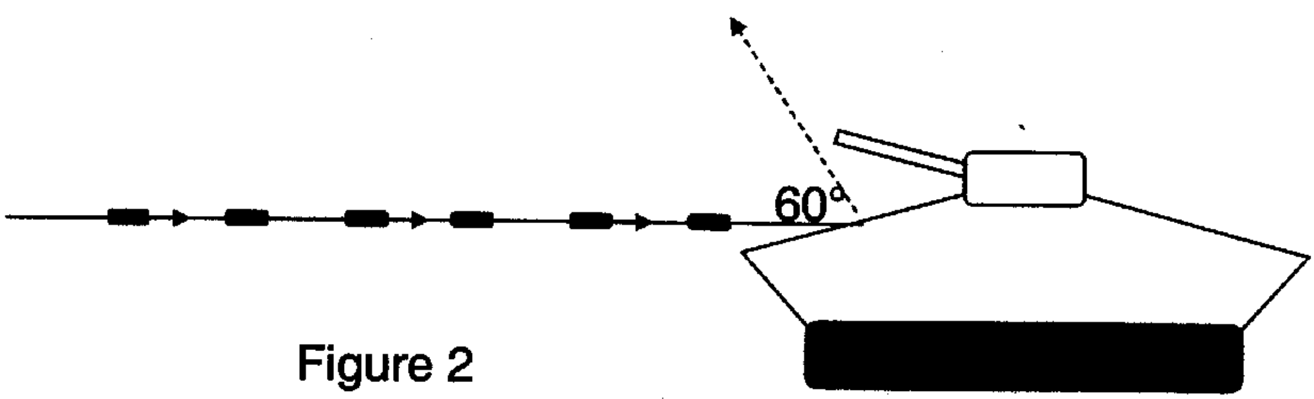

Figure 2

Multi Choice

1. D
2. D
3. D
4. E
5. E
6. B
7. C
8. B
9. D
10. —

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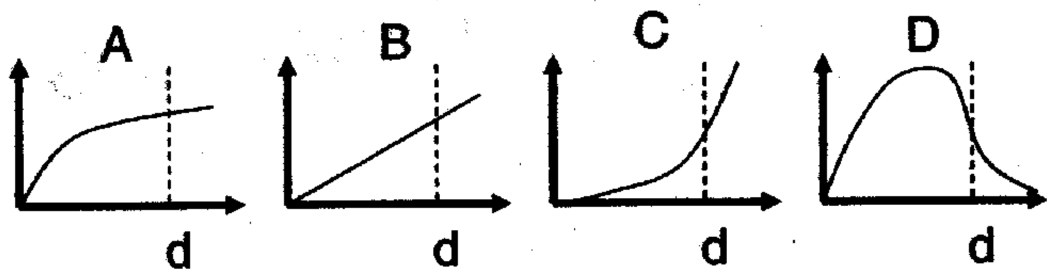
Physics 101 Fall 2002: Test 2—Multiple-Choice Questions

1. The graphs below depict the force required to pull back the string on an archery bow as a function of the distance the string is pulled back for four different bows. All graphs are to the same scale. Assume all the bows have the same efficiency and all potential energy stored in the bow is transferred into the kinetic energy of the arrow. If all four bows are pulled back the same distance d (indicated on the graphs) and arrows are shot at the same initial angle and elevation, the arrow shot from which bow has the greatest range? (Neglect friction and resistance.)



   (a) A
   (b) B
   (c) C
   (d) D
   (e) The arrows from all four bows will have the same range.

2. Mass m_a is traveling on a level, frictionless surface with a speed v_A. In which of the following collisions would the impulse delivered to mass m_a be the greatest?

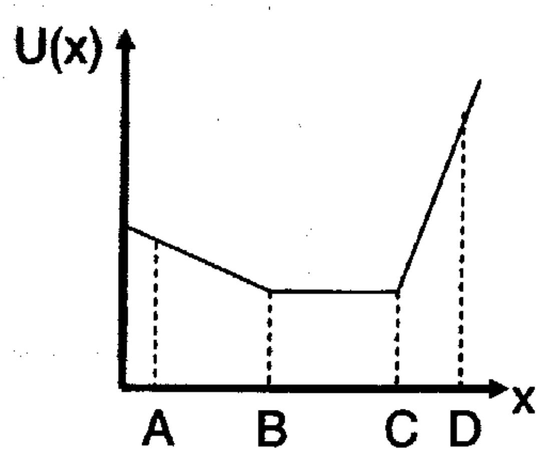
   I. Mass m_a elastically collides with a stationary mass m_b where $m_b < m_a$.

   II. Mass m_a elastically collides with a stationary mass m_c where $m_c = m_a$

   III. Mass m_a elastically collides with a stationary mass m_d where $m_d > m_a$.

   (a) Need more information.
   (b) case I
   (c) case II
   (d) case III
   (e) In cases I, II and III, the same impulse is delivered to m_a.

3. A particle moving along the x-axis is subject to a conservative force acting along the x-axis. The potential energy of this particle is shown as a function of its x-coordinate. Rank the labeled regions according to the magnitude of the force experienced by the particle in that region, from least to greatest.

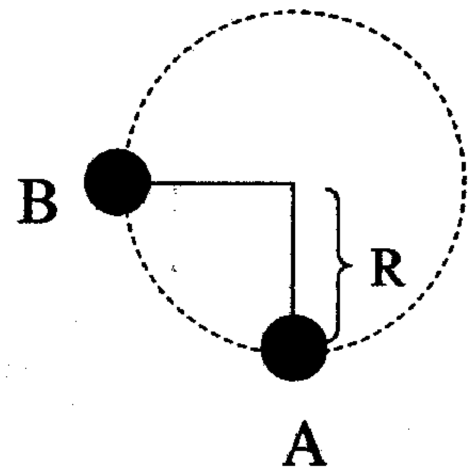


   (a) AB, BC, CD
   (b) AB, CD, BC
   (c) BC, CD, AB
   (d) BC, AB, CD
   (e) CD, BC, AB

4. A spaceship is in circular orbit around the Earth at a distance of R from the center of the Earth. It fires its rocket engines for a period of time and in a particular manner such that it ends up in a new circular orbit at a distance of $2R$ from the center of the Earth. Which of the following statements about the changes in the kinetic and gravitational potential energy of the spaceship are true?

(a) Kinetic energy increases, potential energy decreases.
(b) Kinetic energy increases, potential energy increases.
(c) Kinetic energy remains the same, potential energy increases.
(d) Kinetic energy decreases, potential energy decreases.
(e) Kinetic energy decreases, potential energy increases.

5. A ball of mass m is attached to a cord and spun fast enough so that its path is a vertical circle of radius R. Call v_A the ball's speed at point A and v_B the balls speed at point B. How much work was done by the force of gravity as the ball moves from point A to point B?

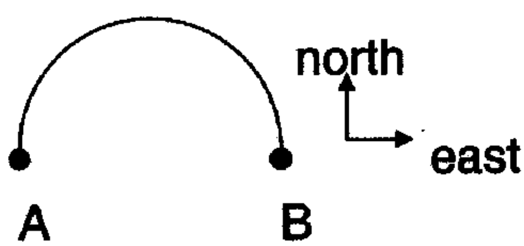


(a) no work
(b) $\frac{1}{2}mv_B^2 - mgR$
(c) $\frac{1}{2}mv_B^2 - \frac{1}{2}mv_A^2 - mgR$
(d) mgR
(e) $-mgR$

6. A car of mass $m = 750$ kg moving at $v_1 = 35$ mph collides with a stationary truck of mass $m = 1500$ kg. What is the ratio of the magnitude of the change of momentum of the car to the magnitude of the change of momentum of the truck?

(a) 2/3
(b) 1
(c) 3/2
(d) 2
(e) Need more information.

7. An object moves from point A to point B in a semicircular path.



Consider the following forces:

- $\vec{\mathbf{F}}_I$ is a constant force pointing north.
- $\vec{\mathbf{F}}_{II}$ is a constant force pointing east.
- $\vec{\mathbf{F}}_{III}$ has a constant magnitude and points along the direction of motion at every point on the path.

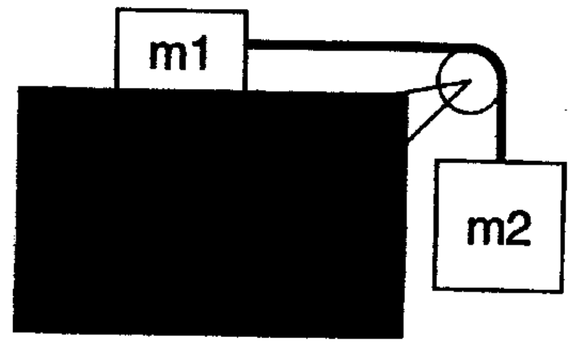
If the magnitudes of the three forces are equal, which force does the most work on the object as it moves from point A to point B?

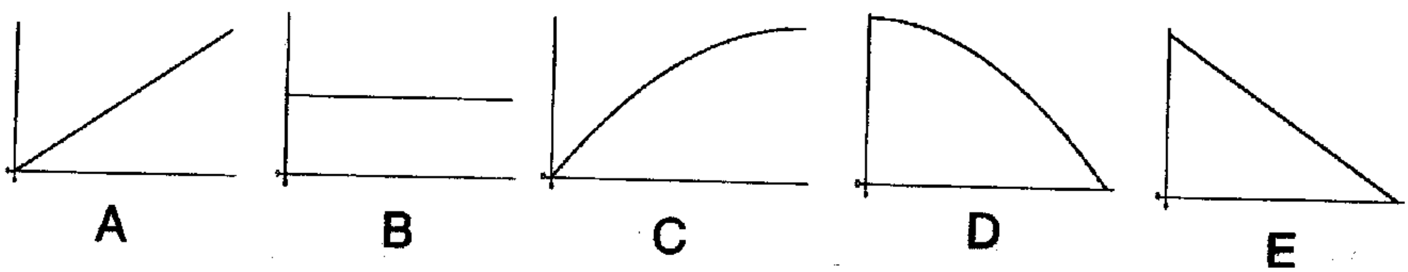
(a) $\vec{\mathbf{F}}_I$
(b) $\vec{\mathbf{F}}_{II}$
(c) $\vec{\mathbf{F}}_{III}$
(d) $\vec{\mathbf{F}}_{II}$ and $\vec{\mathbf{F}}_{III}$ do the same amount of work.
(e) All three do the same amount of work.

8. Compared to the amount of work done to compress an ideal spring from its equilibrium length by an amount x, the work required to compress it from x to $2x$ is

   (a) 4 times as much.
   (b) 3 times as much.
   (c) 2 times as much.
   (d) the same.
   (e) 1/2 times as much.

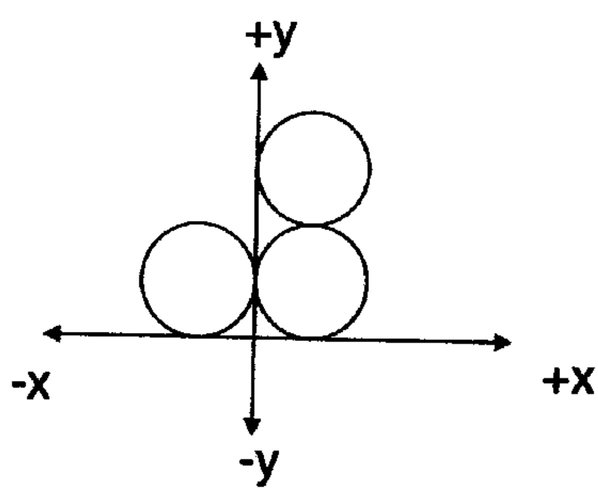
9. The arrangment of two masses m_1 and m_2 attached by a massless, inextensible cord over a massless, frictionless pulley is depicted to the right. When the masses are released, they are observed to accelerate. There is friction between the horizontal surface and m_1. Which of the following graphs could depict the total mechanical energy as a function of time?



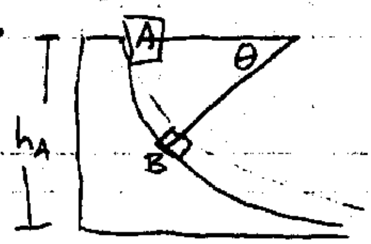


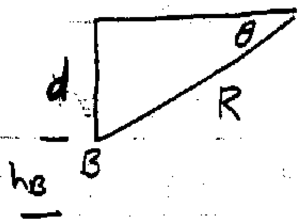
10. Where is the center of mass of the arrangement of disks depicted to the right with respect to the origin shown on the figure? All disks have mass M and radius R.

   (a) $x_{CM} = 0$, $y_{CM} = 2R$
   (b) $x_{CM} = R$, $y_{CM} = 2R$
   (c) $x_{CM} = R/2$, $y_{CM} = R$
   (d) $x_{CM} = \frac{2}{3}R$, $y_{CM} = \frac{5}{3}R$
   (e) $x_{CM} = \frac{2}{3}R$, $y_{CM} = 2R$



1. a.





geometry

$h_A = R \qquad h_B = R - d = R - R\sin\theta = R(1-\sin\theta)$

Use conservation of energy

$U_A = mgh_A \qquad KE_A = 0$

$U_B = mgh_B \qquad KE_B = \frac{1}{2}mv_B^2$

$\Rightarrow \quad \frac{1}{2}mv_B^2 + mgh_B = mgh_A$

$\Rightarrow \quad v_B^2 = 2g(h_A - h_B)$

$\Rightarrow \quad \boxed{v_B = \sqrt{2gR\sin\theta}}$

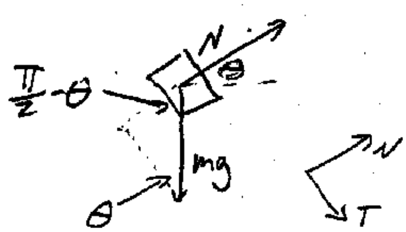
or use work

$W_{net} = \Delta KE$

$\Rightarrow W_N^{\nearrow 0} + W_g = \frac{1}{2}mv_B^2$

$\Rightarrow -\Delta U_g = \frac{1}{2}mv_B^2$

b. Draw F.B.D at point B



use normal, tangential coordinates

Normal:

$\sum F_N = ma_N$

oh, but $\boxed{a_N = \frac{V_B^2}{R} = 2g\sin\theta}$

Tangential

$\sum F_T = mg\cos\theta = ma_T$

$\Rightarrow \boxed{a_T = g\cos\theta}$

c. Use either cons. of energy or work

$U_A = mgh_A \quad KE_A = 0$

$U_C = 0 \quad KE_C = \frac{1}{2}mv_c^2$

$\Rightarrow \quad \frac{1}{2}mv_c^2 = mgh_A$

$$\boxed{v_c = \sqrt{2gR}}$$

d. Must use conse Energy <u>and</u> cons. Moment.

$\underline{V} \rightarrow$ velocity in x-direction of ramp

$v \rightarrow$ velocity in x-direct of blade

$U_A = mgR \quad KE_A = 0$

$U_f = 0 \quad KE_f = \frac{1}{2}mv^2 + \frac{1}{2}M\underline{V}^2$

so $\quad \frac{1}{2}mv^2 + \frac{1}{2}M\underline{V}^2 = mgR$

also $\quad P_i = 0 \quad P_f = mv + M\underline{V}$

so $\quad mv + M\underline{V} = 0$

$\Rightarrow \quad \underline{V} = -\frac{m}{M}v$

$\Rightarrow \quad \frac{1}{2}mv^2 + \frac{1}{2}M\left(\frac{m}{M}v\right)^2 = mgR$

$\Rightarrow \quad mv^2 + \frac{m^2}{M}v^2 = 2mgR$

$\Rightarrow \quad v^2\left(1 + \frac{m}{M}\right) = 2gR$

cont...

1.d. cont.

$$V = \sqrt{\frac{2gR}{1+\frac{m}{M}}} = \sqrt{\frac{2MgR}{M+m}}$$

↑ to right

and then $\quad V = -\frac{m}{M} v$

$$\Rightarrow \quad V = -\frac{m}{M}\sqrt{\frac{2MgR}{M+m}} = -\frac{m}{M}\sqrt{\frac{2gR}{(1+\frac{m}{M})}}$$

↑ i.e. to left

1.e.

$$W_{net} = \Delta KE$$

so

$$W_g + W_t = \frac{1}{2}mv^2$$

W_g: work done by gravity on block

W_t: work done by track on block

$$W_g = -\Delta V = mgR \qquad \frac{1}{2}mv^2 = \frac{1}{2}m\frac{2gR}{1+\frac{m}{M}} = \frac{mMgR}{M+m}$$

$$W_t = \frac{mMgR}{M+m} - mgR = \frac{mgR}{1+\frac{m}{M}} - mgR$$

$$= -\frac{m^2gR}{M+m}$$

or you know $\quad W_t = -W_b$

← W_b: work done by block on track

and $\quad W_b = \Delta KE_t = \frac{1}{2}MV^2$

$$= -\frac{m^2gR}{M+m}$$

No. 937 811E Engineer's Computation Pad STAEDTLER

1. Grading Criteria

10 a. correct answer $V_B = \sqrt{2gR\sin\theta}$ +10

picture +1
$h_A = R$ +1 or $\Delta h = R\sin\theta$ +3 } +3
$h_B = R(1-\sin\theta)$ +2

setting up cons. energy or Work
KE_A, KE_B +1 | $W_{net} = \Delta KE$ +1
V_A +1 | $W_N = 0$ +1
V_B +1 | $W_g = mg\Delta h$ +1
$KE_A + V_A = KE_B + V_B$ +1 | $mg\Delta h = \frac{1}{2}mv_B^2$ +1 } +4

solving for V_B +1
correctly +1

10 b. correct answers

$a_N = 2g\sin\theta$ +4

$a_T = g\cos\theta$ +6

F.B.D +1
Indicating N, T directions +1

$a_N = \frac{v_B^2}{R}$ +2

plugging in for a_N +1

Using N's 2nd for T direct +2
Taking comp of mg +1
correctly +1
solving for a_T +1

1 cont

[5] c. Correct answer $V_c = \sqrt{2gR}$ +5

Using cons of energy or Work +2

setting up $mgR = \frac{1}{2}mv_c^2$ +2

solving for v_c +1

[10] d. correct answers $v = \sqrt{\frac{2MgR}{M+m}}$ +5 (to right)

$V = -\frac{m}{M}\sqrt{\frac{2MgR}{M+m}}$ +5 (to left)

using cons. energy +1
setting up $\frac{1}{2}mv^2 + \frac{1}{2}MV^2 = mgR$ +2

using cons. mom +1
setting up $mv + MV = 0$ +1

expressing $V = -\frac{m}{M}v$ +1

solving for v +1
for V +1

directions right +1

[5] 1.e. correct ans. $W_t = -\frac{m^2gR}{M+m}$ +5

using $W_{net} = \Delta KE_b$ +1

or

using $W_{net} = W_g + W_t$ +1

$W_g = mgR$ +1

$\Delta KE = \frac{1}{2}mv^2$ +1

solving +1

$W_b = \Delta KE_t$ +2

$W_t = -W_b$ +2

2. a.

given $\dfrac{KE_i - KE_f}{KE_i} = .36 \Rightarrow KE_f = (1-.36)KE_i$

$= .64\, KE_i$

therefore

$\frac{1}{2} m v_b'^2 = .64 \; \frac{1}{2} m_b v_b^2$

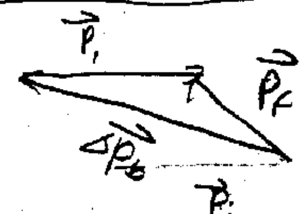
↑ final velocity of bullet

$\Rightarrow v_b'^2 = .64\, v_b^2$

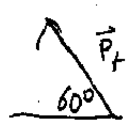
$\Rightarrow v_b' = .8\, v_b$

$= v_b' = 580 \text{ m/s}$

b. using coordinate $Y \uparrow, \rightarrow X$



$\vec{P}_i = m_b v_b \hat{i}$



$|\vec{p}_f| = m_b v_b' = .8\, m_b v_b$

$\Rightarrow \vec{P}_f = -\frac{4}{5} m_b v_b \cos 60\, \hat{i} + \frac{4}{5} m_b v_b \sin 60\, \hat{j}$

$= -\frac{2}{5} m_b v_b \hat{i} + \frac{2\sqrt{3}}{5} m_b v_b \hat{j}$

$\Delta\vec{P}_b = \vec{P}_f - \vec{P}_i = -\frac{7}{5} m_b v_b \hat{i} + \frac{2\sqrt{3}}{5} m_b v_b \hat{j}$

$= (-55.8 \text{ kg m/s})\hat{i} + (27.6 \text{ kg m/s})\hat{j}$

or... maybe expressed as mag, direction

$|\Delta\vec{P}_b| = 62.3 \text{ kg m/s}$

direction: $\theta = \tan^{-1} \dfrac{27.6}{55.8}$

$\theta = 26°$ above horiz., to left

2 c. $\Delta\vec{P}_b = \vec{I}_{t\,on\,b}$ ← Impulse of tank on bullet

$\vec{I}_{t\,on\,b} = -\vec{I}_{b\,on\,t}$

$\vec{F}_{b\,on\,t} = \frac{\vec{I}_{b\,on\,t}}{\Delta t} = \frac{-\Delta\vec{P}_b}{\Delta t} = -R\,\Delta\vec{P}_b$

$= 14.0\,kN\,\hat{i} - 6.9\,kN\,\hat{j}$

or

mag $|\vec{F}_{b\,on\,t}| = 15.6\ kN$

dir $\theta = -26^\circ$ below horiz, to right

d. Easiest to use

$P_t + P_b = 0$ → so no change in KE

$P_t = -P_b = -\vec{F}_{b\,on\,t} \cdot \vec{V}_t$ (since constant force)

$= -(14.0\,kN\,\hat{i} - 6.9\,kN\,\hat{j}) \cdot (-15\,m/s\,\hat{i})$

$= 210\ kW$

2. grading criteria

6 a. correct answer $\boxed{v_b' = 580 m/s}$ +6

expressing $\frac{KE_i - KE_f}{KE_i} = .36$ or similar +1

express $KE_f = .64\, KE_i$ +1

setting $KE_i = \frac{1}{2} m v_b^2$ +1

$KE_f = \frac{1}{2} m v_b'^2$ +1

solving for v_b' +1

units +1

9 b. correct answer $\Delta\vec{p}_b = (-55.8\, kg\, m/s)\hat{i} + (27.6\, kg\, m/s)\hat{j}$ +9

or

$|\Delta\vec{p}_b| = 62.3\, kg\, m/s$ +6

$\theta = 26°$ above horiz, to left +3

$\Delta\vec{p}_b = \vec{p}_f - \vec{p}_i$ +1

$\vec{p}_i = m_b v_b \hat{i}$ +1

$|\vec{p}_f| = .8\, m_b v_b = m v_b'$ +1

drawing picture +1

breaking $\vec{p}_f$ into comp +2

subtracting correctly +1

correctly +1

units +1

or

geometrically solving for $|\Delta p|$ +2

geometrically solving for θ +2

units +1

8 2.c. $\vec{F}_{boat} = 14.0\,kN\hat{i} - 6.9\,kN\hat{j}$ +8

or

$|\vec{F}_{boat}| = 15.6\,kN$ +5

$\theta = -26^\circ$, below horiz, to right +3

$\Delta\vec{p}_b = \vec{I}_{t\ on\ b}$ +1

$\vec{I}_{boat} = -\vec{I}_{t\ on\ b}$ +1

$\vec{F}_{boat} = \frac{\vec{I}_{boat}}{\Delta t}$ +1

$\vec{F}_{boat} = \frac{-\Delta\vec{p}_b}{\Delta t}$ +1

$R = \frac{1}{\Delta t}$ +1

plugging all together +2

units +1

7 2.d. $P_t = 210\ kW$ +7

$\Delta KE_t = 0$ +1

$P_t = -P_b$ +1

$P_b = \vec{F}_{boat} \cdot \vec{V}_t$ +1

using only horiz comp of $\vec{F}$ +1

plugging in +2

units +1